

ΜΑΘΗΜΑ 29

Ταυτότητες

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a + b)(a - b) = a^2 - b^2$$

Στο μάθημα αυτό χρησιμοποιούμε αυτές τις ταυτότητες για να λύσουμε την άσκηση 11 σελ. 50.

Λύση ασκ. 11α,β σελ. 50

$$\begin{aligned}\alpha) \quad (x-4)^2 + (2x+5)^2 &= x^2 - 2 \cdot 4 \cdot x + 4^2 + (2x)^2 + 2 \cdot 5 \cdot 2x + 5^2 \\ &= x^2 - 8x + 16 + 4x^2 + 20x + 25 \\ &= 5x^2 + 12x + 41\end{aligned}$$

$$\begin{aligned}\beta) \quad (x^2-1)^2 - (x^2-3)(x^2+3) &= (x^2)^2 - 2 \cdot 1 \cdot x^2 + 1^2 - ((x^2)^2 - 3^2) \\ &= x^4 - 2x^2 + 1 - (x^4 - 9) \\ &= x^4 - 2x^2 + 1 - x^4 + 9 \\ &= -2x^2 + 10\end{aligned}$$

Λύση ασκ. 11γ,δ σελ. 50

$$\begin{aligned}\gamma) \quad (x+y)^2 - (x-2y)(x+2y) + (2x-y)^2 &= \\ &= x^2 + 2xy + y^2 - (x^2 - (2y)^2) + (2x)^2 - 2(2x)y + y^2 \\ &= x^2 + 2xy + y^2 - x^2 + 4y^2 + 4x^2 - 4xy + y^2 \\ &= 4x^2 - 2xy + 6y^2\end{aligned}$$

$$\begin{aligned}\delta) \quad (3x-4)^2 + (3x+4)^2 - 2(3x-4)(3x+4) &= \\ &= (3x)^2 - 2 \cdot 3x \cdot 4 + 4^2 + (3x)^2 + 2 \cdot 3x \cdot 4 + 4^2 - 2((3x)^2 - 4^2) \\ &= 9x^2 - 24x + 16 + 9x^2 + 24x + 16 - 2(9x^2 - 16) \\ &= 9x^2 - 24x + 16 + 9x^2 + 24x + 16 - 18x^2 + 32 \\ &= 16 + 16 + 32 \\ &= 64\end{aligned}$$

Λύση ασκ. 11ε,στ σελ. 50

$$\begin{aligned}\varepsilon) \quad (2\alpha + 1)^3 + (2\alpha - 1)^3 &= (2\alpha)^3 + 3(2\alpha)^2 \cdot 1 + 3(2\alpha)1^2 + \\ &\quad + 1^3 + (2\alpha)^3 - 3(2\alpha)^2 \cdot 1 + 3(2\alpha)1^2 - 1^3 \\ &= 8\alpha^3 + 3 \cdot 8\alpha^2 + 6\alpha + 1 + 8\alpha^3 - 3 \cdot 8\alpha^2 + 6\alpha - 1^3 \\ &= 8\alpha^3 + 24\alpha^2 + 6\alpha + 1 + 8\alpha^3 - 24\alpha^2 + 6\alpha - 1^3 \\ &= 16\alpha^3 + 12\alpha\end{aligned}$$

$$\begin{aligned}\sigma\tau) \quad (\alpha + 2)^3 - (\alpha + 2)(\alpha^2 - 2\alpha + 4) &= \\ &= \alpha^3 + 3 \cdot \alpha^2 \cdot 2 + 3 \cdot \alpha \cdot 2^2 + 2^3 - (\alpha^3 + 2^3) \\ &= \alpha^3 + 6\alpha^2 + 12\alpha + 8 - \alpha^3 - 2^3 \\ &= \alpha^3 + 6\alpha^2 + 12\alpha + 8 - \alpha^3 - 8 \\ &= 6\alpha^2 + 12\alpha\end{aligned}$$

Λύση ασκ. 11ζ,η σελ. 50

$$\zeta) (\alpha^2 + \alpha)^3 - (\alpha^2 - \alpha)^3 =$$

$$= (\alpha^2)^3 + 3(\alpha^2)^2\alpha + 3\alpha^2\alpha^2 + \alpha^3 - [(\alpha^2)^3 - 3(\alpha^2)^2\alpha + 3\alpha^2\alpha^2 - \alpha^3]$$

$$= \alpha^6 + 3\alpha^4\alpha + 3\alpha^4 + \alpha^3 - (\alpha^6 - 3\alpha^4\alpha + 3\alpha^4 - \alpha^3)$$

$$= \alpha^6 + 3\alpha^5 + 3\alpha^4 + \alpha^3 - \alpha^6 + 3\alpha^5 - 3\alpha^4 + \alpha^3$$

$$= 6\alpha^5 + 2\alpha^3$$

$$\eta) (4\alpha - 1)^3 - \alpha(8\alpha + 1)(8\alpha - 1) =$$

$$= (4\alpha)^3 - 3 \cdot (4\alpha)^2 \cdot 1 + 3 \cdot 4\alpha \cdot 1^2 - 1^3 - \alpha(64\alpha^2 - 1^2)$$

$$= 64\alpha^3 - 3 \cdot 16\alpha^2 + 12\alpha - 1 - 64\alpha^3 + \alpha$$

$$= 64\alpha^3 - 48\alpha^2 + 12\alpha - 1 - 64\alpha^3 + \alpha$$

$$= -48\alpha^2 + 13\alpha - 1$$