

Άσκηση 15

$$\alpha) \frac{\sqrt{50}}{\sqrt{2}} + \sqrt{\frac{49}{81}} = \frac{\sqrt{25}}{\sqrt{1}} + \frac{7}{9} = \frac{5}{1} + \frac{7}{9} = \frac{45}{9} + \frac{7}{9} = \frac{52}{9}$$

$$\beta) \frac{\sqrt{7,5}}{\sqrt{0,3}} - 11\sqrt{\frac{900}{121}} = \sqrt{25} - 10 = 5 - 10 = -5$$

$$\gamma) \sqrt{\frac{5}{3}} \cdot \sqrt{\frac{5}{12}} - \sqrt{\frac{3}{8}} \cdot \sqrt{\frac{2}{3}} = \sqrt{\frac{25}{36}} - \sqrt{\frac{6}{24}} = \frac{5}{6} - \sqrt{\frac{1}{4}} = \frac{5}{6} - \frac{1}{2} = \frac{5}{6} - \frac{3}{6} = \frac{2}{6} = \frac{1}{3}$$

Άσκηση 19

$$\alpha) \sqrt{8} - 3\sqrt{200} - \sqrt{50} = \sqrt{4 \cdot 2} - 3\sqrt{100 \cdot 2} - \sqrt{25 \cdot 2} = 2\sqrt{2} - 30\sqrt{2} - 5\sqrt{2} = (2 - 30 - 5)\sqrt{2} = -33\sqrt{2}$$

$$\beta) \sqrt{12} - \sqrt{75} - 2\sqrt{300} = \sqrt{4 \cdot 3} - \sqrt{25 \cdot 3} - 2\sqrt{100 \cdot 3} = 2\sqrt{3} - 5\sqrt{3} - 20\sqrt{3} = (2 - 5 - 20)\sqrt{3} = -23\sqrt{3}$$

$$\gamma) \sqrt{8} - \sqrt{27} + \sqrt{12} - \sqrt{200} = \sqrt{4 \cdot 2} - \sqrt{9 \cdot 3} + \sqrt{4 \cdot 3} - \sqrt{100 \cdot 2} = 2\sqrt{2} - 3\sqrt{3} + 2\sqrt{3} - 10\sqrt{2} = (-8\sqrt{2}) + (-\sqrt{3}) = -8\sqrt{2} - \sqrt{3}$$

$$\delta) \sqrt{20} - \sqrt{32} + 2\sqrt{45} - 3\sqrt{50} = \sqrt{4 \cdot 5} - \sqrt{16 \cdot 2} + 2\sqrt{9 \cdot 5} - 3\sqrt{25 \cdot 2} = 2\sqrt{5} - 4\sqrt{2} + 6\sqrt{5} - 15\sqrt{2} = (2\sqrt{5} + 6\sqrt{5}) + (-4\sqrt{2} - 15\sqrt{2}) = 8\sqrt{5} - 19\sqrt{2}$$

$$\gamma) \sqrt{8} - \sqrt{27} + \sqrt{12} - \sqrt{200} = \sqrt{4 \cdot 2} - \sqrt{9 \cdot 3} + \sqrt{3 \cdot 4} - \sqrt{2 \cdot 100} = 2\sqrt{2} - 3\sqrt{3} + 2\sqrt{3} - 10\sqrt{2} = \sqrt{2}(2-10) + \sqrt{3}(-3+2) = \sqrt{2}(-8) + \sqrt{3}(-1) = -8\sqrt{2} - \sqrt{3}$$

$$\delta) \sqrt{20} - \sqrt{32} + 2\sqrt{45} - 3\sqrt{50} = \sqrt{4 \cdot 5} - \sqrt{16 \cdot 2} + 2\sqrt{9 \cdot 5} - 3\sqrt{25 \cdot 2} = 2\sqrt{5} - 4\sqrt{2} + 6\sqrt{5} - 15\sqrt{2} = \sqrt{5}(2+6) + \sqrt{2}(-4-15) = 8\sqrt{5} - 19\sqrt{2}$$

21.

$$a) 3^{-5} \cdot 3^8 - \frac{(-5)^7}{3^0} - (2^{-2})^3 \cdot 3^3 - (-5)^2 \cdot 2^2 - 6 \cdot 7 \cdot 2 \cdot 5 - \frac{1}{64} - \frac{2}{1} - \frac{1}{64} = \frac{128}{64} - \frac{1}{64} = \frac{127}{64}$$

$$b) 3^0 \cdot 2^0 - \frac{(-5)^5}{3^0} = 6^0 - 6^0 = 0$$

$$\gamma) 3^{-3} - \left(-\frac{3}{2}\right)^{-3} - \left(\frac{1}{5}\right)^{-1} \cdot \frac{1}{27} + \frac{8}{27} - \frac{3}{27} = \frac{1}{27} - 3 = \frac{1}{3} - 3 = -\frac{9}{3} = -3$$

$$\delta) 100000 \cdot 0,0001 = 10$$

14.

$$a) (x^2-3)^2 = (x^2)^2 - 2x^2 \cdot 3 + 3^2 = x^4 - 6x^2 + 9$$

$$b) (x^5+1)^2 = (x^5)^2 + 2x^5 \cdot 1 + 1^2 = x^{10} + 2x^5 + 1$$

$$\gamma) (3x^2-2)^2 = (3x^2)^2 - 2 \cdot 3x^2 \cdot 2 + 2^2 = 9x^4 - 12x^2 + 4$$

$$\delta) (xy^2+5x)^2 = (xy^2)^2 + 2xy^2 \cdot 5x + (5x)^2 = x^2y^4 + 10x^2y^2 + 25x^2$$

$$\epsilon) (3a^2b-2b)^2 = (3a^2b)^2 - 2 \cdot 3a^2b \cdot 2b + (2b)^2 = 9a^4b^2 - 12a^2b^2 + 4b^2$$

$$\sigma) (5x^3y^2-3x^2y)^2 = (5x^3y^2)^2 - 2 \cdot 5x^3y^2 \cdot 3x^2y + (3x^2y)^2 = 25x^6y^4 - 30x^5y^3 + 9x^4y^2$$

15.

$$a) \left[x - \frac{1}{2}\right]^2 = x^2 - 2x \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 = x^2 - x + \frac{1}{4}$$

$$b) \left(\frac{w}{2} - 1\right)^2 = \left(\frac{w}{2}\right)^2 - 2 \cdot \frac{w}{2} \cdot 1 + 1^2 = \frac{w^2}{4} - w + 1$$

$$\gamma) \left(2x - \frac{1}{3}\right)^2 = (2x)^2 - 2 \cdot 2x \cdot \frac{1}{3} + \left(-\frac{1}{3}\right)^2 = 4x^2 - \frac{4}{3}x + \frac{1}{9}$$

$$25. \alpha. x^3 - (x-1)^3 - 3x(x-2) = x^3 - (x^3 - 3x^2 + 3x - 1) - 3x^2 + 6x =$$

$$\cancel{x^3} - \cancel{x^3} + \cancel{3x^2} - 3x + 1 - \cancel{3x^2} + 6x = +3x + 1$$

$$\beta. 1 - (3x-1)^3 - (x-1) \cdot (3x-2) = 1 - (27x^3 - 3 \cdot (3x)^2 \cdot 1 + 3 \cdot 3x \cdot 1^2 - 1) - (3x^2 - 2x - 3x + 2) =$$

$$1 - (27x^3 - 27x^2 + 9x - 1) - 3x^2 + 2x + 3x - 2 =$$

$$\cancel{1} - 27x^3 + \cancel{27x^2} - 9x + \cancel{1} - 3x^2 + 2x + 3x - \cancel{2} =$$

$$-27x^3 + 24x^2 - 4x$$

$$1. \alpha. 6x^2y - 4xy^2 = 2xy(3x - 2y)$$

$$\beta. 3a^2 - 6a\beta + 12a = 3a(a - 2\beta + 4)$$

$$\gamma. 14x^2y - 7xy^2 - 21xy = 7xy(2x - y - 3)$$

$$\delta. 12a^2\beta^3\gamma - 15a^3\beta = 3a^2\beta(4\beta^2\gamma - 5a)$$

$$5. \alpha. x^3 + 3x^2 + 2x + 6 = x^2(x+3) + 2(x+3) = (x^2+2)(x+3)$$

$$\beta. x^3 - 5x^2 + 3x - 15 = x^2(x-5) + 3(x-5) = (x^2+3)(x-5)$$

$$1. \alpha. 25x^2 - 9y^2 = (5x)^2 - (3y)^2 = (5x-3y)(5x+3y)$$

$$\beta. 4x^2 - 49y^2 = (2x)^2 - (7y)^2 = (2x-7y)(2x+7y)$$

$$\gamma. 36x^2 - 49y^2 = (6x)^2 - (7y)^2 = (6x-7y)(6x+7y)$$

$$30. a. x^2 - \frac{25}{16} = x^2 - \left(\frac{5}{4}\right)^2 = \left(x - \frac{5}{4}\right) \left(x + \frac{5}{4}\right)$$

$$b. y^2 - \frac{1}{4} = y^2 - \left(\frac{1}{2}\right)^2 = \left(y - \frac{1}{2}\right) \left(y + \frac{1}{2}\right)$$

$$g. 9y^2 - \frac{49}{81} = (3y)^2 - \left(\frac{7}{9}\right)^2 = \left(3y - \frac{7}{9}\right) \left(3y + \frac{7}{9}\right)$$

$$36. a. 5x^2 - 20 = 5x^2 - 5 \cdot 4 = 5(x^2 - 4) = 5(x^2 - 2^2) = 5(x-2)(x+2)$$

$$b. 3y^2 - 12 = 3(y^2 - 4) = 3(y^2 - 2^2) = 3(y-2)(y+2)$$

$$g. 8\lambda^2 - 18 = 2(4\lambda^2 - 9) = 2[(2\lambda)^2 - 3^2] = 2(2\lambda-3)(2\lambda+3)$$

$$d. 12x^2 - 75y^2 = 3(4x^2 - 25y^2) = 3[(2x)^2 - (5y)^2] = 3(2x-5y)(2x+5y)$$

$$e. x^3 - x = x(x^2 - 1) = x(x^2 - 1^2) = x(x-1)(x+1)$$

$$67. 4x^3 - x = x(4x^2 - 1) = x[(2x)^2 - 1^2] = x(2x-1)(2x+1)$$

$$45. a. x^2 + 6x + 9 = x^2 + 2 \cdot 3 \cdot x + 3^2 = (x+3)^2$$

$$b. x^2 - 10x + 25 = x^2 - 2 \cdot 5 \cdot x + 5^2 = (x-5)^2$$

$$g. x^2 + 2x + 1 = x^2 + 2 \cdot 1 \cdot x + 1^2 = (x+1)^2$$

$$d. x^2 - 16x + 64 = x^2 - 2 \cdot 8 \cdot x + 8^2 = (x-8)^2$$

$$e. x^2 + 8x + 16 = x^2 + 2 \cdot 4 \cdot x + 4^2 = (x+4)^2$$

$$a. x^2 - 12x + 36 = x^2 - 2 \cdot 6 \cdot x + 6^2 = (x-6)^2$$

10. α. $\frac{x^2 - 2x + 1}{x^2 - x} = \frac{(x-1)^2}{x(x-1)} = \frac{(x-1)}{x} = \frac{x-1}{x}$, ορίζεται για $x \neq 0, x \neq 1$
 $\hookrightarrow x \neq 0$
 $x \neq 1$

β. $\frac{x^3 - 9x}{x^2 - 6x + 9} = \frac{x(x^2 - 9)}{(x-3)^2} = \frac{x(x-3)(x+3)}{(x-3)^2} = \frac{x(x+3)}{x-3}$, ορίζεται για $x \neq 3$
 $\hookrightarrow x \neq 3$

γ. $\frac{3x^4 + 15x^3}{x^3 - 25x} = \frac{3x^3(x+5)}{x(x^2 - 25)} = \frac{3x^3(x+5)}{x(x-5)(x+5)} = \frac{3x^2}{x-5}$, ορίζεται για $x \neq 0, x \neq 5, x \neq -5$
 $\hookrightarrow x \neq 0$
 $x \neq 5$
 $x \neq -5$

2. α. $\frac{5x-10}{3x-3} \cdot \frac{x^2-x}{x^2-4} = \frac{5(x-2)}{3(x-1)} \cdot \frac{x(x-1)}{(x-2)(x+2)} = \frac{5x}{3(x+2)}$, ορίζεται για $x \neq 1, x \neq 2, x \neq -2$
 $\hookrightarrow x \neq 1$
 $\hookrightarrow x \neq 2$
 $x \neq -2$

β. $\frac{x^2-7x}{3x-15} \cdot \frac{x^2-10x+25}{x^2-49} = \frac{x(x-7)}{3(x-5)} \cdot \frac{(x-5)^2}{(x-7)(x+7)} = \frac{x(x-5)}{3(x+7)}$, ορίζεται για $x \neq 5, x \neq 7, x \neq -7$
 $\hookrightarrow x \neq 5$
 $\hookrightarrow x \neq 7$
 $x \neq -7$

5. α. $\frac{x+1}{3} : \frac{x+1}{6} = \frac{x+1}{3} \cdot \frac{6}{x+1} = 2$, ορίζεται για $x \neq -1$
 $\hookrightarrow x \neq -1$

β. $\frac{3a-2}{\beta+5} : \frac{2-3a}{5+\beta} = \frac{3a-2}{\beta+5} \cdot \frac{5+\beta}{-(2+3a)} = -1$, ορίζεται για $\beta \neq -5, a \neq \frac{2}{3}$
 $\hookrightarrow \beta \neq -5$
 $\hookrightarrow a \neq \frac{2}{3}$

γ. $\frac{x^2-49}{x^2} : \frac{ax-7a}{x} = \frac{(x-7)(x+7)}{x^2} \cdot \frac{x}{a(x-7)} = \frac{x(x+7)}{x^2 a} = \frac{x+7}{xa}$, ορίζεται για $x \neq 0, x \neq 7, a \neq 0$
 $\hookrightarrow x \neq 0$
 $\hookrightarrow a \neq 0$
 $x \neq 7$

δ. $\frac{10x^2-5x}{1+4x+4x^2} : \frac{25x}{2-8x^2} = \frac{5x(2x-1)}{(1+2x)^2} \cdot \frac{2(1-4x^2)}{25x} = \frac{5x(2x-1)}{(1+2x)^2} \cdot \frac{2(1-2x)(1+2x)}{25x} = \frac{2(2x-1)(1-2x)}{(2x+1) \cdot 5} = \frac{2(2x-1)^2}{5(2x+1)}$, ορίζεται για $x \neq \pm \frac{1}{2}, x \neq 0$
 $\hookrightarrow x \neq \pm \frac{1}{2}$
 $\hookrightarrow x \neq 0$

$$\alpha. \frac{2x}{2x-4} - \frac{2}{x-2} = \frac{2x}{2(x-2)} - \frac{2}{x-2} = \frac{2x}{2(x-2)} - \frac{4}{2(x-2)} = \frac{2x-4}{2(x-2)} = \frac{2(x-2)}{2(x-2)} = 1, \text{ ορίζεται για } x \neq 2$$

$$\hookrightarrow \text{ΕΚΠ}(2(x-2), (x-2)) = 2(x-2)$$

$$\beta. \frac{5x+6}{x^2+3x} - \frac{3}{x+3} = \frac{5x+6}{x(x+3)} - \frac{3}{x+3} = \frac{5x+6}{x(x+3)} - \frac{3x}{x(x+3)} = \frac{2x+6}{x(x+3)} = \frac{2(x+3)}{x(x+3)} = \frac{2}{x}, \text{ ορίζεται για } x \neq 0, x \neq -3$$

$$\hookrightarrow \text{ΕΚΠ}(x(x+3), (x+3)) = x(x+3)$$

$$\gamma. \frac{x}{x^2-1} + \frac{1}{2-2x} = \frac{x}{(x-1)(x+1)} + \frac{1}{2(1-x)} = \frac{x}{(x-1)(x+1)} - \frac{1}{2(x-1)} = \frac{-2x}{-2(x-1)(x+1)} + \frac{(x+1)}{-2(x-1)(x+1)} = \frac{-2x + (x+1)}{-2(x-1)(x+1)} = \frac{-x+1}{-2(x-1)(x+1)} = \frac{x-1}{2(x-1)(x+1)} = \frac{1}{2(x+1)}, \text{ ορίζεται για } x \neq -1, x \neq 1$$

$$\hookrightarrow \text{ΕΚΠ}((x-1)(x+1), -2(x-1)) = -2(x-1)(x+1)$$

$$\frac{2x}{2(x-1)(x+1)} - \frac{x+1}{2(x-1)(x+1)} = \frac{2x - (x+1)}{2(x-1)(x+1)} = \frac{x-1}{2(x-1)(x+1)} = \frac{1}{2(x+1)}, \text{ ορίζεται για } x \neq -1, x \neq 1$$

$$\hookrightarrow x \neq 1, x \neq -1$$

$$\delta. \frac{1}{a\beta - a^2} + \frac{2}{a^2 - \beta^2} = \frac{1}{a(\beta - a)} + \frac{2}{(a - \beta)(a + \beta)} = \frac{1}{-a(a - \beta)} + \frac{2}{(a - \beta)(a + \beta)} = \frac{a + \beta}{-a(a - \beta)(a + \beta)} + \frac{-2a}{-a(a - \beta)(a + \beta)} = \frac{a + \beta - 2a}{-a(a - \beta)(a + \beta)} = \frac{-a + \beta}{-a(a - \beta)(a + \beta)} = \frac{a - \beta}{a(a - \beta)(a + \beta)} = \frac{1}{a(a + \beta)}, \text{ ορίζεται για } a \neq 0, a \neq \beta, a \neq -\beta$$

$$\hookrightarrow a \neq 0, \beta \neq a, \beta \neq -a$$

6. α. $\frac{x-1}{x} + \frac{2}{x-1} - \frac{2}{x^2-x} = \frac{x-1}{x} + \frac{2}{x-1} - \frac{2}{x(x-1)} = \frac{(x-1)^2}{x(x-1)} + \frac{2x}{x(x-1)} - \frac{2}{x(x-1)} =$
 $\begin{matrix} \xrightarrow{x \neq 0} & \xrightarrow{x \neq 1} \\ \text{ΕΚΠ}(x, x-1, x(x-1)) = x(x-1) \end{matrix}$

$$\frac{x^2 - 2x + 1 + 2x - 2}{x(x-1)} = \frac{x^2 - 1}{x(x-1)} = \frac{(x-1)(x+1)}{x(x-1)} = \frac{x+1}{x}, \text{ ορίζεται για } x \neq 0, x \neq 1$$

$$\frac{(x-1)(x+1)}{x(x-1)} = \frac{x+1}{x}, \text{ ορίζεται για } x \neq 0, x \neq 1$$

β. $\frac{x^2}{x-2} + \frac{2x}{x+2} + \frac{8x}{x^2-4} = \frac{x^2}{x-2} + \frac{2x}{x+2} + \frac{8x}{(x-2)(x+2)} = \frac{x^2(x+2)}{(x-2)(x+2)} + \frac{2x(x-2)}{(x-2)(x+2)} + \frac{8x}{(x-2)(x+2)} =$
 $\begin{matrix} \xrightarrow{x \neq 2} & \xrightarrow{x \neq -2} \\ \text{ΕΚΠ}(x-2, x+2, (x-2)(x+2)) = (x-2)(x+2) \end{matrix}$

$$\frac{x^3 + 2x^2 + 2x^2 - 4x + 8x}{(x-2)(x+2)} = \frac{x^3 + 4x^2 + 4x}{(x-2)(x+2)} =$$

$$\frac{x(x^2 + 4x + 4)}{(x-2)(x+2)} = \frac{x(x+2)^2}{(x-2)(x+2)} = \frac{x(x+2)}{x-2}, \text{ ορίζεται για } x \neq 2, x \neq -2$$

γ. $\frac{x}{x^2-4} - \frac{1}{2-x} + \frac{2}{3x+6} = \frac{x}{(x-2)(x+2)} - \frac{1}{x-2} + \frac{2}{3(x+2)} = \frac{3x}{3(x-2)(x+2)} - \frac{3(x+2)}{3(x-2)(x+2)} + \frac{2(x-2)}{3(x-2)(x+2)} =$
 $\begin{matrix} \xrightarrow{x \neq 2} & \xrightarrow{x \neq -2} \\ \text{ΕΚΠ}((x-2)(x+2), x-2, 3(x+2)) = 3(x-2)(x+2) \end{matrix}$

$$\frac{3x - 3(x+2) + 2(x-2)}{3(x+2)(x-2)} = \frac{3x - 3x - 6 + 2x - 4}{3(x+2)(x-2)} = \frac{2x - 10}{3(x+2)(x-2)} =$$

$$\frac{2(x-5)}{3(x-2)(x+2)}, \text{ ορίζεται για } x \neq 2, x \neq -2$$