6:05 | Solving Problems Using Equations

prep quiz 6:05

What is the sum of:

1 3 and 4? 2 x and 3? 3 x and y?

What is the product of:

4 3 and 4? 5 x and 3? 6 x and y?

7 I have \$50. I spend \$$x$. How much do I have left?

8 There are x books and each one costs \$5. What is the total cost of the books?

9 There are x cars with y people in each car. How many people are there altogether?

10 A man is x years old. How old will he be in 5 years time?

Consider the following simple problem.

'I think of a number. If I add 7 to the number the result is 22. What is the number?'

- This problem can be solved by forming an equation. If the missing number is represented by the pronumeral x, then the equation $x + 7 = 22$ represents the information given in the problem. Solving the equation then yields the answer to the original problem.

worked examples

Translate the following into number sentences.
In all cases use the 'x' to represent the unknown number.

1 I multiply a number by 2 and the result is 50.
2 If I add 6 to a number the answer is 11.
3 I subtract a number from 6 and the answer is 2.
4 A certain number is multiplied by 3 then 6 is added and the result is 17.

■ We often use 'x' to represent an unknown number.

Solutions

1 I multiply a number by 2 and the result is 50.
$\underbrace{\quad}_{2 \times x}$ $\underset{= 50}{\downarrow\downarrow}$
The equation is $2x = 50$.

2 If I add 6 to a number the answer is 11.
$\underbrace{\quad}_{6 + x}$ $\underset{= 11}{\downarrow\downarrow}$
The equation is $6 + x = 11$.

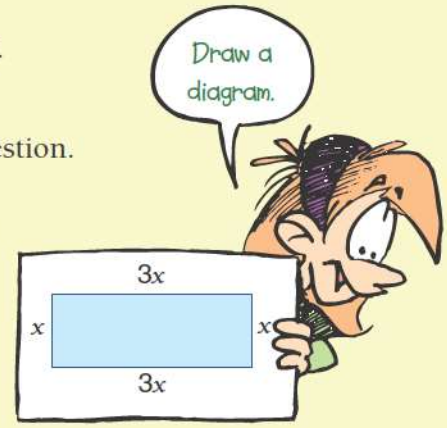
3 I subtract a number from 6 and the answer is 2.
$\underbrace{\quad}_{6 - x}$ $\underset{= 2}{\downarrow\downarrow}$
The equation is $6 - x = 2$.

4 A certain number is multiplied by 3, then 6 is added and the result is 17.
$\underbrace{\quad}_{x \times 3}$ $\underbrace{\quad}_{+ 6}$ $\underset{= 17}{\downarrow\downarrow}$
The equation is $3x + 6 = 17$.

To use equations to solve problems we must be able to analyse a written problem, translate it into an equation and then solve it.

Approach

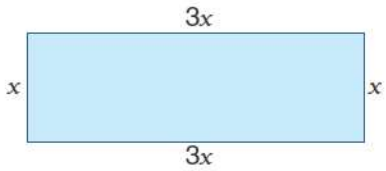
- Read the problem carefully, examining the wording of the question.
- Establish what is to be found and what information is given.
- Ask yourself whether any other information can be assumed, eg that a pack of cards mentioned is a standard pack.
- Try to connect the given information to form an equation. This will often require a knowledge of a formula or the meaning of mathematical terms.



worked examples

Example 1

A rectangle is three times longer than it is wide. If it has a perimeter of 192 m, what are its dimensions?



Solution 1

Let the width be x metres.

$\therefore$ The length $= 3 \times x$ metres
$= 3x$ metres

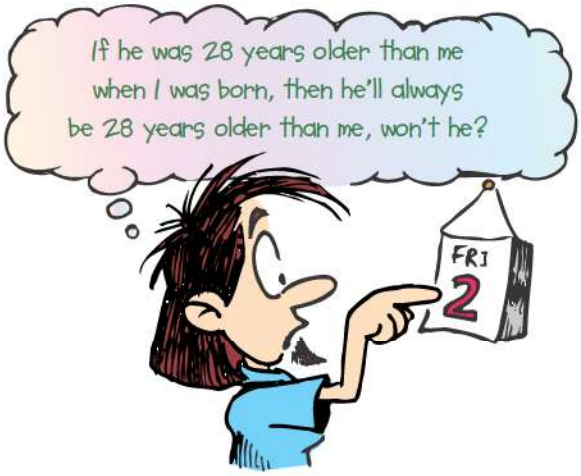
Now perimeter means the sum of the lengths of the sides (or the distance around the outside of the figure).

$\therefore 3x + x + 3x + x = 192$
$\therefore 8x = 192$
$\therefore x = 24$
$\therefore$ The width $= 24$ m
and the length $= 3 \times 24$ m
$= 72$ m

■ In the first line of each solution, indicate what the pronumeral represents.

Example 2

My father was 28 years old when I was born. If he is now three times as old as I am, what are our present ages?



Solution 2

Let my present age be x years.

$\therefore$ My father's present age is $3 \times x$ years.
When I was born my father was 28.
$\therefore$ The difference in our ages is 28 years.
$\therefore$ Father's age − my age always equals 28 years.
$\therefore 3x - x = 28$
$2x = 28$
$x = 14$
$\therefore$ I am 14 years old and my father is 42 years old (ie 3×14 years).

Example 3

Car A left London for Manchester at 6:00 am and travelled at an average speed of 80 km/h. At 7:30 am car B left London for Manchester. If car B travels at an average speed of 100 km/h, at what time will it catch car A?

continued →→→

Solution 3

Car B will catch car A when both have travelled the same distance
and distance travelled = average speed × time.

Now *let car B catch up to car A t hours after car B starts.*
∴ Car B has been travelling for t hours.
∴ Car A has been travelling for $(t + 1\frac{1}{2})$ hours (since it started at 6 am).
∴ Distance travelled by car A $= 80 \times (t + 1\frac{1}{2})$ Distance travelled by car B $= 100 \times t$

$$\therefore 80(t + 1\tfrac{1}{2}) = 100t$$
$$\therefore 80t + 80 \times 1\tfrac{1}{2} = 100t$$
$$\therefore 120 = 20t$$
$$\therefore t = 6$$

∴ Car B catches car A 6 hours after it starts, ie at 1:30 am.

CAR A
Speed = 80 km/h
Starts at 6:00 am
Travels for $(t + 1\frac{1}{2})$ hours
Distance travelled = $80(t + 1\frac{1}{2})$

CAR B
Speed = 100 km/h
Starts at 7:30 am
Travels for t hours
Distance travelled = $100t$

Exercise 6:05

'I subtract 3 from a certain number' translates to $x - 3$.

'I subtract a certain number from 3' translates to $3 - x$.

Also $x - 3$ is not the same as $3 - x$.

Foundation Worksheet 6:05

Solving problems using equations

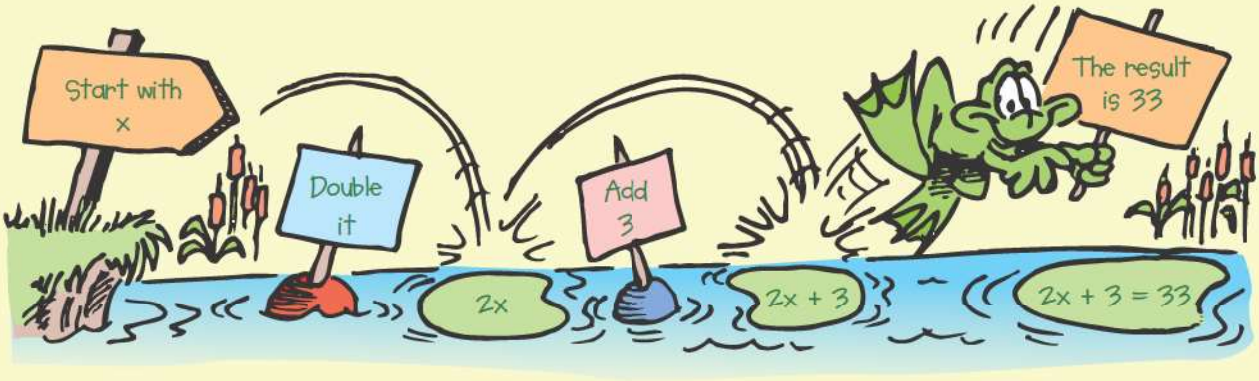
1 Write equations for:
  a the sum of a and 5 equals 8
  b the product of 7 and x equals 42.
2 Write equations for the following and solve.
  a If I subtract 9 from 3 times a number the result is 27.

1 Translate the following sentences into equations, using the pronumeral 'x' to represent the unknown number. Then solve the equation to find the value of the unknown number.

a If 5 is added to a number the answer is 22.
b If I subtract 3 from a certain number the result is 10.
c I multiply a number by 8 and the result is 32.
d Dividing a certain number by 8 gives an answer of 7.
e A number is multiplied by 2, then 6 is added and the result is 14.
f Three times a certain number is added to 5 and the result is 20.
g A certain number is multiplied by 5, then 8 is subtracted and the result is 22.
h If 5 is added to a certain number and the result is multiplied by 4 the answer is 56.
i When 5 is subtracted from half of a number the result is 3.

2 For each of the following problems form an equation and then solve it.

a I think of a number, double it, add 3 and the result is 33. What is the number?



b I think of a number and multiply it by 4. If I then subtract 3, the answer is 25. Find the number.

c I think of a number, add 3 and then double that result. If the answer is 22, find the number.

d I think of a number. After dividing it by 4 and subtracting 7 the result is 1. What is the number?

e I think of a number. If I add 4 and then divide by 3 the result is 8. Find the number.

3 Solve each of the following problems by first forming an equation.

a If 5 is added to 3 times a certain number the result is 38. What is the number?

b If I subtract 6 from 5 times a certain number the result is 29. What is the number?

c If 5 is subtracted from a certain number and that result is then halved, the answer is 6. What is the number?

d A number is doubled and then 5 is added. When this is divided by 3 the result is 7. What is the number?

4 **a** My father is three times as old as I am. If he is 26 years older than I, what are our ages?

b Two men have $560 between them. If one man has six times as much money as the other, how much has each man?

c Jalena has $7 less than Mina. Together they have $43. How much does each girl have?

d Prizemoney of $500 is divided between Alex and Zarko so that Alex receives $170 more than Zarko. How much does each receive?

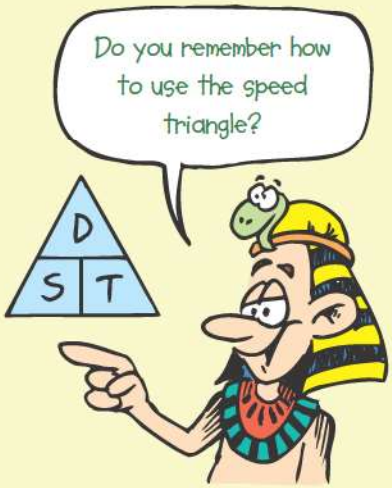
e If a father is five times as old as his son at present, how old is he if he was 32 years old when his son was born?

5 **a** In a class of 32 students, it is known that there are 6 more boys than girls. How many girls are there in the class?

b A rectangle is 6 cm longer than it is wide. Find its dimensions if its perimeter is 64 cm.

c If a quarter of the weight of a leg of lamb is lost in roasting, what weight of lamb should be bought in order to have 3 kg of roasted meat?

d A town B is between towns A and C. B is five times as far from C as it is from A. The distance from A to C is 144 km. How far is it from A to B?



6 **a** Six kilograms of an inferior tea is mixed with 3 kilograms of tea that costs \$2 a kilogram more. The total price of the mixture is \$24.
What was the price of the inferior tea?

b Two bike riders X and Y both start at 2 pm riding towards each other from 40 km apart. X rides at 30 km/h, Y at 20 km/h. If they meet after t hours, find when and where they meet.

c A man is twice as old as his son. If 9 years ago the sum of their ages was 66 years, find their present ages.

d A man notices that a tank is half full. After emptying 600 litres from the tank, he observes that it is now one-third full. How much does the tank hold when it is full?

7 **a** Franco is 25 years older than Alejandro and, in 5 years, he will be twice as old as Alejandro. Find their present ages.

b A bank teller notices that he has 50 coins all of which are 5c or 10c pieces. He finds that the value of the coins is \$4.20. How many of each must he have?

c A tennis player has won 36 out of 54 matches. His sponsor says that he must win 60% of his total number of matches to qualify for a bonus. If there are 26 matches remaining on the tour, how many more must he win to collect his bonus?

d One tank holds 300 litres more than another. If the smaller is two-thirds full, it holds as much as the larger when it is half full. What is the capacity of each?

e A certain journey took 40 min to complete. If half the distance was travelled at an average speed of 100 km/h and the other half at an average speed of 60 km/h, what was the length of the journey?

60 km/h | 100 km/h
A | B

$$\text{Time for A} + \text{Time for B} = 40 \text{ min}$$
$$40 \text{ min} = \frac{2}{3} \text{ h}$$

8 **a** Pump A delivers water at twice the rate of pump B. If both pumps operate together, a tank of 18 000 litres capacity can be filled in 30 minutes. Find the pumping rate of each pump, in litres per minute.

b A car travels between A and B at an average speed of 60 km/h. If the car increased its average speed to 100 km/h it would take 10 minutes less to make the trip. How far is it between the towns?

c Car A is travelling along a freeway at 100 km/h when it is passed by car B. If both cars maintain a constant speed and the end of the freeway is 10 km away, find the speed at which car B must travel to beat car A to the end of the freeway by 1 minute.

d A sum of money is divided between A, B and C in the ratio 1 : 2 : 3. However, before the money is divided C dies and it is decided to divide his share between A and B in the ratio 1 : 3. If After C's share is divided B has \$2000 more than A, how much money was there altogether?

e Rectangles A and B are both four times as long as they are wide and the length of rectangle A is three times the length of rectangle B. If the difference in the perimeters is 16 cm, find the dimensions of each rectangle.