Looking at an Angle

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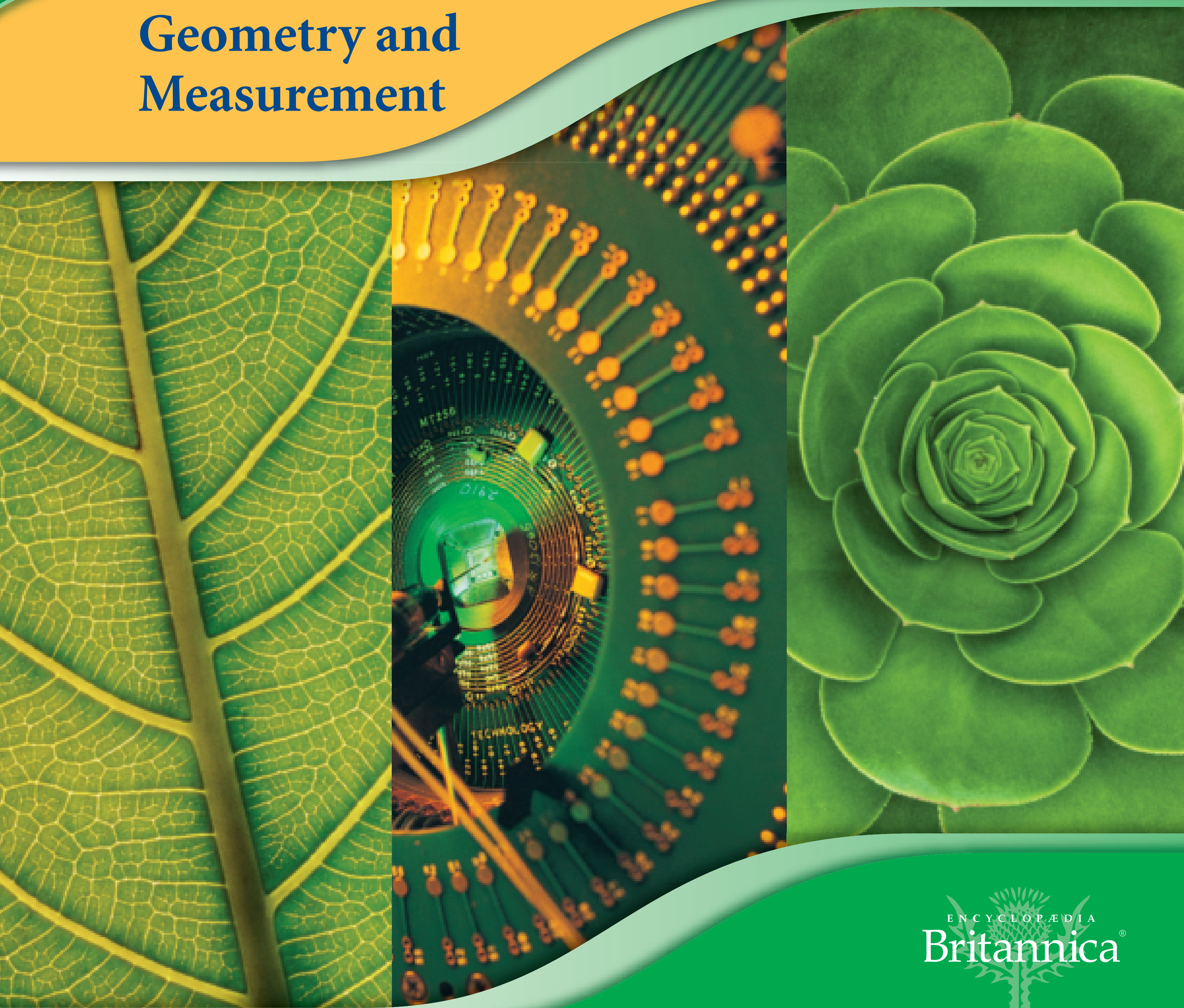
Geometry and Measurement



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Contents

Letter to the Student vi

Section A — Now You See It, Now You Don't

The Grand Canyon 1
The Table Canyon Model 3
Ships Ahoy 6
Cars and Blind Spots 9
Summary 10
Check Your Work 10

Section B — Shadows and Blind Spots

Shadows and the Sun 13
Shadows Cast by the Sun and Lights 18
A Shadow is a Blind Spot 20
Summary 22
Check Your Work 22

Section C — Shadows and Angles

Acoma Pueblo 25
Summary 30
Check Your Work 30

Section D — Glide Angles

Hang Gliders 32
Glide Ratio 34
From Glide Ratio to Tangent 37
Summary 42
Check Your Work 42

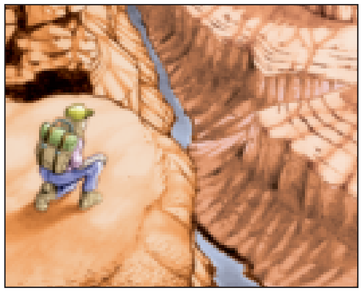
Section E — Reasoning with Ratios

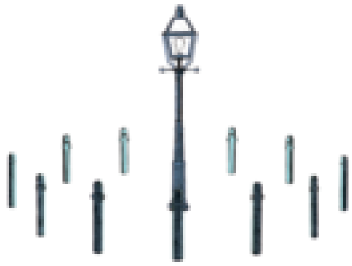
Tangent Ratio 45
Vultures Versus Gliders 46
Pythagoras 47
The Ratios: Tangent, Sine, Cosine 50
Summary 54
Check Your Work 54

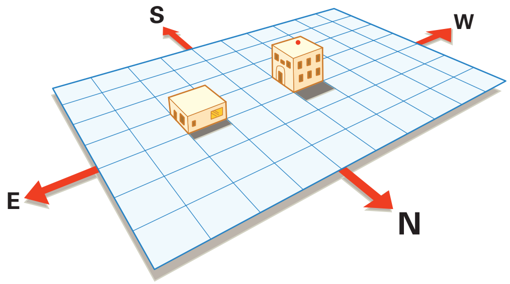
Additional Practice 56

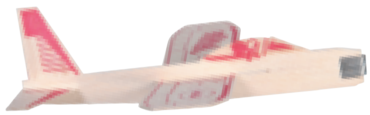
Answers to Check Your Work 61

Appendix A 68







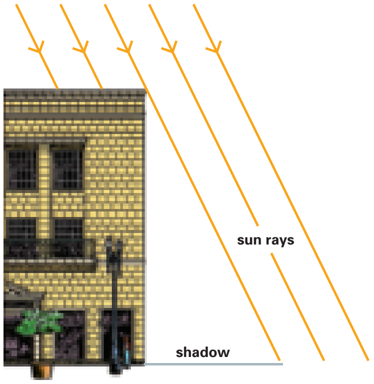


Dear Student,

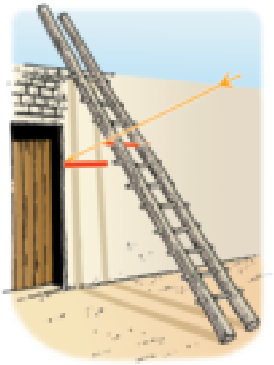
Welcome to *Looking at an Angle*!

In this unit, you will learn about vision lines and blind areas. Have you ever been on one of the top floors of a tall office or apartment building? When you looked out the window, were you able to see the sidewalk directly below the building? If you could see the sidewalk, it was in your field of vision; if you could not see the sidewalk, it was in a blind spot.

The relationship between vision lines and rays of light and the relationship between blind spots and shadows are some of the topics that you will explore in this unit. Have you ever noticed how the length of a shadow varies according to the time of day? As part of an activity, you will measure the length of the shadow of a stick and the corresponding angle of the sun at different times of the day. You will then determine how the angle of the sun affects the length of a shadow.



Besides looking at the angle of the sun, you will also study the angle that a ladder makes with the floor when it is leaning against a wall and the angle that a descending hang glider makes with the ground. You will learn two different ways to identify the steepness of an object: the angle the object makes with the ground and the tangent of that angle.



We hope you enjoy discovering the many ways of "looking at an angle."

Sincerely,

The Mathematics in Context Development Team

A Now You See It, Now You Don't

The Grand Canyon

The Grand Canyon is one of the most famous natural wonders in the world. Located on the high plateau of northwestern Arizona, it is a huge gorge carved out by the Colorado River. It has a total length of 446 kilometers (km). Approximately 90 km of the gorge are located in the Grand Canyon National Park. The north rim of the canyon (the Kaibab Plateau) is about 2,500 meters (m) above sea level.

This photograph shows part of the Colorado River, winding along the bottom of the canyon.

1. Why can't you see the continuation of the river on the lower right side of the photo?

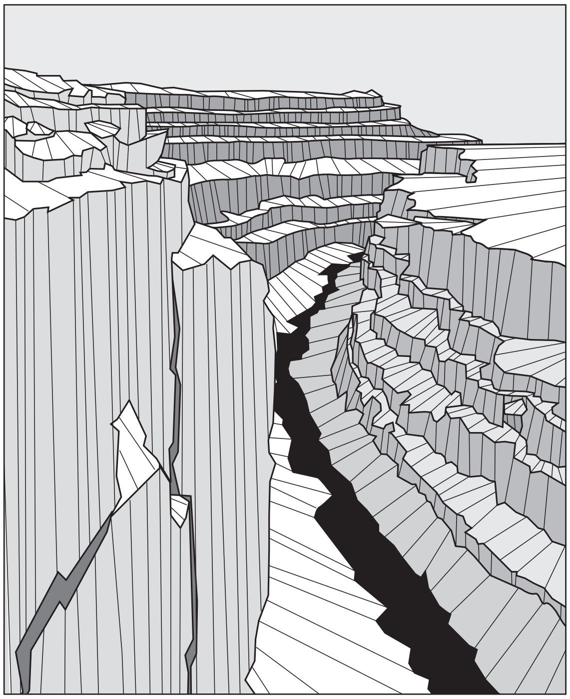
A Now You See It, Now You Don't

The Colorado River can barely be seen from most viewpoints in Grand Canyon National Park.

This drawing shows a hiker on the north rim overlooking a portion of the canyon.

2. Can the hiker see the river directly below her? Explain.

Here you see a photograph and a drawing of the same area of the Grand Canyon. The canyon walls are shaped like stairs in the drawing.



3. Describe other differences between the photo and the drawing.

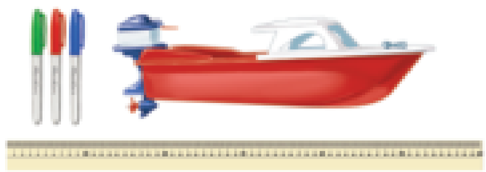
Activity

The Table Canyon Model

In this activity, you will build your own "table canyon" to investigate how much of the "river" can be seen from different perspectives. To do this activity, you will need at least three people: two viewers and one recorder.

Materials

- two tables
- two large sheets of paper
- a meter stick
- markers
- a boat (optional)



- Place two tables parallel to each other, with enough room between them for another table to fit.
- Hang large sheets of paper from the tables to the floor as shown in the photograph above. The paper represents the canyon walls, and the floor between the two tables represents the river.
- Sit behind one of the tables, and have a classmate sit behind the other. Each of you is viewing the canyon from a different perspective.
- Have another classmate mark the lowest part of the canyon wall visible to each of you viewing the canyon. The recorder should make at least three marks along each canyon wall.

Measure the height of the marks from the floor with the meter stick, and make notes for a report so that you can answer the following.

4. **a.** Can either of you see the river below? Explain.

   b. On which wall are the marks higher, yours or your classmate's? Explain.

   c. Are all the marks on one wall the same height? Explain.

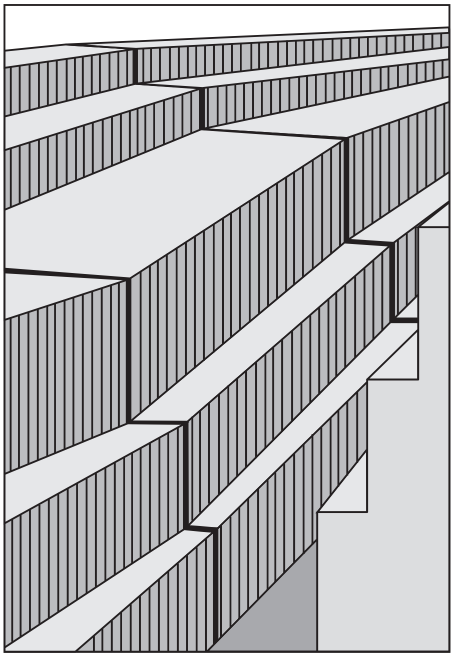
   d. What are some possible changes that would allow you to see the river better? Predict how each change affects what you can see.

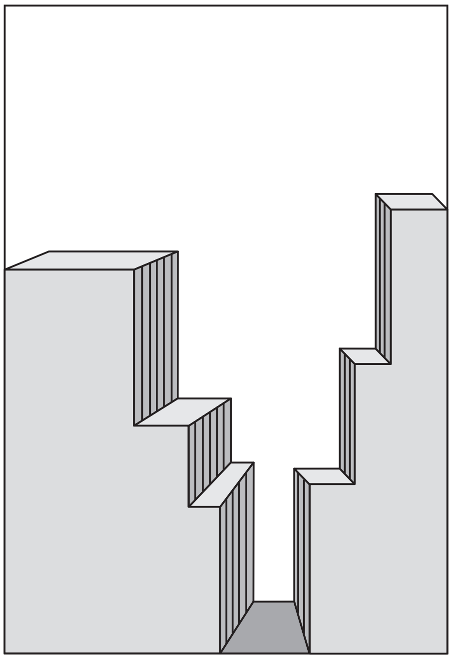
   e. Where would you place a boat on the river so that both of you can see it?

   f. What would change if the boat were placed closer to one of the canyon walls?

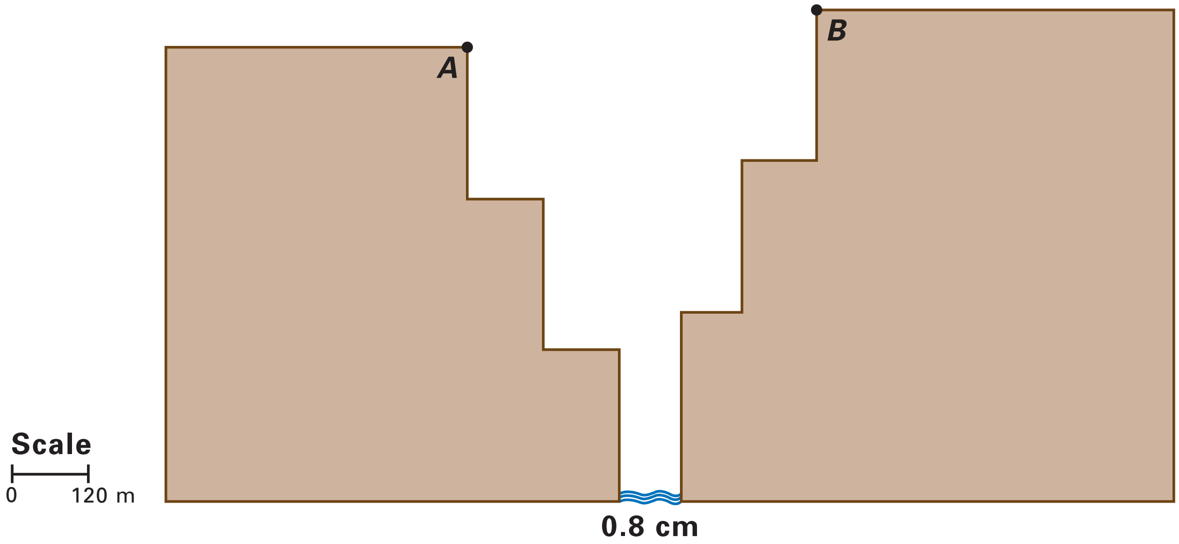
5. Write a report on this activity describing your investigations and discoveries. You may want to use the terms *visible, not visible*, and *blind spot* in your report.

These drawings show two schematic views of the canyon. The one on the right looks something like the table canyon from the previous activity.





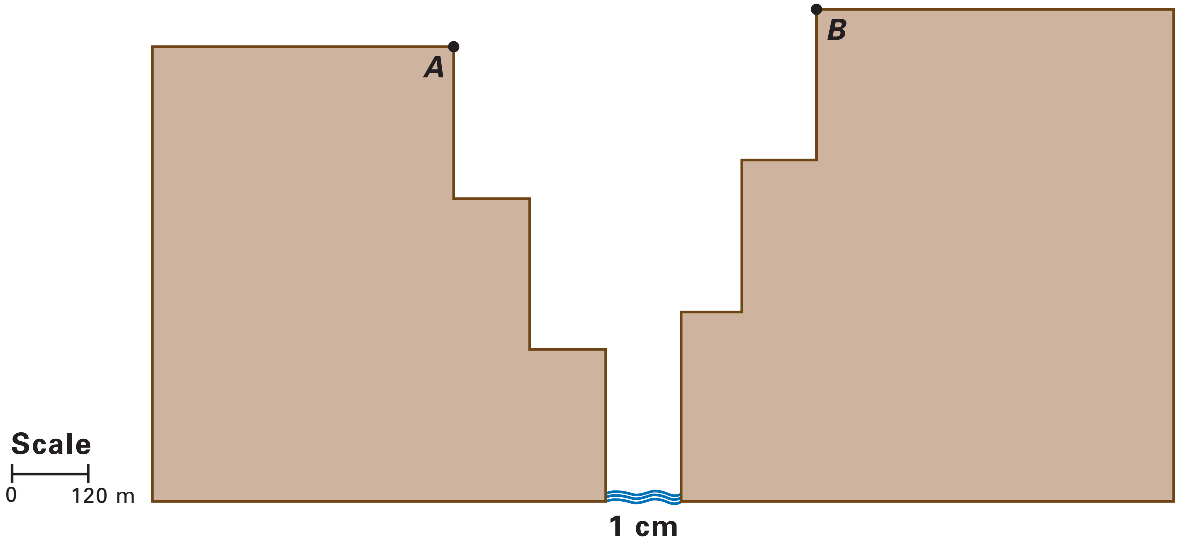
We will look more closely at that drawing on the right. Now we see it in a scale drawing of the *cross-section* of the canyon.



6. Is it possible to see the river from point A on the left rim? Why or why not?

7. What is the actual height of the left canyon wall represented in the scale drawing?

8. If the river were 1.2 centimeters (cm) wide in the scale drawing, could it be seen from point A?

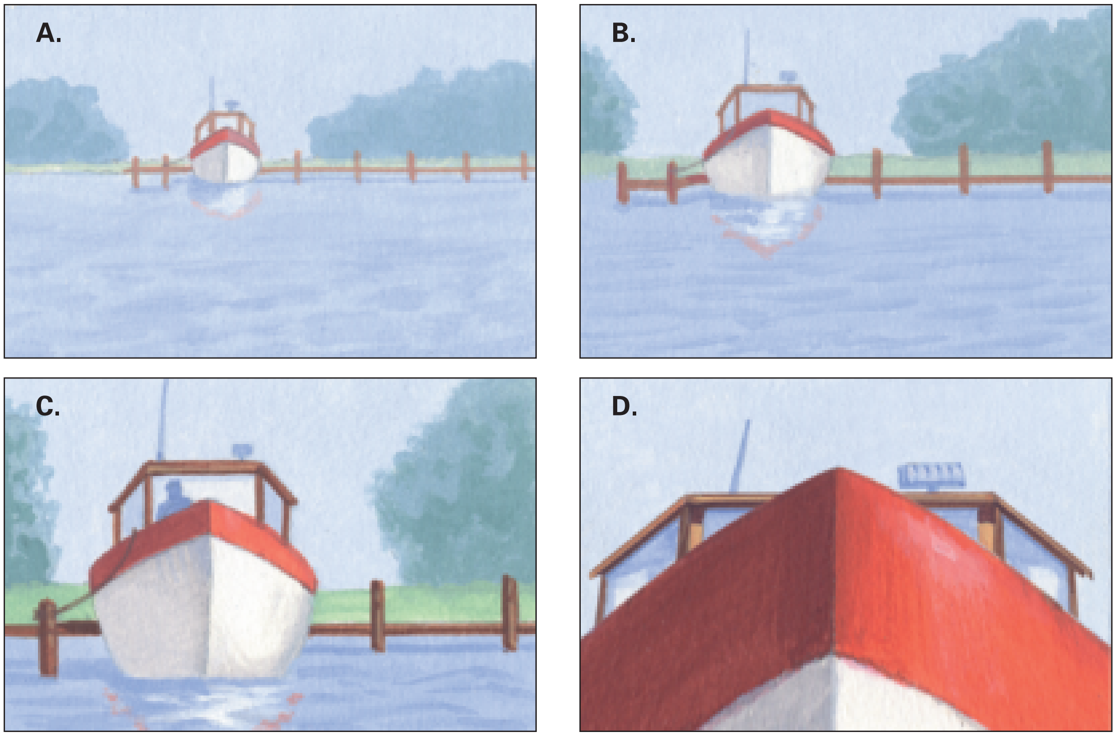


9. In the scale drawing above, the river is now 1 cm wide. Is it possible to see the river from point B? If not, which ledge is blocking your view? Explain.

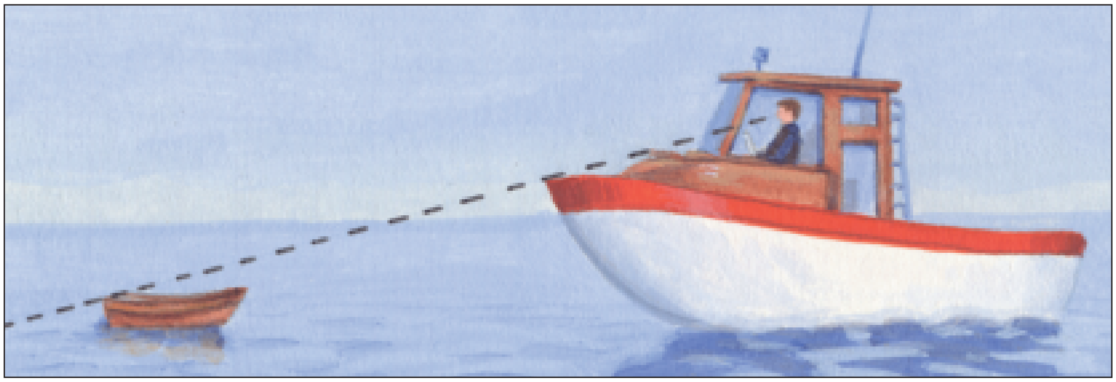
Ships Ahoy

Picture yourself in a small rowboat rowing toward a ship that is tied to a dock. In the first picture, the captain at the helm of the ship is able to see you. As you get closer, at some point the captain is no longer able to see you.

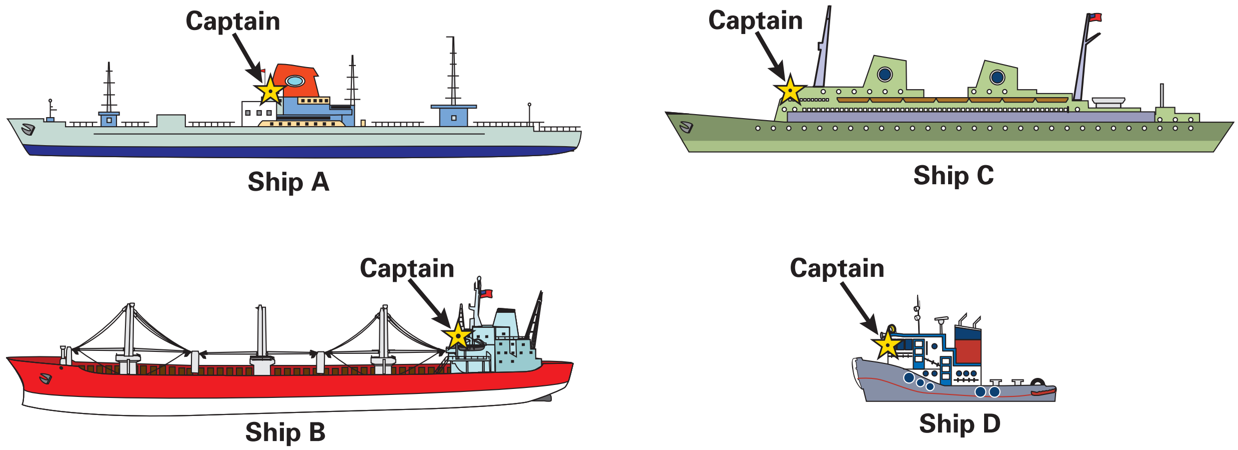
10. Explain why the captain cannot see you in the fourth picture.



The captain's height and position in the ship determine what the captain can and cannot see in front of the ship. The shape of the ship will also affect his field of vision. To find the captain's field of vision, you can draw a **vision line**. A vision line is an imaginary line that extends from the captain's eyes, over the edge of the ship, and to the water.



11. For each ship shown on **Student Activity Sheet 1**, draw a vision line from the captain, over the front edge of the ship, to the water. Measure the angle between the vision line and the water. (A star marks the captain's location.)



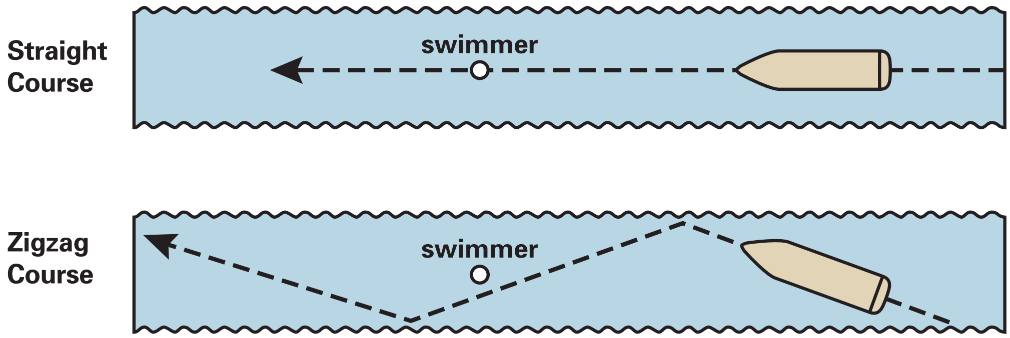
12. Compare the ships on **Student Activity Sheet 1**.

a. On which ship is the captain's blind area the smallest? Explain.

b. How does the shape of the ship affect the captain's view?

c. How does the angle between the vision line and the water affect the captain's view?

Suppose that you are swimming in the water and a large boat is coming toward you. If you are too close to the boat, the captain may not be able to see you! In order to see a larger area of the water, a captain can travel in a zigzag course.



13. Explain why the captain has a better chance of seeing something in front of the boat by traveling in a zigzag course.

Activity

For this activity, each group of students needs a piece of string and a toy boat. The boat can be made of either plastic or wood, but it must have a flat bottom.

Line up all the boats in the front of the classroom. For each boat, assign a number and determine the captain's location.

14. Without measuring, decide which boat has the largest blind spot and which has the smallest blind spot. Explain your decisions.

When comparing blind spots, you have to take into account the size of the boat. A large boat will probably have a large blind spot, but you must consider the size of the blind spot relative to the size of the boat.

In your group, use the following method to measure your boat's blind spot.

Place your boat on the **Student Activity Sheet 2** graph paper. Trace the bottom of the boat. Attach a piece of string to the boat at the place where the captain is located. The string represents the captain's vision line.

Using the string and a pencil, mark the spot on the graph paper where the captain's vision line hits the water. Make sure the vision line is stretched taut and touches the edge of the boat.

Mark several places on the graph paper where the captain's vision line hits the water so that you can determine the shape of the blind spot (the captain looks straight ahead and sideways). If the graph paper is not large enough, tape several pieces together. Draw the blind spot on the graph paper.

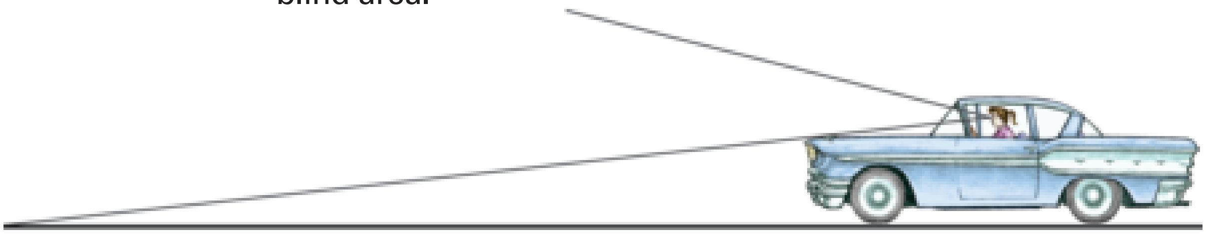
Find the area of the blind spot. Note: Each square on the graph paper is one square centimeter.

15. Make a list of the data for each boat. Decide which boat has the largest blind spot relative to its size and which has the smallest blind spot relative to its size.

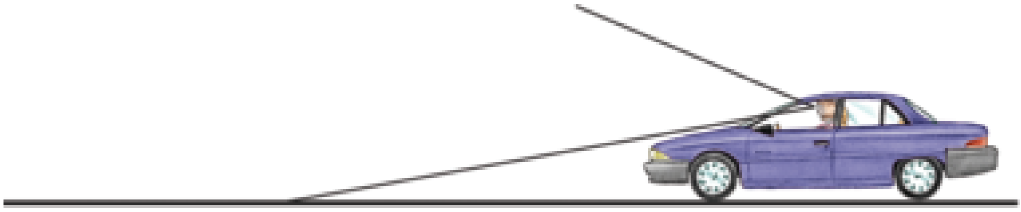
Cars and Blind Spots

This photograph is of a 1958 Pontiac Star Chief. This car is 5.25 m long.

Here is a side view of the car with vision lines indicating the blind area.



Today cars are designed so that the blind area in front of the car is much smaller. The car shown below is a 1997 Buick Skylark that is 4.7 m long. Notice how the vision line touches the hood of this car.



16. Find the length of road in front of each car that cannot be seen by the driver.

17. Which car has the longest relative blind spot?

18. What does the vision line that extends upward from each car indicate? Why is it important that this vision line be as close to vertical as possible?

19. Describe a situation from your daily life which involves a blind spot. Include a drawing of the situation with the blind spot clearly indicated.

A Now You See It, Now You Don't

Summary

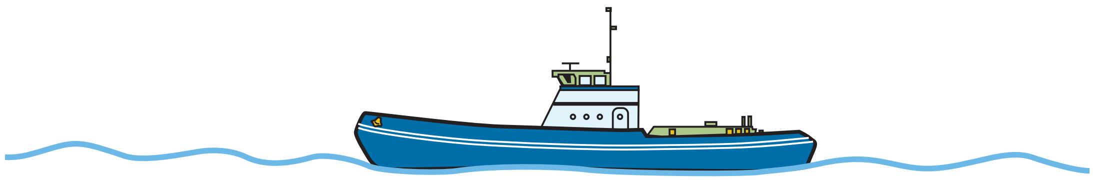
When an object is hidden from your view because something is in the way, the area that you cannot see is called the **blind area** or **blind spot**.

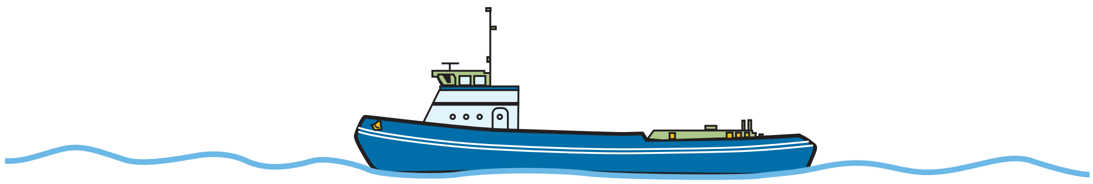
Vision lines are imaginary lines that go from a person's eyes to an object. Vision lines show what is in a person's line of sight, and they can be used to determine whether or not an object is visible.

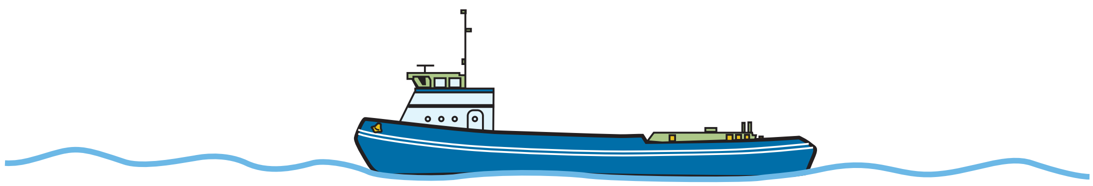
In this section, you used vision lines to discover that the Colorado River is not visible in some parts of the Grand Canyon. You also used vision lines to find the captain's blind area for ships of various sizes.

Check Your Work

These drawings on **Student Activity Sheet 3** show three different ways a ship's bridge, or steering house, can be positioned. The dot on each boat is the front of the boat.







1. **a.** Draw the vision lines to show the blind spots of the captain in each of the three cases.

   b. Measure the angle between the vision line and the horizon in each case.

   c. How does the blind spot at the back of ship change if you move the bridge forward?

Vision lines, such as the ones you drew on **Student Activity Sheet 1**, do not show everything that captains can and cannot see. For example, some ships' bridges, the area from which the captain navigates the ship, are specially constructed to improve the captain's view. The captain can walk across the bridge, from one side of the boat to the other side, to increase his or her field of vision.

Below is a photograph of a large cruise ship. Notice how the bridge, located between the arrows, has wings that project out on each side of the ship.

2. Explain how the wings of the bridge give the captain a better view of the water in front of the ship.

Hydrofoils have fins that raise the boat out of the water when it travels at high speeds.

3. Make two side-view drawings of a hydrofoil: one of the hydrofoil in the water traveling at slow speed and one of it rising out of the water and traveling at high speed. Use vision lines to show the difference between the captain's view in each drawing. (You may design your own hydrofoil.)

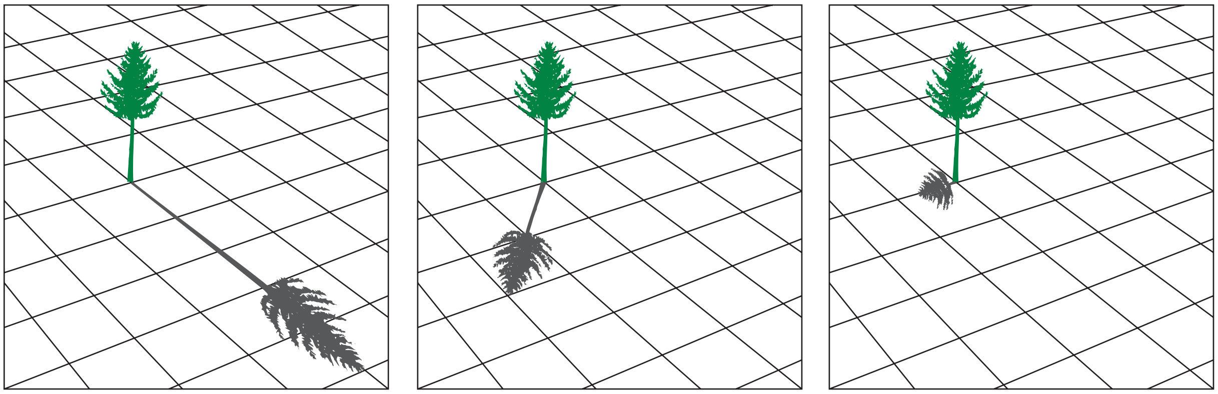
For Further Reflection

When you approach a town from afar, you sometimes see a tall tower or building. As you move closer to the town, the tall object seems to disappear. Make a drawing with vision lines to show why the tower or building seem to disappear when you get closer to town.

B Shadows and Blind Spots

Shadows and the Sun

When the sun is shining, it casts shadows. The length of the shadow varies throughout the day. Sometimes shadows are very short (when the sun is "high"), and sometimes they are very long (when the sun is rising or setting).



6:00 A.M.   9:00 A.M.   12:00 NOON

Here are three sketches of a tree and its shadow in the early morning, mid-morning, and noon.

1. Sketch how the pictures would look at 3.00 P.M. and 6.00 P.M.

The tree is two meters high. The tiles are one meter wide.

2. At what time do you think the tree's shadow will be two meters long?

The sun rises in the east.

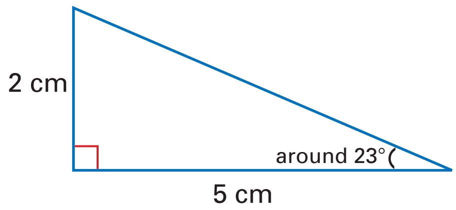
3. Indicate east, west, north, and south in your sketch.

Here is a table to organize and record information.

Time of Day	Direction of Sun	Length of Shadow	Angle of Sun's Ray
6:00 A.M.	E	5 m	?
9:00 A.M.			
12:00 P.M.			

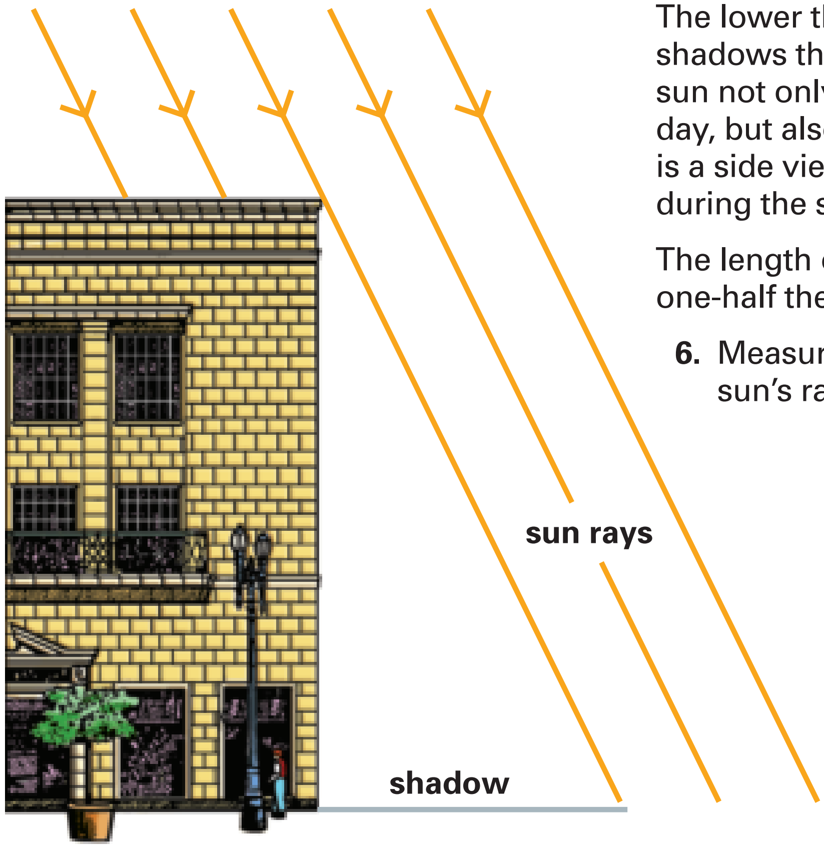
In order to find the angle of the sun's rays, you can make a scale drawing of a right triangle showing the 2-m tree and the length of the shadow. You can then use your protractor or compass card to measure the angle of the sun's ray.

Here is a scale drawing for the 6:00 A.M. picture.



4. Use the pictures on the previous page to create scale drawings for the two remaining pictures. Use this information to copy and complete the table.

5. Fill in the values for 3:00 P.M. and 6:00 P.M., assuming that the sun is at the highest point at noon.



The lower the sun is, the longer the shadows that are cast. The height of the sun not only depends on the time of the day, but also on the season. Shown here is a side view of a building around noon during the summer.

The length of the building's shadow is one-half the height of the building.

6. Measure the angle between the sun's rays and the ground.

Around noon during the winter, the length of this building's shadow is two times the height of the building.

7. a. Draw a side view of the building and its shadow around noon during the winter.

b. Measure the angle between the sun's rays and the ground.

Around noon during the spring, the angle between the sun's rays and the ground is 45°.

8. a. Draw a side view of the building and its shadow around noon during the spring.

b. If the building is 40 m tall, how long is its shadow?

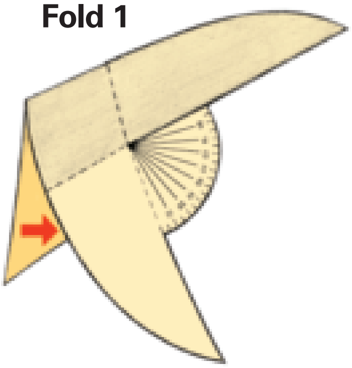
9. Describe the changes in the length of the shadow and the angle of the sun's rays from season to season.

Activity

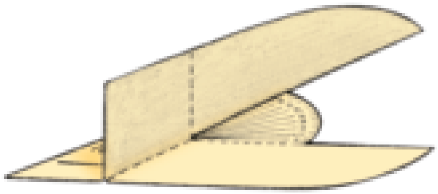
In this activity, you will investigate the shadows caused by the sun. On a sunny day, you will measure the shadow and the angle of the sun's rays.

First, you need to assemble your angle measuring tool (AMT). Cut out the figure on **Student Activity Sheet 4** along the solid lines. Make the first fold as shown here and glue the matching shaded pieces together. Continue to fold your AMT in the order shown.

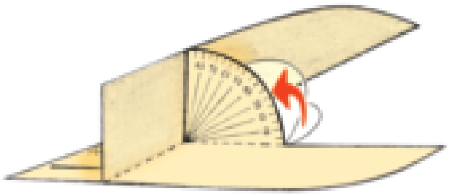
Fold 1



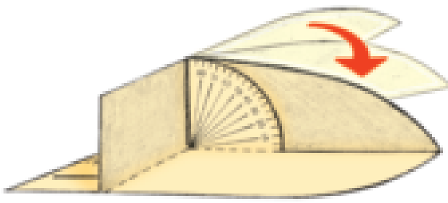
Fold 2



Fold 3

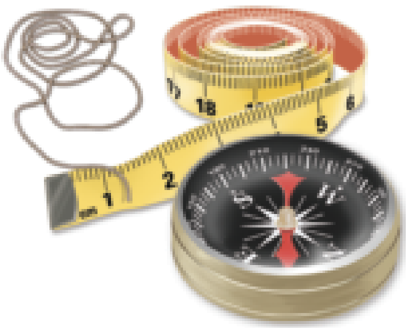


Fold 4



You will need the following items:

- a stick about 1.2 m long
- a stick about 0.7 m long
- a metric tape measure
- several meters of string
- your AMT
- a directional compass

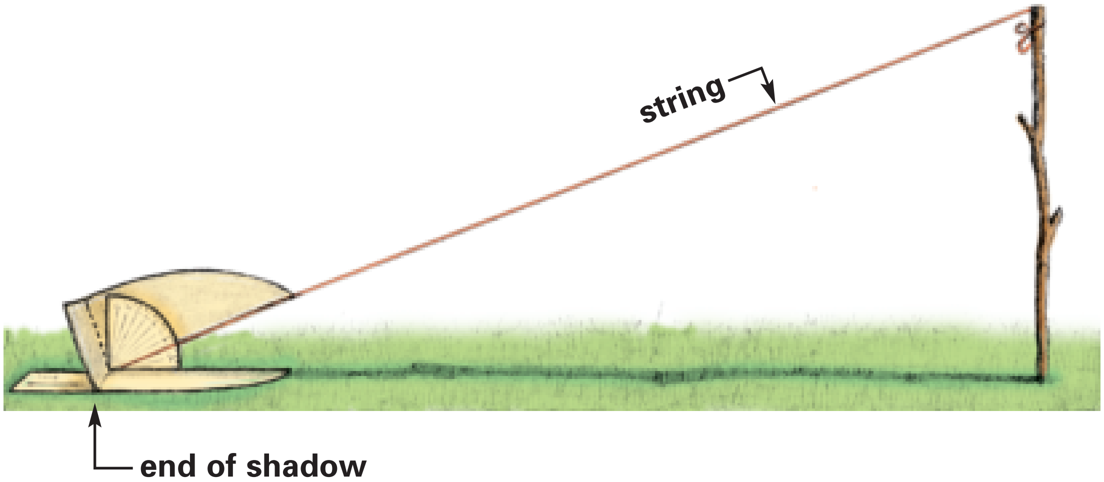


Drive both sticks into the ground about 2 m apart. The longer stick should have a height of 1 m above the ground, and the shorter stick should have a height of 0.5 m above the ground. The sticks should be perfectly vertical.

In your notebook, copy the following table. Take measurements at least five different times during the day and fill in your table. Add more blank rows to your table as needed.

		0.5-meter Stick		1-meter Stick	
Time of Day	**Direction of Sun**	**Length of Shadow (in cm)**	**Angle of Sun's Rays**	**Length of Shadow (in cm)**	**Angle of Sun's Rays**

Use the compass to determine the direction from which the sun is shining. Use the tape measure to measure the lengths of the shadows of both sticks, and use your AMT and string (as shown below) to measure the angle between the sun's rays and the ground for both sticks. Be sure to stretch the string to where the shadow ends and place your AMT there.



Use your data from the table you made in the activity on pages 16 and 17 to answer the following problems.

10. **a.** Describe the movement of the sun during the day.

b. Describe how the direction of the shadow changes throughout the day. How are the shadows related to the direction from which the sun is shining?

c. Describe the changes in the length of the shadow throughout the day. When are the shadows longest and when are they shortest?

Compare the shadows of the longer stick with the shadows of the shorter stick.

11. **a.** Describe the relationship between the length of the shadow and the height of the stick.

b. Were the shadows of the two sticks parallel at all times? Explain.

Compare the angle of the sun's rays for each stick at any moment during the day.

12. **a.** Describe how the angle of the sun's rays changed during the day. When is the angle the greatest, and when is it the smallest?

b. How is the size of the angle of the sun's rays related to the length of the shadows?

Shadows Cast by the Sun and Lights

The sun causes parallel objects to cast parallel shadows. In this photograph, for example, the bars of the railing cast parallel shadows on the sidewalk.

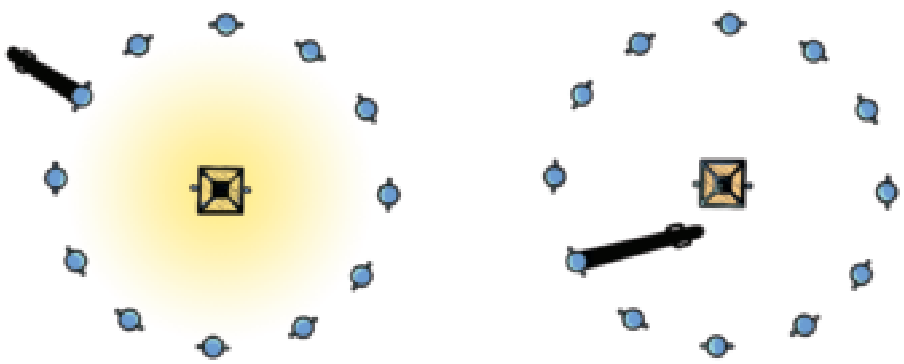
A streetlight causes a completely different picture.

13. Explain the differences between the shadows caused by the streetlight and the shadows caused by the sun and the reasons for these differences.

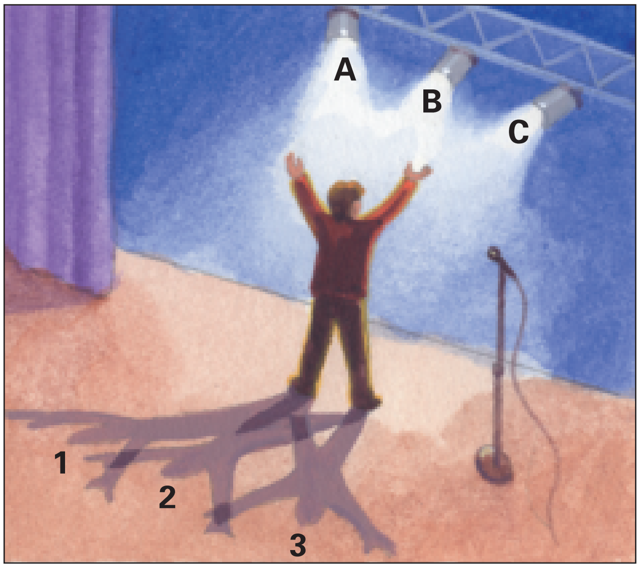
This is a picture of a streetlight surrounded by posts.

14. On **Student Activity Sheet 5**, draw in the missing shadows. It is nighttime in top view A, so the streetlight is shining. It is daytime in top view B, so the streetlight is off, and the sun is shining.

Top View A **Top View B**



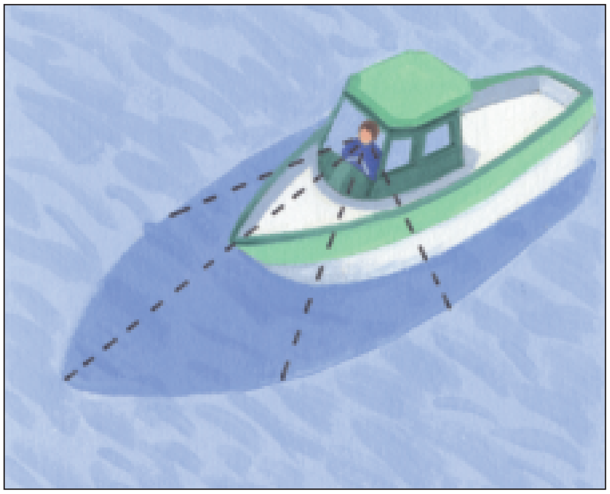
Nighttime Streetlight **Daytime Sunlight**



This is a picture of a singer on stage. Three different spotlights are used in the performance. Three shadows are formed on the stage.

15. Which light creates which shadow?

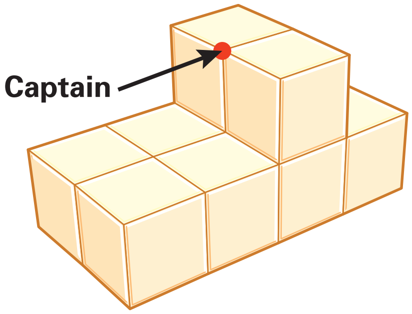
A Shadow Is a Blind Spot



Here are two boats.

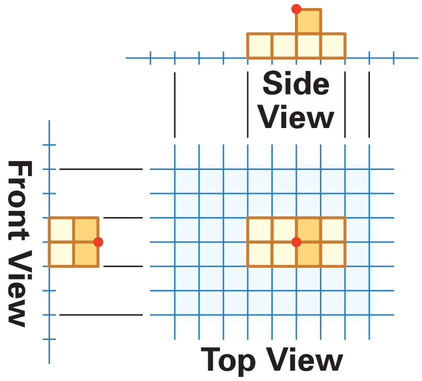
One picture shows the blind spot of the captain on the boat during the day. The other picture shows the shadow of the boat at nighttime, caused by a searchlight.

16. Explain why these pictures are almost exactly the same.

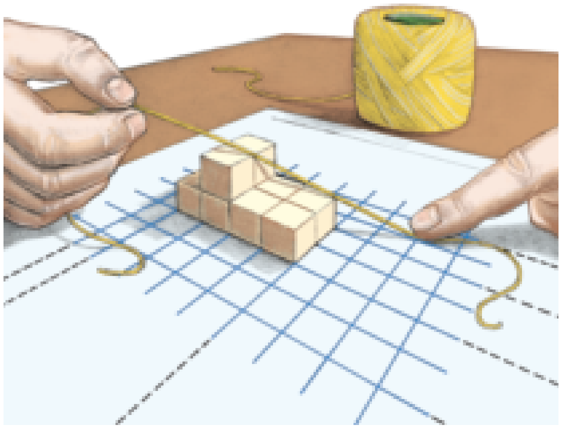


In this activity, you will investigate the blind area of a tugboat.

- Use 1-cm blocks to build a model of the tugboat.



- Place your boat on the top-view outline on **Student Activity Sheet 6**.

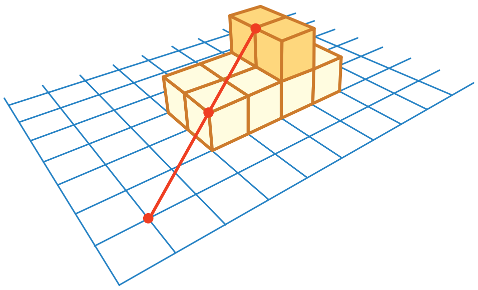


- Use string to represent the captain's vision line.

On **Student Activity Sheet 6**, draw the captain's vision lines for the side, top, and front views.

In the top view, shade the area of the graph paper that represents the blind area of the boat.

On **Student Activity Sheet 7**, draw vision lines and shade the blind area for the view shown. One vision line has already been drawn.



B Shadows and Blind Spots

Summary

Shadows can be caused by two kinds of light:

- light that is nearby, such as a streetlight;
- light that is very far away, such as the sun.

When the light comes from the sun, the rays of light are parallel, and the shadows of parallel lines are parallel.

When the light comes from a lamp, the shadows are cast in different directions. They resemble vision lines.

For that reason, shadows are similar to blind spots or blind areas.

As the sun moves, shadows will too.

A sun low in the sky casts long shadows.

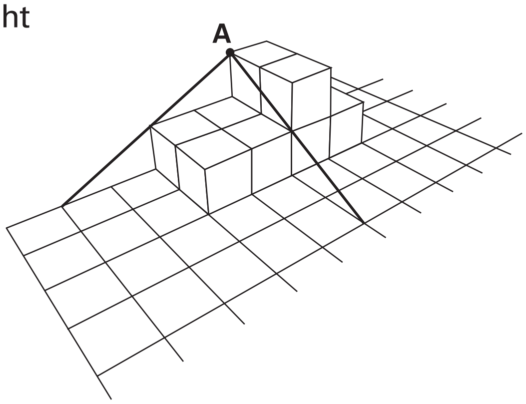
A sun high in the sky casts short shadows.

The shadows caused by the sun do not only change in length, they also change in direction. In the morning shadows will stretch toward the west.

Check Your Work

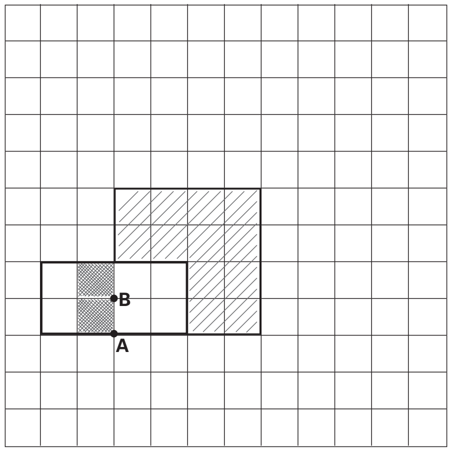
The model of a tugboat has a searchlight at point A.

In order to show the shadow caused by the searchlight, two rays of light are drawn.



1. Use **Student Activity Sheet 8** to draw and shade in the shadow of the tugboat caused by the searchlight.

Here is a top view of the same tugboat. The shadow caused by searchlight A is shaded.



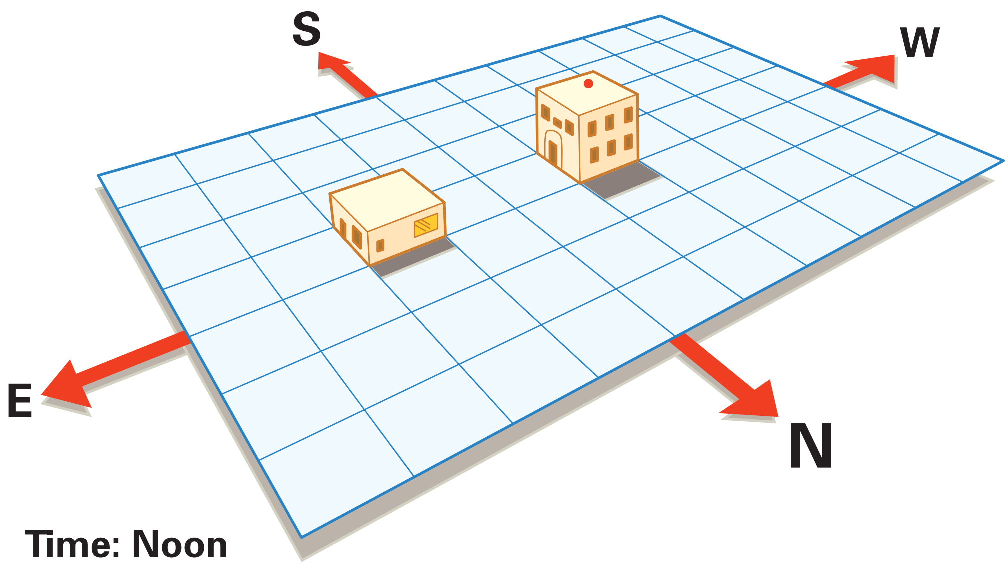
2. Check whether this shadow is correct and explain why or why not.

3. Shade in the shadow (in **Student Activity Sheet 8**) caused by searchlight B.

4. Is the blind area now smaller?

5. Where would you place the searchlight?

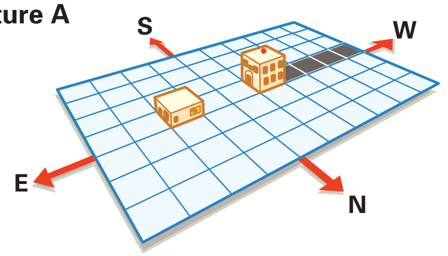
The picture here shows the shadows of two buildings at noon. The sun is shining from the south. One building is twice as tall as the other.



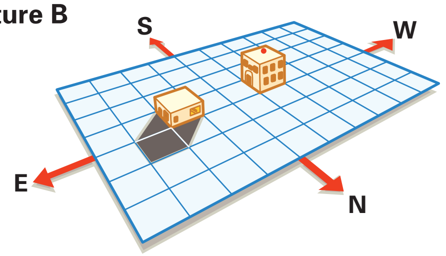
6. Study the shadows of the buildings shown here. Describe the direction and length of the shadows.

Now here are two buildings drawn at four different times of day.

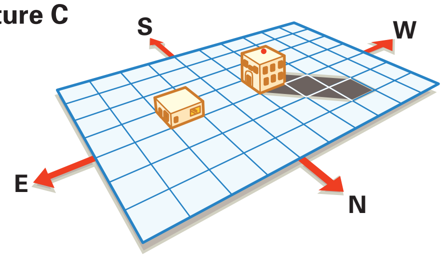
Picture A



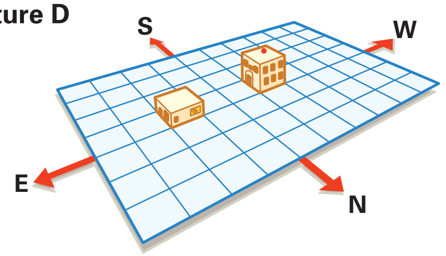
Picture B



Picture C



Picture D



7. **a.** On **Student Activity Sheet 9**, draw and shade in the shadows that are missing. Note: Picture D needs both shadows shaded in. Assume that the sun is setting in Picture D.

   b. Label each picture with an appropriate time of day.

For Further Reflection

For a classmate, explain the meaning of each phrase or word. You may use drawings for your explanation.

- vision line
- blind spot
- shadow

C

Shadows and Angles

Acoma Pueblo

The Acoma Pueblo is considered the oldest continually inhabited village in the United States. This drawing is of the Acoma Pueblo as it might have looked over 100 years ago. Located near Albuquerque, New Mexico, it is famous for its beautiful pottery and architecture. By analyzing the pottery, archaeologists have determined that this village was settled about 1,000 years ago.

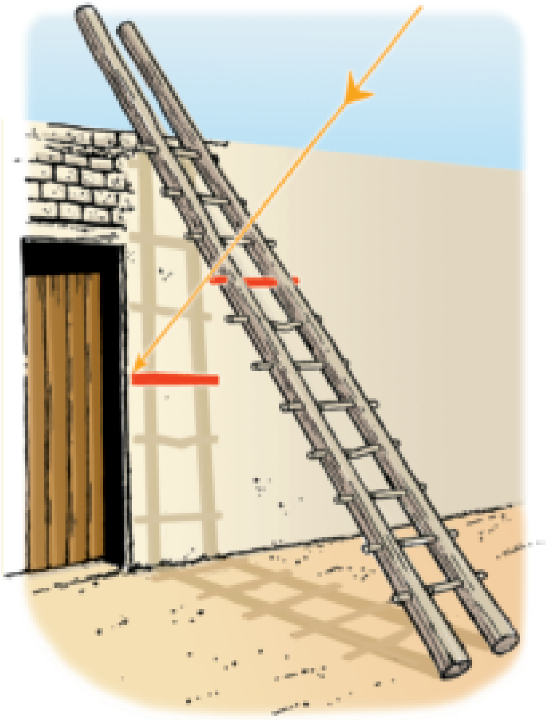
The photograph on the left shows the typical architecture of a main street of the village. This picture was taken in the morning.

1. Describe how the shadows will be different at noon.

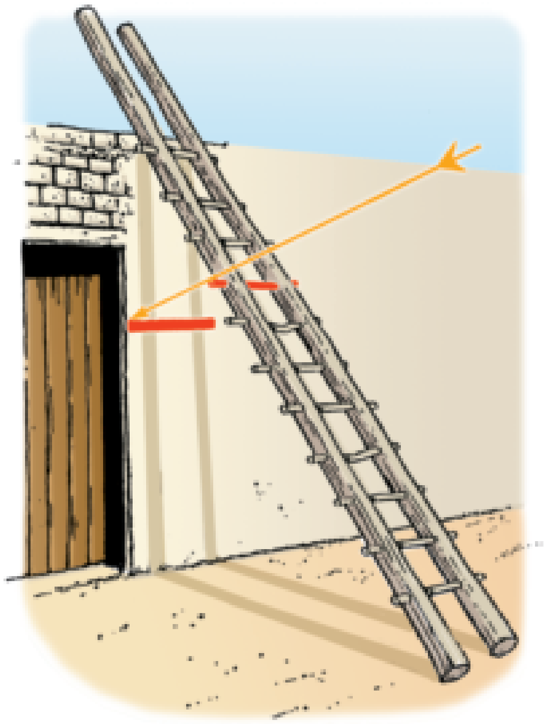
Originally, the houses in the Acoma Pueblo had no front doors; ladders were used to enter the houses on the second floor. Ladders propped against the houses formed different angles. The steepness of the ladders can be measured several ways.

Recall from Section B that the sun's rays are parallel. The drawing marked Picture A shows a ladder and its shadow. The drawing also shows how the shadow of one rung in the ladder is cast by a ray of sunlight.

Picture A



Picture B

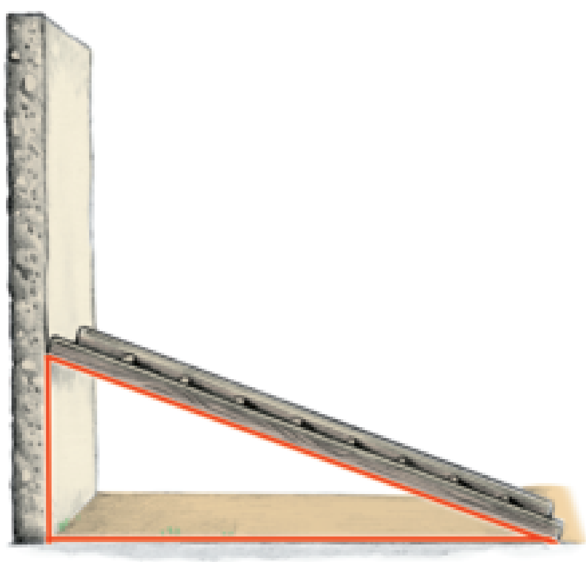


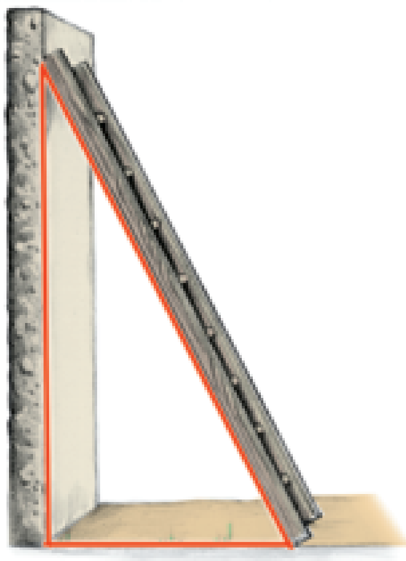
2. Use **Student Activity Sheet 10** to draw rays of sunlight that cast a shadow for each of the other ten rungs of Picture A.

The drawing marked Picture B shows the same ladder in the same position, but at a different time of day.

3. Use **Student Activity Sheet 10** to draw rays of sunlight and the corresponding shadow for each of the other ten rungs of Picture B.

Here are drawings of two side views of the same ladder leaning against a wall.



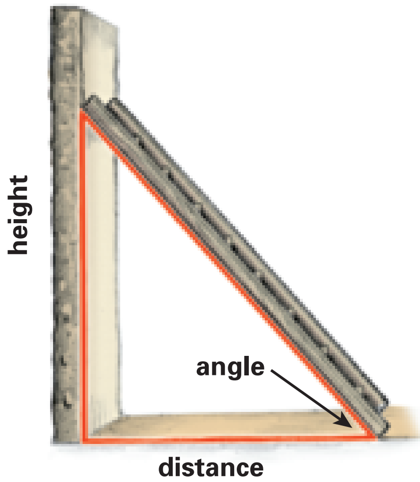


4. Describe differences between the positions of the ladder against the wall in the drawings.

5. a. What problems might occur if the ladder is very steep?
   b. What problems might occur if the ladder is not steep enough?

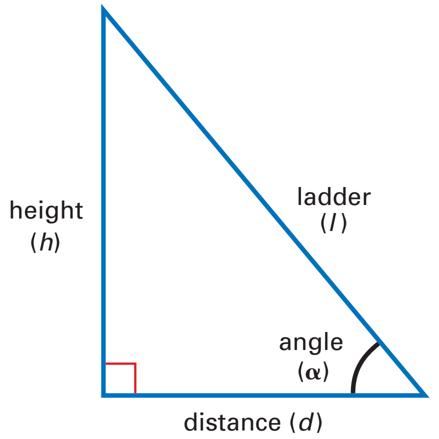
As the steepness of the ladder changes, the following measurements also change:

- the height on the wall that can be reached by the top of the ladder;
- the distance between the foot of the ladder and the wall;
- the angle between the ladder and the ground.



6. Investigate different degrees of steepness by using a ruler or pencil to represent a ladder and an upright book or box to represent a wall. Describe your discoveries. You may use drawings.

Here is a drawing of a ladder leaning against a wall. Angles are often given names. Sometimes the name of the angle is a letter of the Greek alphabet. The first letter in the Greek alphabet is α (alpha), the second letter is β (beta), and the third letter is γ (gamma).




This symbol is used to indicate a 90°, or right, angle.

7. Why must the angle between the height (h) and the distance (d) be 90°?

8. Measure angle α in the drawing.

There are several ways to measure the steepness of a ladder. You can measure angle α, or you can find the ratio of the height to the distance. The ratio of height to distance can be expressed as a ratio, a fraction, or a decimal.

9. What happens to angle α as the ratio of the height to distance increases?

10. Use a compass card or a protractor and a ruler to make side-view drawings to scale of a ladder leaning against a wall for each of the following situations. Also, label α, h, and d with their measurements, and find the height-to-distance ratio.

a. $\alpha = 45°$

b. $h = 2$, $d = 1$

c. $\alpha = 30°$

d. $h = 1$, $d = 2$

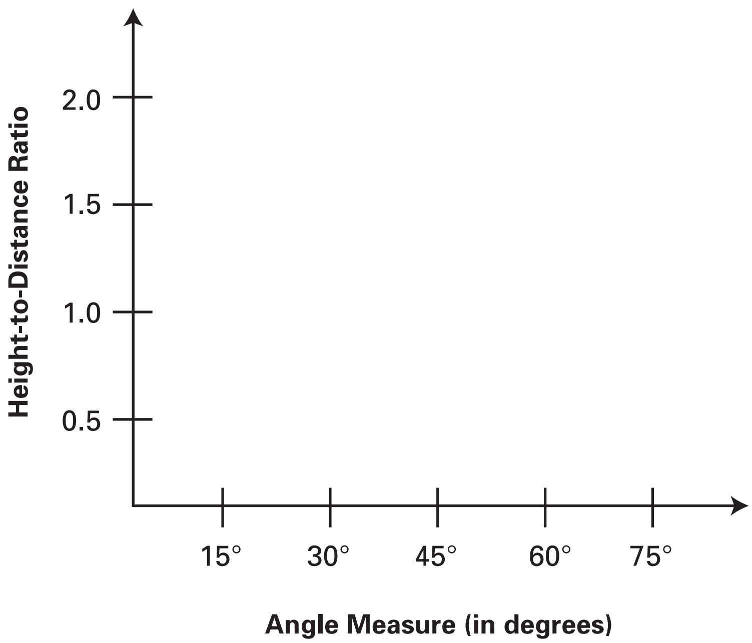
e. $\alpha = 60°$

11. Copy the following table and fill it in using your data from problem 10. Arrange your entries so that the angle measurements increase from left to right.

Steepness Table					
α (angle measure in degrees)					
$h:d$ (ratio of height to distance)					

12. Use the table from problem 11 to make a graph of the height-to-distance ratio for a ladder leaning against a wall. Label your graph as shown here.

Steepness Graph



13. Explain the information shown in your graph. Compare your graph to your answer to problem 9.

Suppose that it is safe to be on a ladder when the ratio *h:d* is greater than two and smaller than three.

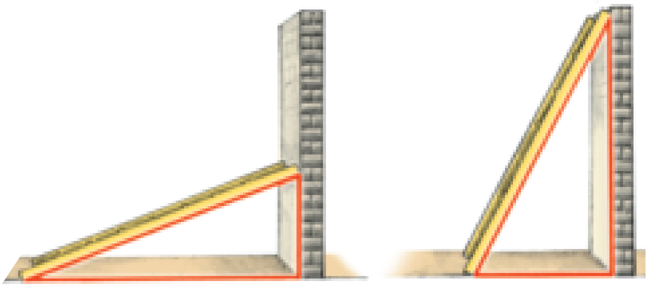
14. Give a range of angles for which a ladder can be positioned safely.

C Shadows and Angles

Summary

As the angle between a ladder and the ground increases, the height of the position of the ladder on the wall increases. At the same time, the distance between the foot of the ladder and the wall decreases.

In the same way, as the angle between a ray of sunlight and the ground increases, a shadow on the ground becomes shorter.



The steepness of a ladder can be measured in the following two ways:

- by the angle (the greater the angle, the steeper the ladder);
- by the ratio of height to distance, or *h:d* (the greater the ratio, the steeper the ladder).

Check Your Work

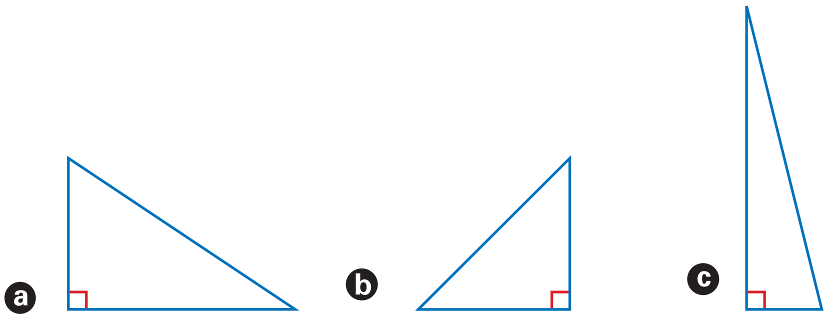
1. Use a compass card or a protractor and a ruler to make scale drawings of a ladder leaning against a wall for each of the following situations:

a. $\alpha = 60°$

b. $h = 3$, $d = 1$

c. Measure and record α from problem **b.**

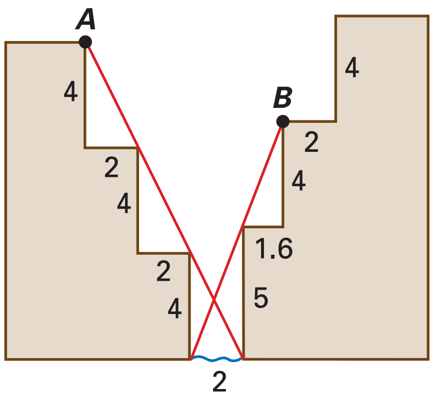
Here are three different scale drawings of right triangles, each representing a "ladder situation."



2. For each ladder situation, use the scale drawing to find α, *h, d,* and *h:d*.

Here is a drawing of a cross-section of another canyon model, like the one you worked with in Section A. The numbers indicate the scale of the height and the width of the ledges and the width of the river.

3. Which vision line is steeper, the one from point A down to the river or the one from point B down to the river? Support your answer with information about the angle between the vision line and the river and the ratio of the height to the distance.



For Further Reflection

Explain how you could use shadows to estimate the height of a tower.

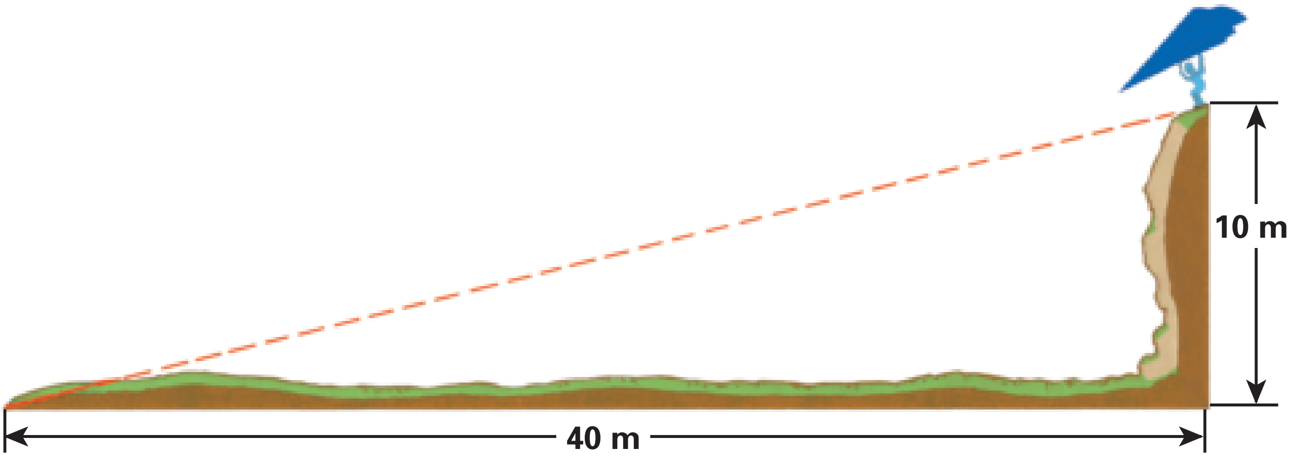
D Glide Angles

Hang Gliders

Hang gliders are light, kite-like gliders that carry a pilot in a harness. The pilot takes off from a hill or a cliff into the wind. The hang glider then slowly descends to the ground.

When pilots make their first flight with a new glider, they are very careful because they do not know how quickly the glider will descend.

Marianne, the pilot in this picture, decides to make her first jump from a 10-m cliff. She glides along a straight line, covering 40 m of ground as shown in the drawing.



After several successful flights, she decides to go to a higher cliff. This cliff is 15 m high.

1. How much ground distance does the glider cover from the higher cliff? Note: Assume that the steepness of the flight path remains the same.

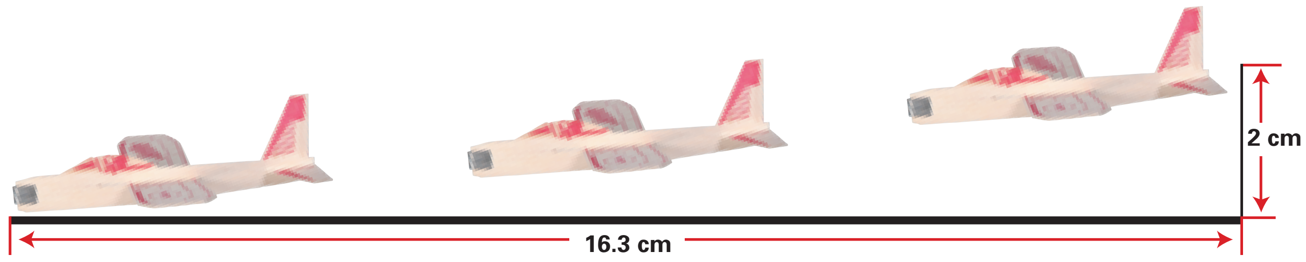
2. Marianne makes flights from three cliffs that are 20 m, 50 m, and 100 m high. How much ground distance does the glider cover on each flight?

Marianne has designed a glider that can travel farther than her first one. With the new glider, Marianne claims, "When I jump from a 10-m cliff, I can cover 70 m of ground!"

3. a. Draw a side view of Marianne's flight path with the new glider.

b. Copy the table below and complete it for the new glider.

Height (in m)	10	25	100			
Distance (in m)	70			245		1,000



This picture is based on three separate photographs, taken one after the other. It shows a model glider that is used in laboratory experiments. By taking three pictures within a short period of time, you can determine the path of the glider.

4. In your notebook, trace the path of this glider and make a scale drawing similar to the drawing on top of page 33. Use your scale drawing to answer the following questions.

a. If the glider in the picture is launched from a height of 5 m, how far will the glider fly before landing?

b. How far will the glider fly from a 10-m cliff?

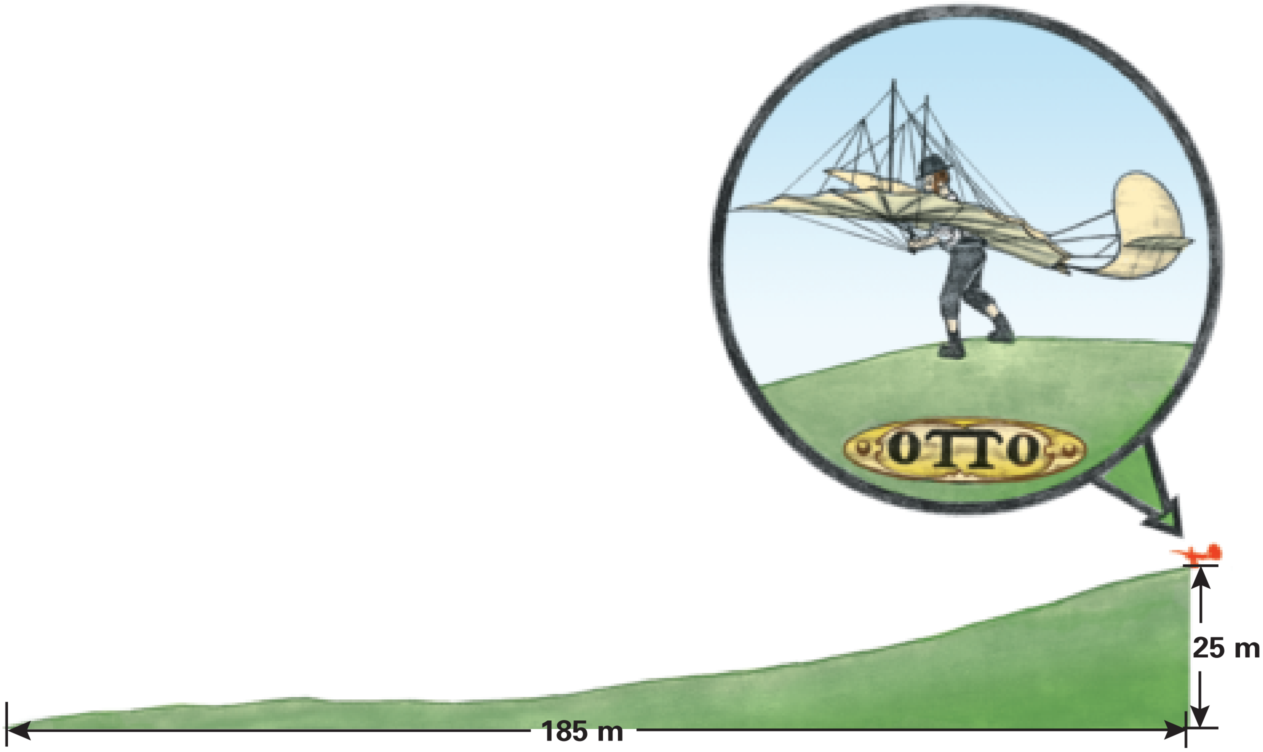
c. Compare the distances covered by Marianne's two hang gliders and this model glider. If all three are launched from 10 m, which one flies the farthest? Explain.

Glide Ratio

To determine which hang glider travels farther, you can consider the **glide ratio**. Marianne's first glider flew 40 m from a 10-m cliff. This glider has a glide ratio of 1:4 (one to four). Marianne's second glider flew 70 m from a 10-m cliff.

The second glider has a glide ratio of 1:7.

5. What do you think a glide ratio is?



Otto Lilienthal made more than 2,000 flights with hang gliders at the end of the 19th century. Suppose that on one of his flights from the Rhinower Hills near Berlin, Germany, he started from a height of 25 m and covered 185 m of ground distance as shown here. On his next flight, suppose he redesigned his glider a little, started from a height of 20 m, and traveled a ground distance of 155 m.

6. What were the glide ratios of Otto's two gliders? Which glider could travel farther?

7. Suppose that a glider has a glide ratio of 1:8. It takes off from a cliff and covers 120 m of ground distance. How high is the cliff?

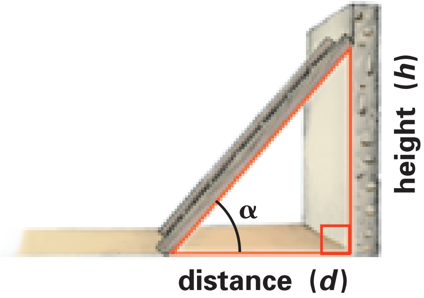
8. Make scale drawings to represent the following glide ratios.

a. 1:1 **b.** 1:2 **c.** 1:4

d. 1:10 **e.** 1:20

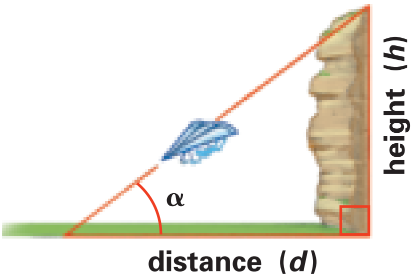
D Glide Angles

In Section C when you studied ladders at different angles, you made a table similar to the one below, showing the angle between the ladder and the ground and the ratio of the height to the distance.



Ladder Steepness					
α	27°	30°	45°	60°	63°
h:*d*	0.5	0.6	1	1.7	2

You can organize your information about the steepness of the glide path of a hang glider with a similar table. The angle that the hang glider makes with the ground as it descends is called a **glide angle**.



Glide Path Steepness					
Glide Angle α					
Glide Ratio *h*:*d*	1:1	1:2	1:4	1:10	1:20

9. Copy this table in your notebook. Fill in the missing glide angles by using the scale drawings you made for problem 8. Measure the angles using a compass card or protractor.

Glide ratios can also be expressed as fractions or decimals.

10. Which of the following glide ratios are equivalent?

1:25	$\frac{4}{100}$	1:20	3:75	$\frac{1}{20}$
$\frac{1}{30}$	2:40	0.04	$\frac{1}{25}$	4:100
0.05	$\frac{1}{4}$	0.20	4:120	$\frac{2}{50}$

Suppose that it is safe to fly gliders that have a glide ratio smaller than 1:10.

11. What is the largest glide angle that is safe?

Suppose three gliders have the following glide ratios.

- Glider 1: 1:27
- Glider 2: 0.04
- Glider 3: $\frac{3}{78}$

12. Which glider is the safest? Explain.

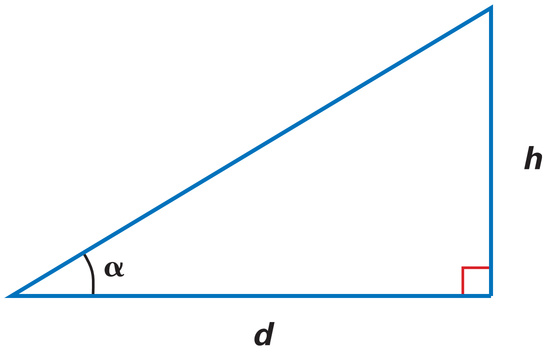
From Glide Ratio to Tangent

The relationship between the glide ratio and the glide angle is very important in hang gliding, as well as in other applications, such as the placement of a ladder. For this reason, there are several ways to express this ratio and angle.

glide ratio = $h{:}d$

glide angle = α

The ratio $h{:}d$ is also called the **tangent** of angle α, or $\tan \alpha = \frac{h}{d}$.



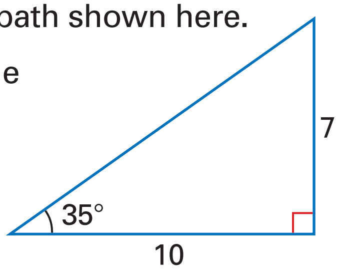
For a glide ratio of 1:1, the glide angle is 45°, so $\tan 45° = \frac{1}{1} = 1$.

Suppose that another one of Otto's hang gliders has a glide ratio of 1:7. This means that the tangent of the glide angle is 1 to 7 (or $\frac{1}{7}$).

13. Describe in your own words the relationship between the glide ratio, glide angle, and tangent.

D Glide Angles

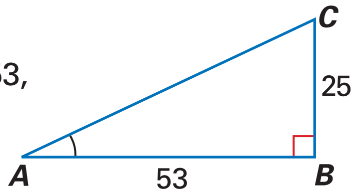
Suppose that a glider follows the flight path shown here.



From this information, you know that the glide ratio of a 35°angle is 0.7 (or 7:10). You can write this information in the following way.

$\tan 35° = \frac{7}{10} = 0.7$

Here is a different glider situation.

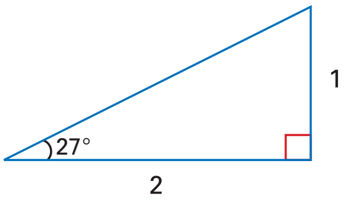


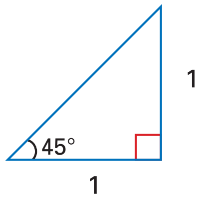
You know the tangent of angle *A* is 25:53, or 0.47.

You can write this information in the following way.

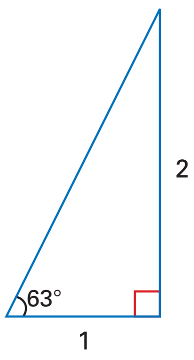
$\tan A = \frac{25}{53} = 0.47$

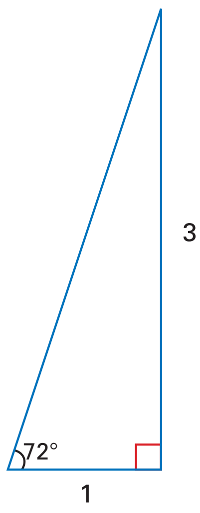
14. Complete the statements below for each of the following right triangles.





a. tan __?°__ = __?__ **b.** tan __?°__ = __?__





c. tan __?°__ = __?__ **d.** tan __?°__ = __?__

Peter wants to buy a balsa wood model glider for his nephew, but he is not sure which one to buy. The salesperson at the hobby store claims, "The smaller the tangent of the glide angle, the better the glider."

15. Is the salesperson correct? Explain.

16. Suppose for triangle *ABC*, the measure of angle *B* is 90° and $\tan A = \frac{3}{5}$.

a. Make a scale drawing of triangle *ABC*.

b. Suppose you drew triangle *ABC* so that side *AB* measures 10 cm. What is the length of side *BC*?

c. What is the measure of angle *A* in triangle *ABC*? Is it the same size in both of the triangles drawn?

The following table lists some angles and the approximate measurements of their tangents.

Angle (in degrees)	0°	1°	2°	3°	4°	5°		31°	32°	33°	34°	35°
Tangent of Angle (as a decimal)	0	0.02	0.04	0.05	0.07	0.09		0.60	0.63	0.65	0.68	0.70

Use the table to answer the following problems.

17. a. Draw a side view of the flight path for a glider whose glide angle is 5°.

b. What is the glide ratio for this glider?

18. If the glide angle is 35°, how much ground distance does a glider cover from a height of 100 m?

19. If a ladder makes an 80° angle with the ground, what can you determine about the position of the ladder if you know tan 80° = 5.7?

The table in Appendix A shows the relationship between the size of an angle and its tangent value. You can also use a scientific calculator to find the tangent of an angle. Since calculators differ, you may want to investigate how to use the tangent key on your calculator. You can use the table in the appendix to verify your work.

You can also use a scientific calculator to find angle measurements if you know the tangent ratio.

Use either the table or the tangent key on your scientific calculator to answer the following problems.

20. What do you know about a glider with a glide angle of 4°? A glide angle of 35°?

21. Explain why tan 45° = 1.

22. Which angle has a tangent value of 2? Of 3? Of 4?

23. How much does the measurement of the angle change when the tangent value changes in these ways?

a. From 0 to 1

b. From 1 to 2

c. From 2 to 3

d. From 3 to 4

e. From 4 to 5

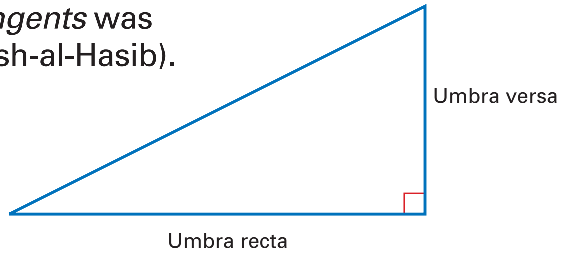
Math History

Shadows and Gliders

Shadow reckoning was an early device for finding heights. If you see the sun's rays under an angle of 45°, you can measure the shadow of a tower to know its height.

Around 400 B.C., the Hindus understood the use of shadows to measure heights. Albategnius made the first *tables of shadows* around 920 A.D.

Around 860, the first table of *tangents* was constructed by the Arabs (Habash-al-Hasib). Arab writers described the straight shadow *umbra recta* (horizontal distance) and the turned shadow or *umbra versa* (vertical distance).



As soon as the first glider planes were invented, their performances were compared by using glide ratios. The German aviator Otto Lilienthal (around 1890) made a kind of hang glider with glide ratios of around 1 to 9, which is also the glide ratio of NASA's space shuttle.

There is always confusion about glide ratios: In Europe, people identify 1:40 as the glide ratio of a good contemporary sailplane (sometimes even 1:60!), but in the United States you can often see 42:1 or even just 42.

Gliding is also a common adaptation in mammals. In a scientific article by Brian Stafford and others, the glide ratios of Japan's flying squirrels are compared. The scientists conclude that the glide paths of these animals vary greatly. The best gliders among those squirrels have a glide ratio close to 1:3.5. Not bad! The worst flyers barely reach 1:1. Hard landings can be expected.

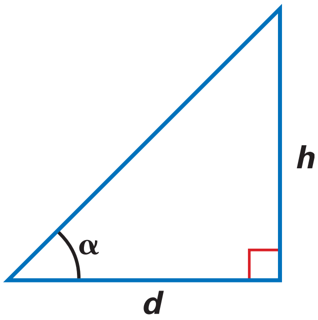
D Glide Angles

Summary

The steepness of a ladder, the angle of the sun's rays, and the flight path of a hang glider can all be modeled by a right triangle such as the one here.

Steepness can be measured as the angle α or as the ratio $h:d$.

The ratio $h:d$ is also called the tangent of angle α, or $\tan \alpha = \frac{h}{d}$.



Check Your Work

On a calm day, a glider pilot wants to make a flight that covers 120 km. The glider has a glide ratio of 1:40.

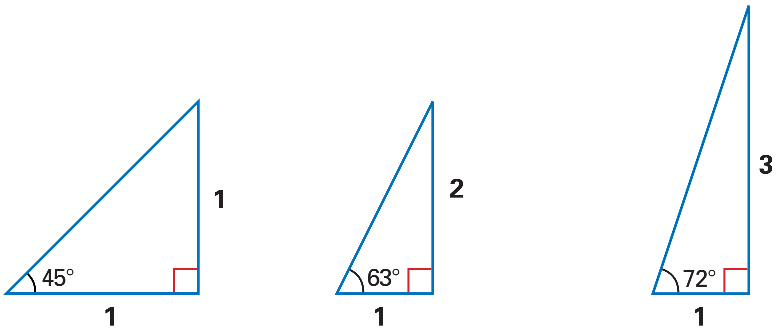
1. From what height does the glider have to be launched?

Source: © 1997, Encyclopædia Britannica, Inc.

A glider with a glide ratio of 1:28 is launched after being pulled by an airplane to 1,200 m above Lake Havasu City in Arizona.

2. Indicate on the map on **Student Activity Sheet 11** how far the glider can fly if there is no wind.

Here are some of the right triangles you worked with in problem 14.



You used tangent notation to describe each situation:

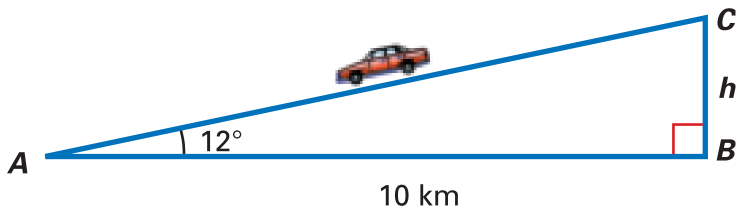
a. tan 45° = 1 **b.** tan 63° = 2 **c.** tan 72° = 3

3. **a.** Suppose these right triangles were ladder situations. What conclusions can you make about these ladders?

b. What can you conclude about the length of a ladder described by tan 86° = 14?

D Glide Angles

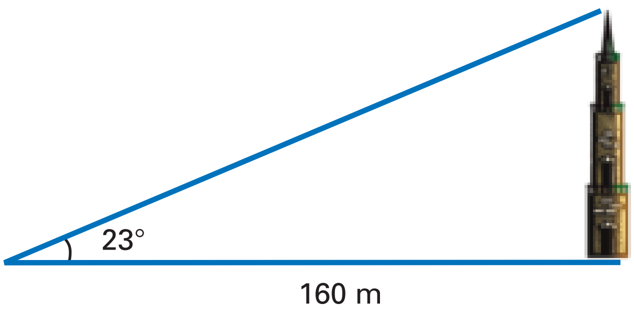
A road has been constructed with a fairly large angle between the road and the horizontal plane.



4. **a.** Use the tangent table or your calculator to find tan 12°.

   b. Use this to compute height *h.*

At a distance of 160 m from a tower, you look up at an angle of 23° and see the top of the tower.



5. What is the height of the tower? Hint: Use tan 23°.

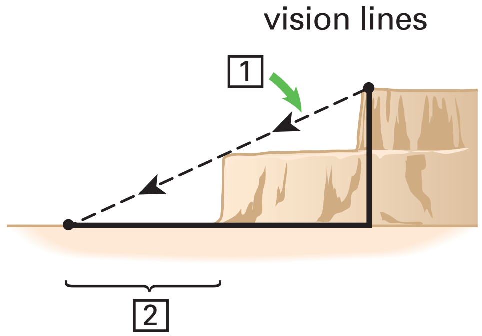
For Further Reflection

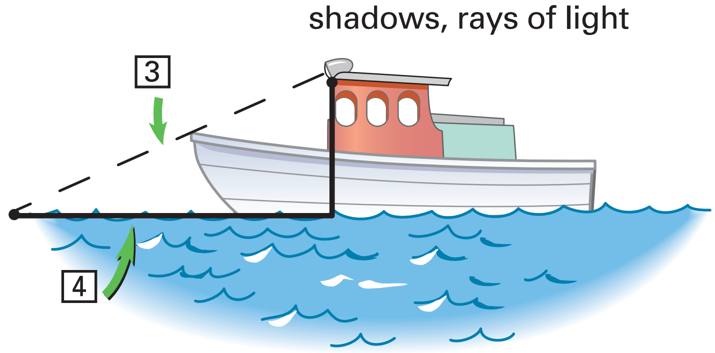
Write a short paragraph about the similarities you see between the situations involving ladders and hang gliders. In your description, use the terms *steepness, height-to-distance ratio*, and *angle*.

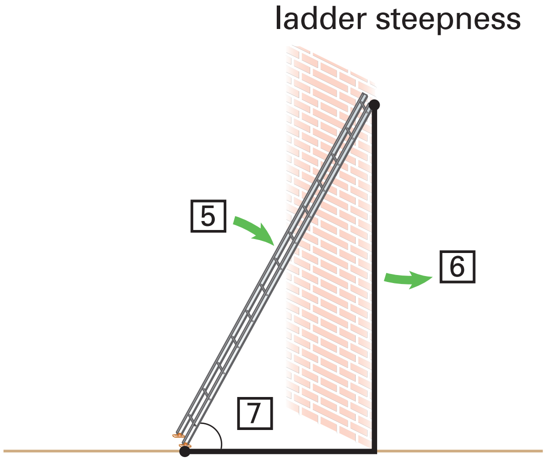
E Reasoning with Ratios

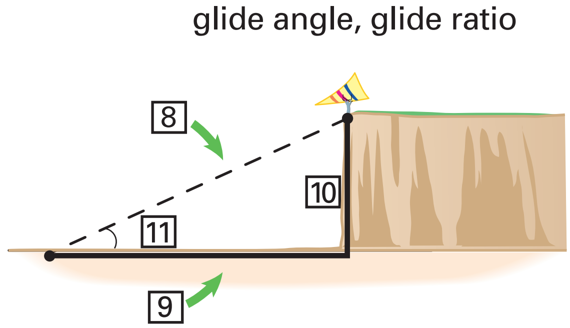
Tangent Ratio

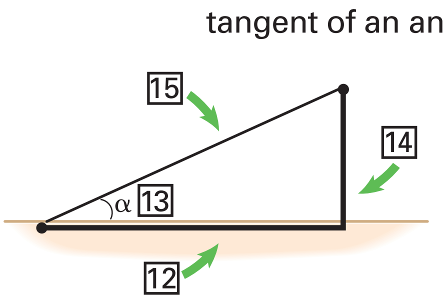
So far, you have worked with situations like these.









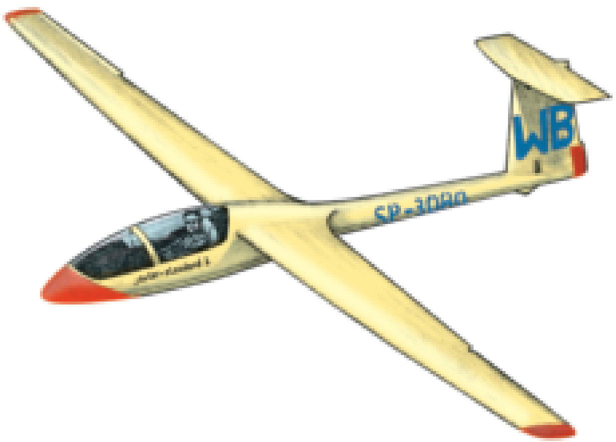


1. The elements of these situations are labeled with numbers from 1 to 15. Name each element. For example, a line labeled with a number might be a ray of light or a flight path.

All the situations on the previous page have a right triangle in common. In many situations, the ratio of the height to distance plays an important role, tan $\alpha = \frac{h}{d}$.

This ratio is a measurement for *steepness*, as is the angle. A small angle corresponds to low steepness.

Vultures Versus Gliders



While searching for food, vultures use updraft hot air currents, or thermals, to gain altitude. They rise up circling and after reaching a certain altitude, glide down to catch a new thermal column. Vultures look for thermals so that they can gain altitude without expending energy. Like vultures, gliders also rely on thermals.

Vultures and gliders can be compared in terms of their glide ratios.

For the vulture, the glide ratio is 1:10, or tan $\alpha = 0.1$.

For the glider, tan $\alpha = 0.03$.

2. **a.** Who is the better "glider?"

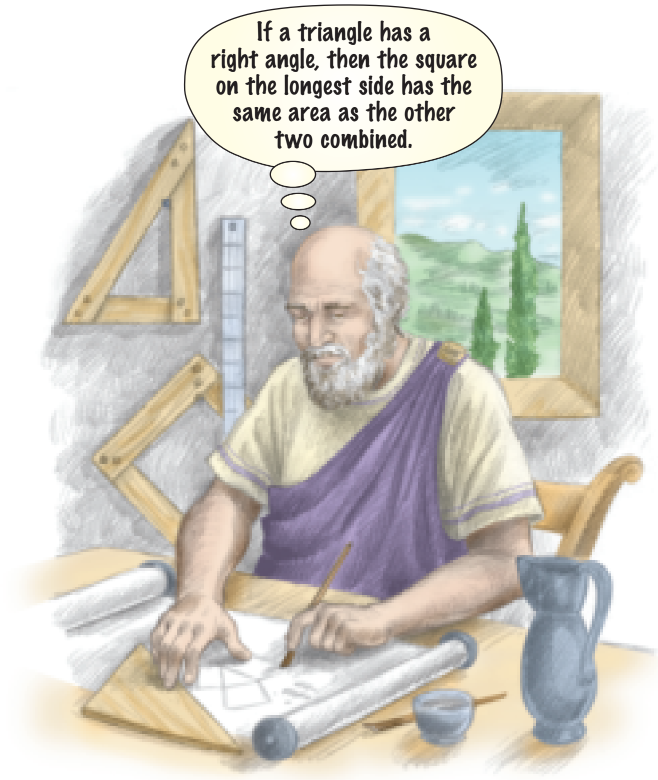
b. What distance can each fly, starting from a vertical height of 1 km?

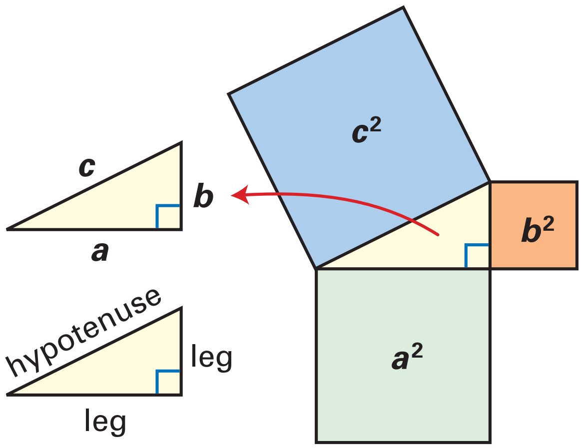
c. What is the size of the two glide angles?

d. Which flight path is steeper?

Pythagoras

You may remember a theorem that is closely connected to right triangles. It is called the **Pythagorean theorem**. Pythagoras was born in Samos, Ionia, in the sixth century B.C. The Pythagorean theorem is used to calculate the length of any one side of a right triangle if the lengths of the other two sides are known.

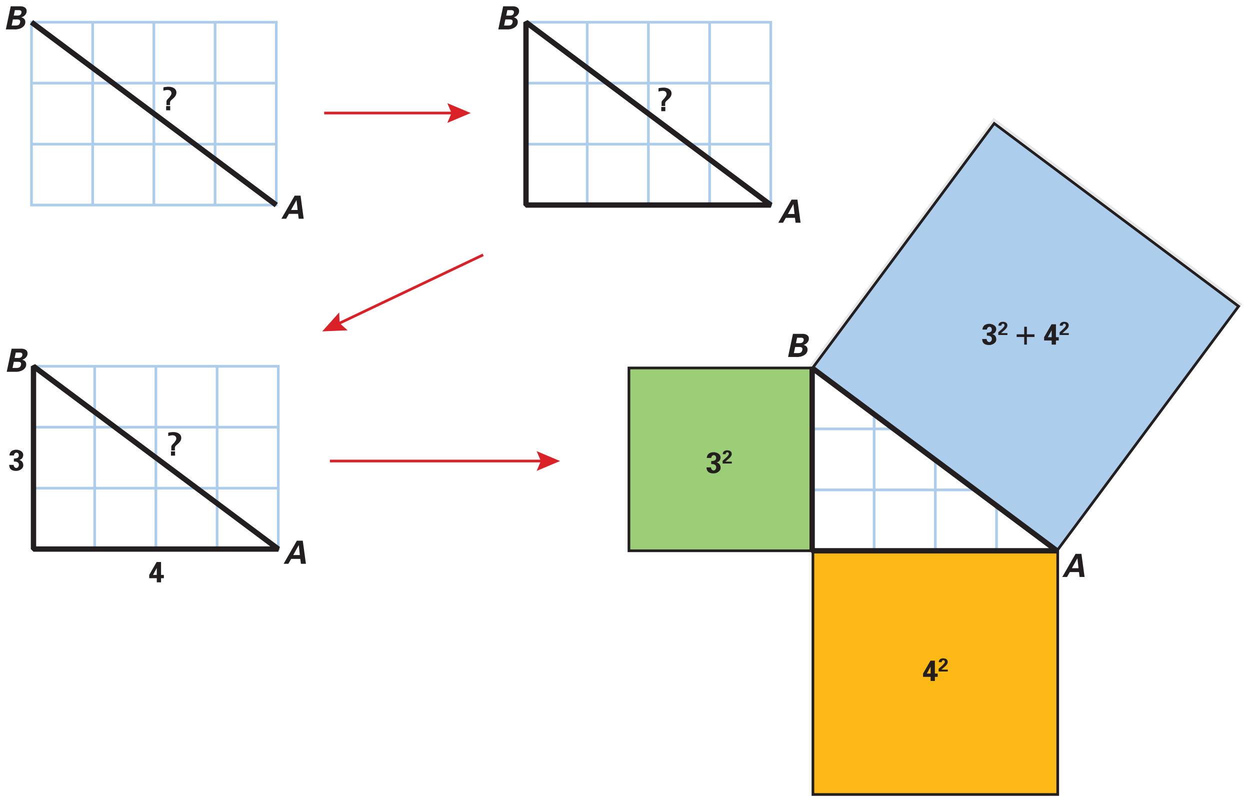




You can write the Pythagorean theorem as an equation, $a^2 + b^2 = c^2$, where *a* and *b* represent the two short sides, called **legs**, of a right triangle, and *c* represents the longest side, called the **hypotenuse**. The hypotenuse is always the side opposite the right angle.

If you know the measurements of all three sides of a triangle, you can use the Pythagorean theorem to find out whether the triangle is a right triangle. If $a^2 + b^2 \neq c^2$, then the triangle is not a right triangle.

The Pythagorean theorem is used to find distances. Look carefully at the following picture story.

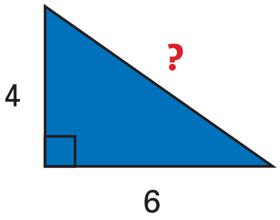


3. **a.** Explain the picture story.

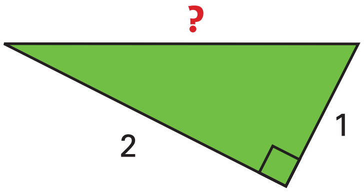
b. What is the length of segment *AB*?

4. Find the missing distances in the triangles below. Show your work.

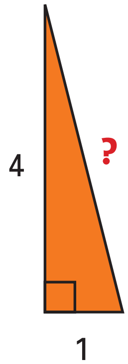
a.



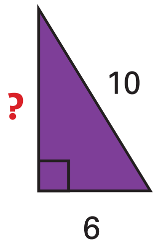
b.



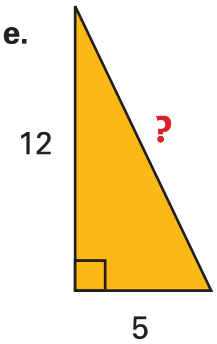
c.



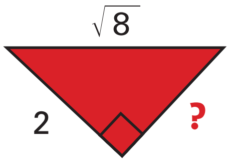
d.

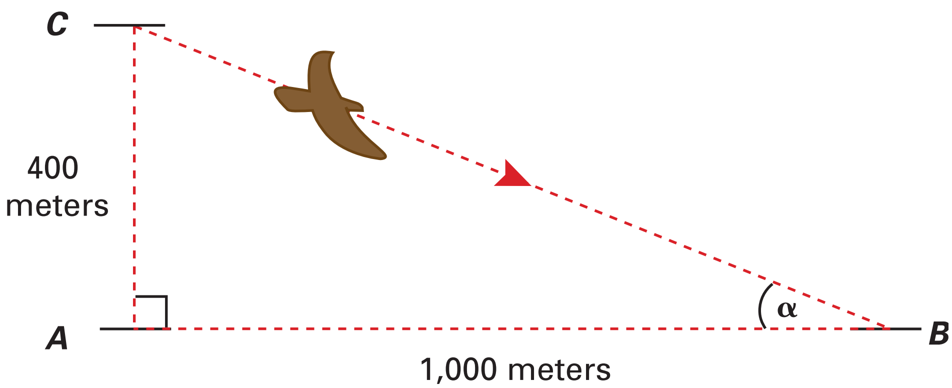


e.



f.





The vulture does not have to fly at a glide ratio of 1:10. It can glide at much steeper angles, especially when diving for food.

5. a. What is the vulture's glide ratio in the picture above?

b. Use your calculator to find the size of the glide angle.

c. Use the Pythagorean theorem to calculate the length of the vulture's flight path (*BC*).

If the glide angle is very small, lengths *AB* and *BC* will be like they are in problem 5. What happens if the glide angle is greater?

Consider a right triangle with an angle of 40°.

6. a. Use a ruler and a protractor to draw a right triangle *ABC*. Angle *A* is the right angle, angle *B* = 40°, and the length of *AB* = 5 cm. *AB* represents a length of 1,000 m in reality.

b. Use your calculator or the appendix to find tan 40° and calculate the length of *AC* in the drawing. Round the answer to whole centimeters.

c. Find the length of *BC*. Are *AC* and *BC* about the same length in the drawing? In reality?

Nicole wants to draw a right triangle *ABC* in which angle *A* is the right angle and angle *B* = 80°.

d. Explain why it is not possible to make this drawing in your notebook. What is the length of *BC* in this triangle? In reality?

e. What is your conclusion about the lengths of *AC* and *BC* if the size of the glide angle increases?

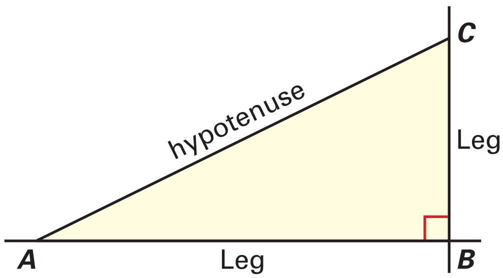
Directions for painters state that a ladder is placed safely against a wall if the angle with the ground is about 70°. Consider a ladder that is 10 m long.

7. a. Make a drawing to scale of this ladder where it is placed safely against the wall. Show what you did to make the drawing.

b. About how far from the wall is this 10-m ladder?

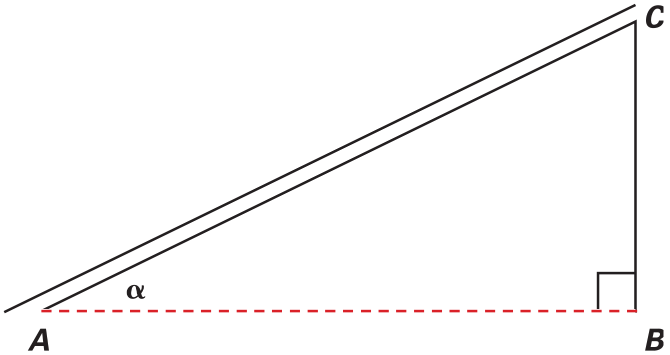
The Ratios: Tangent, Sine, Cosine

In right triangles, we have three sides: the hypotenuse (the ladder) and the legs of the right triangle (the height and distance) that help define the glide ratio.



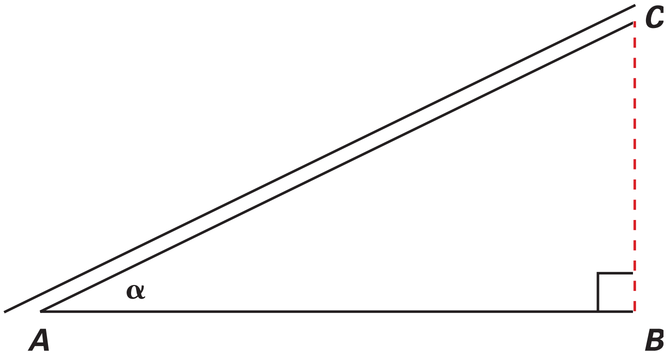
In addition to the tangent ratio, there are two other ratios which describe a relationship between the sides of a right triangle.

One is the **sine ratio**, abbreviated sin α.

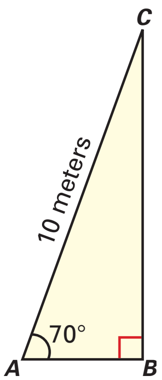


$$\sin \alpha = \frac{\textit{side opposite } \alpha}{\textit{hypotenuse}} = \frac{BC}{AC}$$

The other is the **cosine ratio**, abbreviated cos α.



$$\cos \alpha = \frac{\textit{side adjacent to } \alpha}{\textit{hypotenuse}} = \frac{AB}{AC}$$



Notice that for both the sine and the cosine ratios, the hypotenuse is the "denominator."

Refer back to your scale drawing of the ladder situation in problem 7 where angle $A = 70°$ and the length of the ladder is 10 m.

$$\cos 70° = \frac{\textit{side adjacent to } \alpha}{\textit{hypotenuse}} = \frac{AB}{AC} = \frac{\textit{horizontal distance of the ladder to the wall}}{\textit{length of the ladder}}$$

8. a. Find the value of cos 70°. Use the table from the appendix or your calculator.

b. Use the cosine value to find the distance of the ladder to the wall. Show your calculation.

c. How high does this ladder reach on the wall? Use the sin 70° to make a calculation.

Answer the following questions. You can either use drawings, the table from the appendix, or your calculator.

9. Complete each ending.

If the angle is small,

a. the tangent ratio is …

b. the sine ratio is …

c. the cosine ratio is …

10. The tangent ratio can reach any positive value.

True or not true?

11. The sine ratio can reach any value.

True or not true?

12. The cosine ratio can never exceed 1.

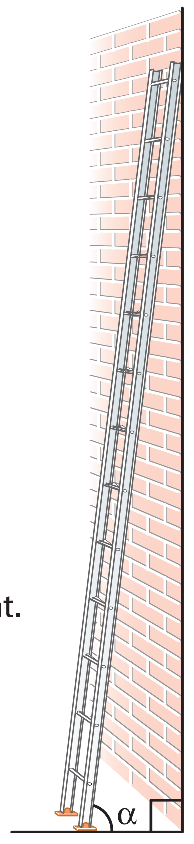
True or not true?

A ladder is placed very steeply against a wall:

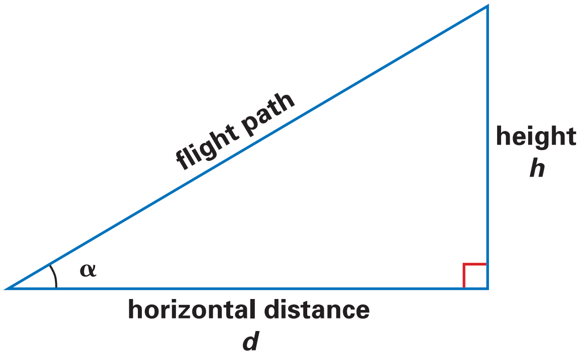
13. a. Explain why sin α is very close to 1 in this situation.

b. Explain why sin α is always smaller than 1.

c. Explain why tan α can be as large as you want.



Glide angle, tangent of the glide angle, glide ratio, and slope are all measurements for the steepness of the flight path.



α = glide angle

tan $\alpha = \frac{h}{d}$

glide ratio = h:d or $\frac{h}{d}$

slope = h:d or $\frac{h}{d}$

This table compares several flight paths with varying degrees of steepness.

Flight Path	Height (in km)	Distance (in km)	Slope $\left(\frac{h}{d}\right)$	α
1	6	11		
2		8	0.5	
3	4		$\frac{2}{5}$	
4		4.5		20°

14. **a.** In your notebook, copy and complete the table.

b. List the flight paths in order from the steepest to the least steep.

c. Find the length of each flight path. Use the sine ratio, the cosine ratio, and the Pythagorean theorem at least one time each when you calculate the lengths.

E Reasoning with Ratios

Summary

In this unit, many different situations play a role—vision lines, shadows and light rays, steepness of ladders, glide paths, glide angle, and glide ratios. All situations involve a right triangle, which plays a vital role.

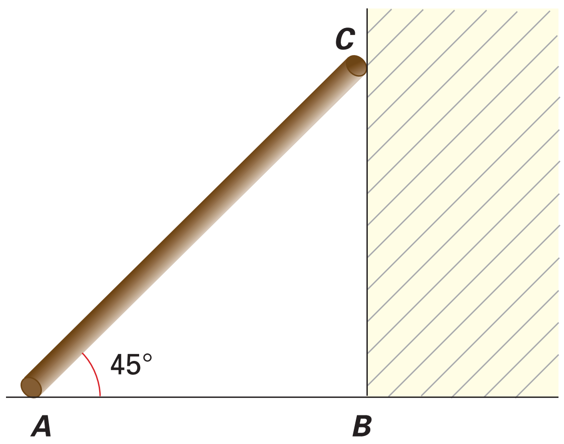
The steepness of a ladder and the glide ratio can be expressed with the mathematical ratio called the tangent ratio.

Right triangles were explored somewhat further.

- You made use of the Pythagorean theorem to find an unknown side's length.
- You investigated two new ratios, the sine ratio and cosine ratio.

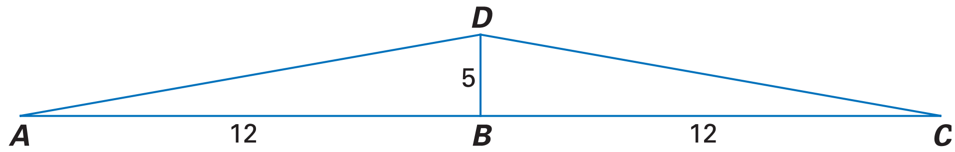
Check Your Work

A pole 7 m long is placed against a wall at an angle of 45°.



1. How high is *BC*?

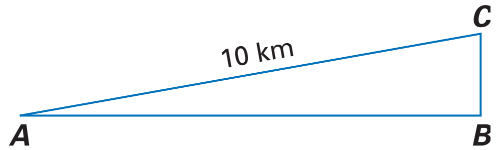
A roof is constructed according to this diagram:



2. a. How long should AD and DC be?

   b. How large is angle A?

The road AC is 10 km long and angle $A = 10°$.



3. What is height BC at the end of the road?

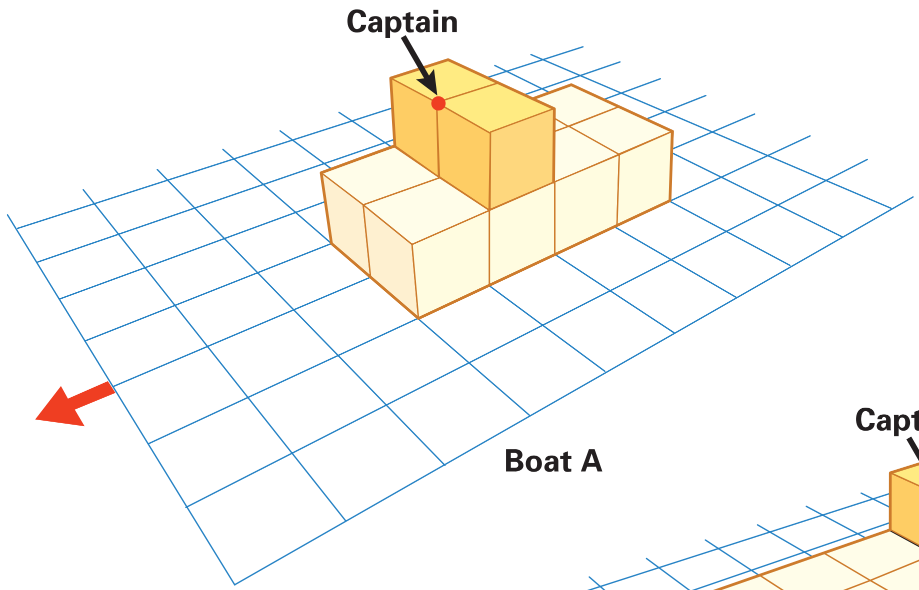
For Further Reflection

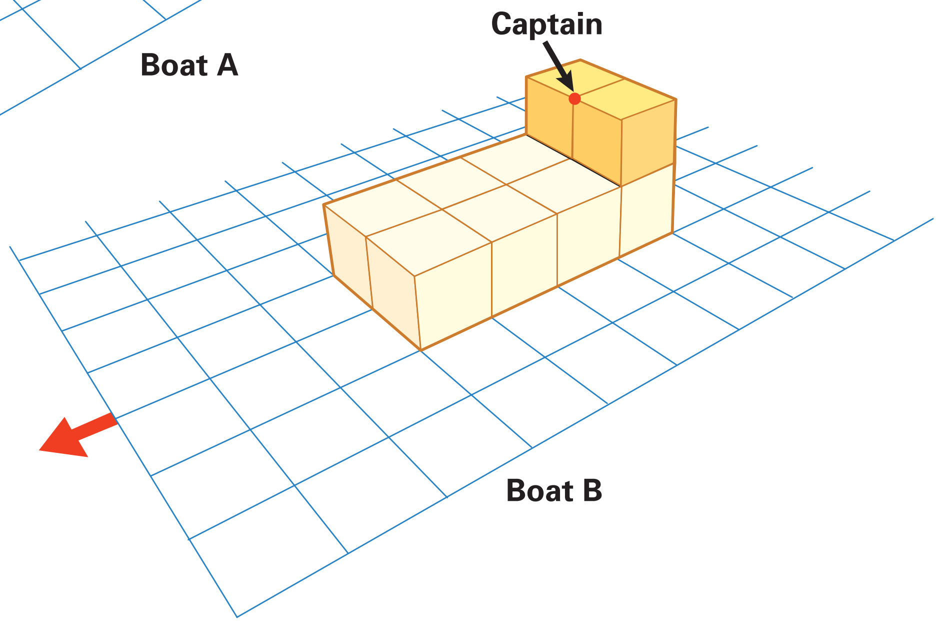
Make a drawing of a right triangle, labeled ΔPQR. The angle measurement of $\angle Q = 90°$.

Make an overview of the side ratios—sine, cosine, and tangent—for this triangle.

Additional Practice

Section A Now You See It, Now You Don't





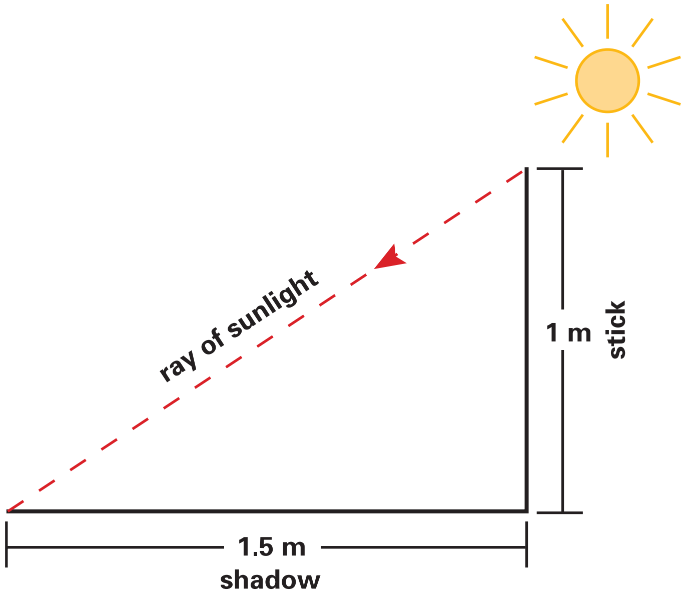
The drawings show two boat models made with 1-cm blocks. Imagine that the boats are sailing in the direction shown by the arrows.

1. On graph paper or **Student Activity Sheet 2**, make side-view and top-view drawings of each boat.
2. On your drawings, include vision lines for the captain, who can look straight ahead and sideways, and shade in the blind area.
3. How many square units is the blind area of boat A? boat B?
4. On your side-view drawings of each boat, measure and label the angle between the water and the vision line.
5. On which boat is the captain's view the best? Explain.

Section B Shadows and Blind Spots

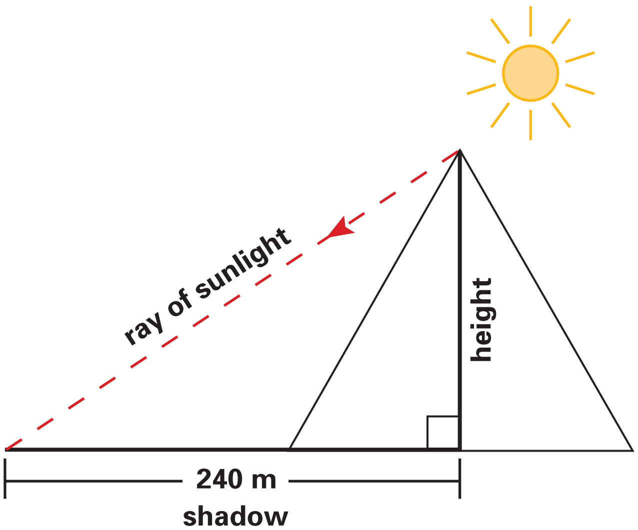
The height of a pyramid can be determined by studying the shadows caused by the sun.

Suppose that you put a stick into the ground near a pyramid. As shown in the drawing, the length of the stick above ground is 1 m, and its shadow caused by the sun is 1.5 m long.



1. a. If the shadow of the pyramid is pointing northeast, what direction is the shadow of the stick pointing?

   b. From what direction is the sun shining?



The picture to the left shows the pyramid and its shadow at the same time of day. The length of the pyramid's shadow, measured from the center of the pyramid, is 240 m.

2. Compare the height of the stick and the length of its shadow to find the height of the pyramid. Explain your reasoning.

Section C Shadows and Angles

1. Use a compass card or a protractor and a ruler to make side-view drawings to scale of the following ladders. Each ladder is leaning against a wall.

   Ladder A

   - The distance between the foot of the ladder and the wall is 3 m.
   - The angle between the ladder and the ground is 60°.

   Ladder B

   - The distance between the foot of the ladder and the wall is 4 m.
   - The ladder touches the wall at a height of 6 m.

2. Determine the height-to-distance ratio for each ladder.

3. What is the angle between ladder B and the ground?

4. Which ladder is steeper, ladder A or ladder B? Explain.

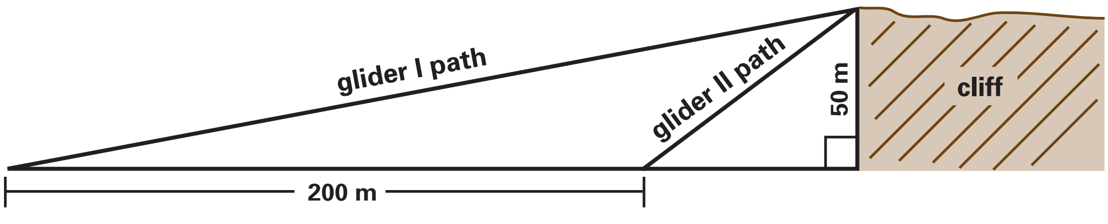
Section D Glide Angles

Use your calculator or the table in the appendix to solve the following problems.

1. **a.** If $\tan A = \frac{1}{20}$, what is the measurement of $\angle A$?

   b. If $\tan B = 20$, what is the measurement of $\angle B$?

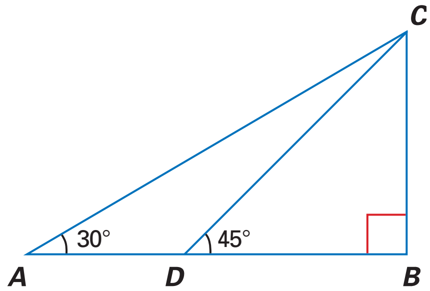
Marco is comparing two hang gliders. He takes one test flight with each glider from a cliff that is 50 m high. The following picture shows the path for each flight. Note: The picture is not drawn to scale.



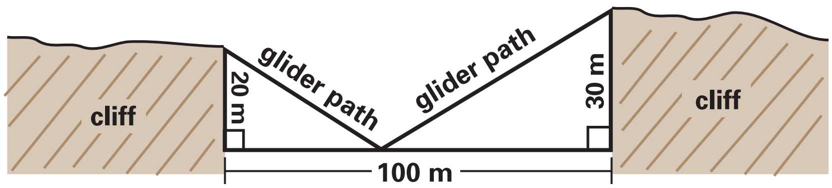
The glide ratio of glider I is 1:20, and glider I travels 200 m farther than glider II.

2. What is the glide ratio of glider II?

3. In the picture below, the measurement of $\angle D$ is 45° and the measurement of $\angle A$ is 30°. If the length of side BD is 10 cm, how long is side AB?



The following picture shows two cliffs that are 100 m apart. One cliff is 20 m high and the other is 30 m high. Imagine that a hang glider takes off from the top of each cliff. The two hang gliders have the same glide ratio and land at the same location.



4. How far from each cliff do the gliders land?

5. Suppose that a glider has a glide ratio of 5%.

   a. What do you think a glide ratio of 5% means?

   b. What is the glide angle for this glider?

Section E Reasoning with Ratios

1. Make a drawing of a right triangle with an angle of 45°. Use this drawing to show that sin 45° = cos 45°.

2. Use the drawing from problem 1. Sides *AB* and *BC* are 1.

   Use the Pythagorean theorem to find the value of sin 45°.

3. Complete the following table:

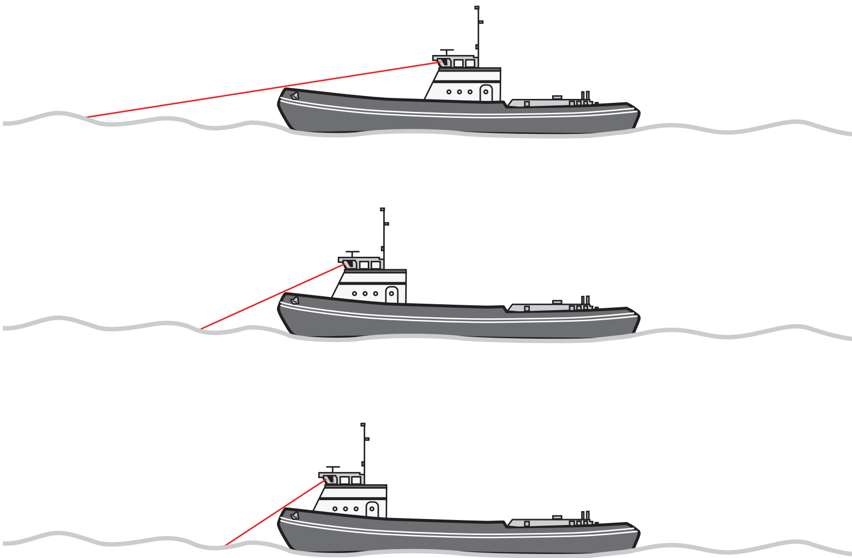
α	sin α	cos α
10°		
20°		
30°		
40°		
50°		
60°		
70°		
80°		

4. Explain the results of the table by comparing the values for sine and cosine.

Answers to Check Your Work

Section A Now You See It, Now You Don't

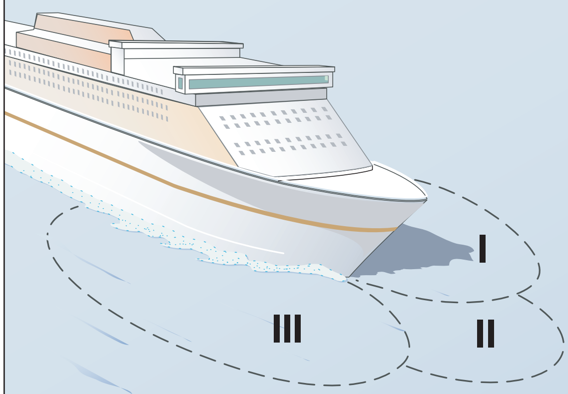
1. a.



b. 8°–10°, 20°, 30°

c. The blind spot gets smaller in front but larger at the back.

2. The captain can walk to the ends of the wings and increase the area he or she can see directly in front and on the sides of the ship. You can make a drawing showing how the blind spot moves as the captain walks from one side of the bridge to the other, as shown here.

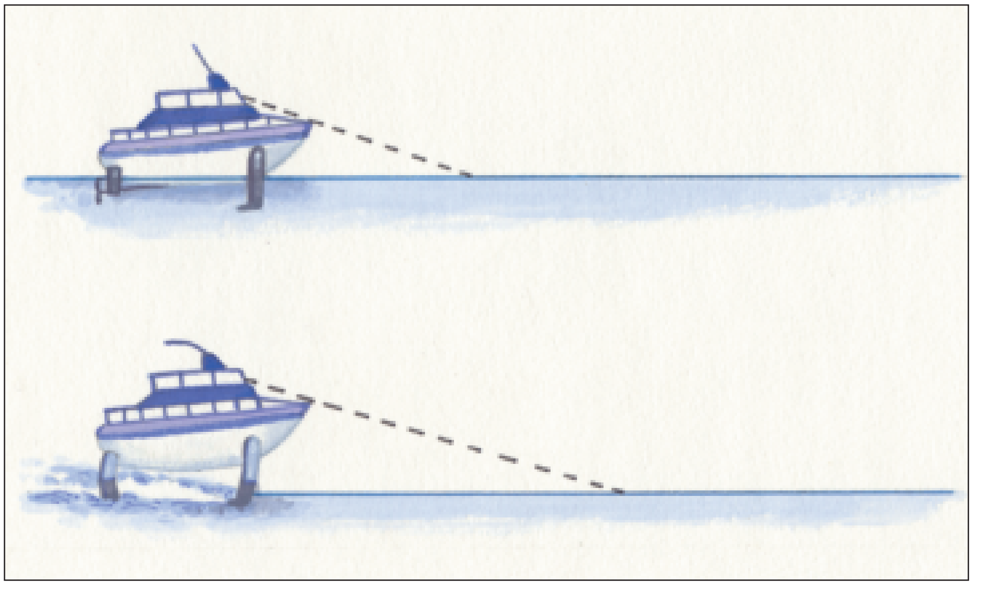


Area I indicates the blind spot when standing on the left side of the bridge.

Area II indicates the blind spot when standing in the middle of the bridge.

Area III indicates the blind spot when standing on the right side of the bridge.

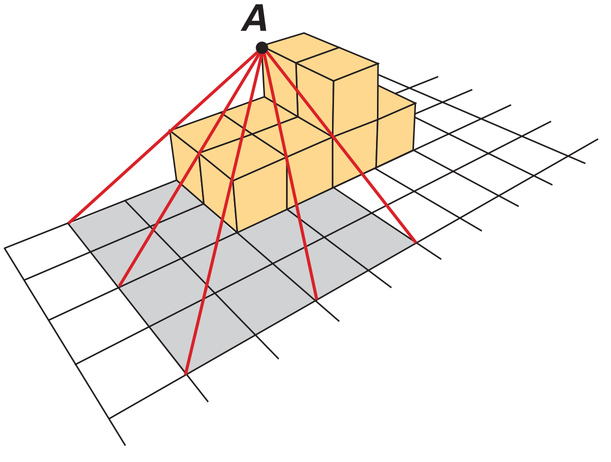
3. Your drawings may differ from the ones shown here.



The higher the ship, the larger the captain's blind spot.

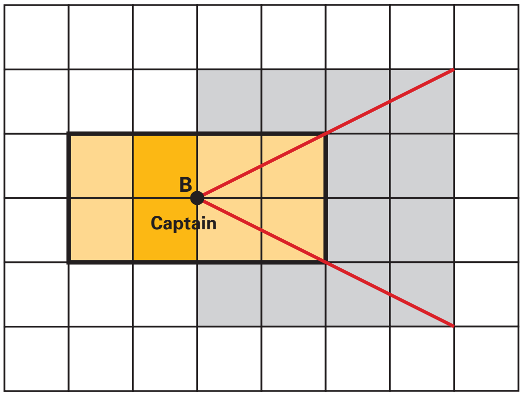
Section B Shadows and Blind Spots

1.



2. From the answers for problem 1, it is clear that the shadow is part of a four-by-four square minus four squares.

3.



4. No, the blind area is still a four-by-four square minus four squares.

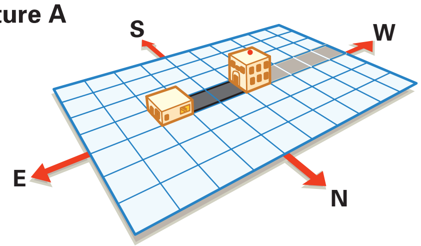
5. In the middle because that is the place where the captain will be.

6. Your description may differ from this sample.

   The sun is shining from the south, so the shadows fall toward the north. The shadow of the shorter building is half as long as the shadow of the taller building.

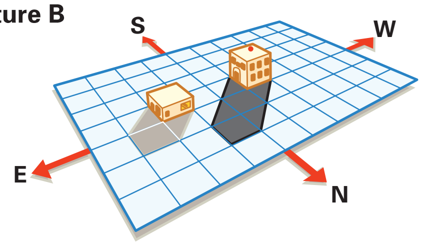
7. a.

Picture A



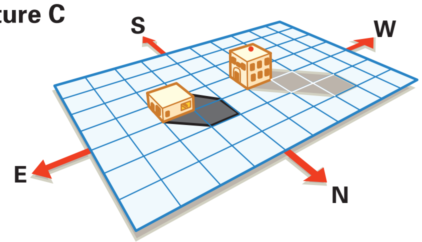
Time: 7:00 A.M. (sunrise)

Picture B



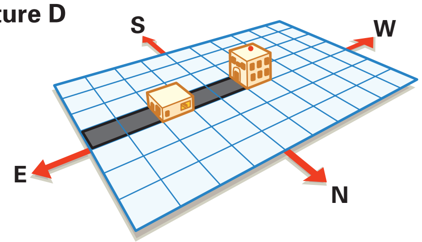
Time: 2:30 P.M. (mid-afternoon)

Picture C



Time: 9:30 A.M. (mid-morning)

Picture D



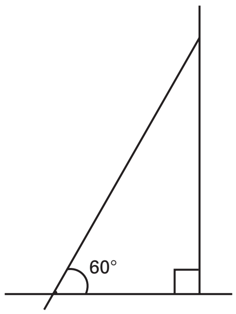
Time: 5:00 P.M. (sunset)

b.

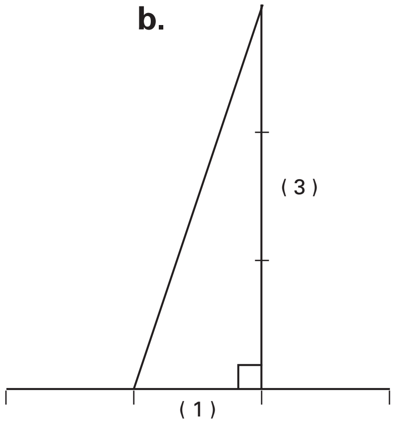
Picture A	7:00 A.M. sunrise
Picture B	2:30 P.M. mid-afternoon
Picture C	9:30 A.M. mid-morning
Picture D	5:00 P.M. sunset

Section C Shadows and Angles

1. a.



b.



c. $a \approx 72°$

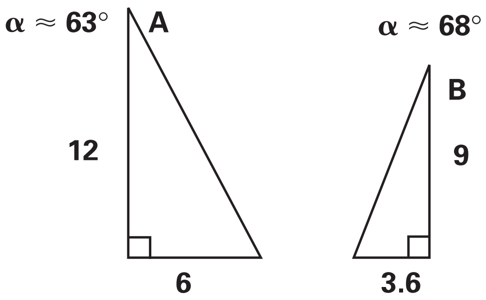
2.

	α	h	d	$\frac{h}{d}$
a.	34°	2	3	$\frac{2}{3}$
b.	45°	2	2	1
c.	76°	4	1	4

3. Vision line from ledge B is steeper.

Explanations may vary. Here are two.

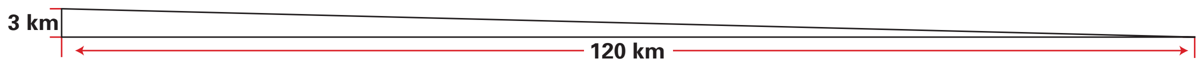
Using the canyon scale information, I constructed a right triangle for each situation and found α. It is larger toward ledge B, making it steeper.



I compared *h*:*d* for each. 12:6 versus 9:3.6. This is like 2:1 compared to 2.5:1, since this 2.5:1 is larger; the line to B goes up faster and is steeper.

Section D Glide Angles

1. The glider must be launched from a height of 3 km.



Since the glide ratio is 1:40, and 120 is three times 40 km, you only need to triple the height.

Height (in km)	**1**	3
Distance (in km)	**40**	**120**

2. 3,600 m or 33.6 km

Here is a ratio table with the 1:28 glide ratio, building up to 1,200 m height.

Height (in km)	**1**	**10**	1,000	20	200	**1,200**
Distance (in km)	**28**	280	28,000	560	5,600	33,600

Using the map scale line, you should draw a circle around Lake Havasu City.

Source: © 1997, Encyclopædia Britannica, Inc.

3. a. Some students will focus on the ladder position on the wall; others might focus on the relative lengths of the ladders. Two examples of student responses are: first, as the ladder was moved higher up the wall, the angle at the base increased. Second, if you want to keep the ladder 1 m from the wall, then you need to get taller and taller ladders to make the specified angle.

b. The length of the ladder is probably greater than 14 m.

4. 1 km

Strategies may vary.

tan 12° = 0.21, which means that you need a height of 21 km to travel 100 km. The car needs to descend a ground distance of 10 km, so I only need to divide 21 by 10 to get an answer of 2.1 km. This keeps the same glide ratio and angle of 12°.

5. 67.2 m

Sample strategy:

tan 23° ≈ 0.42, which means that you need a height of 42 m to travel 100 m along the ground. Then I used a ratio table and placed the glide ratio in the first column. My goal was to build up to a distance of 160 m.

Height (in m)	**42**	21	4.2	67.2
Distance (in m)	**100**	50	10	**160**

Section E Reasoning with Ratios

1. $BC \approx 4.9$, or 5

Calculation:

$\sin 45° = \frac{BC}{7}$

$0.707 = \frac{BC}{7}$

$0.707 \times 7 = BC$

$BC = 4.949 \approx 4.9$, or 5

2. a. $AD = 13$

Calculation, using the Pythagorean theorem:

$AD^2 = 5^2 + 12^2$

$AD^2 = 25 + 144$

$AD^2 = 169$

$AD = \sqrt{169} = 13$

b. 23°

Calculation:

$\tan \angle A = \frac{5}{12}$

$\angle A \approx 22.6°$, or 23°

3. $BC = 1.74$, or 2

Calculation:

$\sin 10° \approx \frac{BC}{10}$

$0.174 = \frac{BC}{10}$

$10 \times 0.174 = BC$

$BC = 1.74$, or 2

Appendix A

Angle Degree	Sine	Cosine	Tangent
0°	0.000	1.00	0.000
1°	0.017	1.00	0.017
2°	0.035	0.999	0.035
3°	0.052	0.999	0.052
4°	0.070	0.998	0.070
5°	0.087	0.996	0.087
6°	0.105	0.995	0.105
7°	0.122	0.993	0.123
8°	0.139	0.990	0.141
9°	0.156	0.988	0.158
10°	0.174	0.985	0.176
11°	0.191	0.982	0.194
12°	0.208	0.978	0.213
13°	0.225	0.974	0.231
14°	0.242	0.970	0.249
15°	0.259	0.966	0.268
16°	0.276	0.961	0.287
17°	0.292	0.956	0.306
18°	0.309	0.951	0.325
19°	0.326	0.946	0.344
20°	0.342	0.940	0.360
21°	0.358	0.934	0.384
22°	0.375	0.927	0.404
23°	0.391	0.921	0.424
24°	0.407	0.914	0.445
25°	0.423	0.906	0.466
26°	0.438	0.899	0.488
27°	0.454	0.891	0.510
28°	0.469	0.883	0.532
29°	0.485	0.875	0.554
30°	0.500	0.866	0.577

Appendix A

Angle Degree	Sine	Cosine	Tangent
31°	0.515	0.857	0.601
32°	0.530	0.848	0.625
33°	0.545	0.839	0.649
34°	0.559	0.829	0.675
35°	0.574	0.819	0.700
36°	0.588	0.809	0.727
37°	0.602	0.799	0.754
38°	0.616	0.788	0.781
39°	0.629	0.777	0.810
40°	0.643	0.766	0.839
41°	0.656	0.755	0.869
42°	0.669	0.743	0.900
43°	0.682	0.731	0.933
44°	0.695	0.719	0.966
45°	0.707	0.707	1.000
46°	0.719	0.695	1.036
47°	0.731	0.682	1.072
48°	0.743	0.669	1.111
49°	0.755	0.656	1.150
50°	0.766	0.643	1.192
51°	0.777	0.629	1.235
52°	0.788	0.616	1.280
53°	0.799	0.602	1.327
54°	0.809	0.588	1.376
55°	0.819	0.574	1.428
56°	0.829	0.559	1.483
57°	0.839	0.545	1.540
58°	0.848	0.530	1.600
59°	0.857	0.515	1.664
60°	0.866	0.500	1.732
61°	0.875	0.485	1.804

Appendix A

Angle Degree	Sine	Cosine	Tangent
62°	0.883	0.469	1.881
63°	0.891	0.454	1.963
64°	0.899	0.438	2.050
65°	0.906	0.423	2.145
66°	0.914	0.407	2.246
67°	0.921	0.391	2.356
68°	0.927	0.375	2.475
69°	0.934	0.358	2.605
70°	0.940	0.342	2.748
71°	0.946	0.326	2.904
72°	0.951	0.309	3.078
73°	0.956	0.292	3.271
74°	0.961	0.276	3.487
75°	0.966	0.259	3.732
76°	0.970	0.242	4.011
77°	0.974	0.225	4.332
78°	0.978	0.208	4.705
79°	0.982	0.191	5.145
80°	0.985	0.174	5.671
81°	0.988	0.156	6.314
82°	0.990	0.139	7.115
83°	0.993	0.122	8.144
84°	0.995	0.105	9.514
85°	0.996	0.087	11.43
86°	0.998	0.070	14.30
87°	0.999	0.052	19.08
88°	0.999	0.035	28.64
89°	1.00	0.017	57.29
90°	1.00	0.000	