

ΑΝΙΣΩΣΕΙΣ 2^{ου} ΒΑΘΜΟΥ
ΛΥΣΕΙΣ ΑΣΚΗΣΕΩΝ

(1)

Άσκηση 1

(α) $x^2 - 5x + 6 \quad \Delta = 25 - 24 = 1 \quad x_{1,2} = \frac{5 \pm 1}{2} < \begin{matrix} 3 \\ 1 \end{matrix}$
 $= (x-3)(x-1)$

(β) $x^2 + 6x + 9 = (x+3)^2$

(γ) $x^2 - x + 2 \quad \Delta = 1 - 8 < 0 \quad \text{Δεν παραγοντοποιείται}$

(δ) $2x^2 - x - 10 \quad \Delta = 1 + 80 = 81 \quad x_{1,2} = \frac{1 \pm 9}{4} < \begin{matrix} \frac{5}{2} \\ -2 \end{matrix}$
 $= 2(x - \frac{5}{2})(x+2) = (2x-5)(x+2)$

(ε) $-2x^2 + 3x + 9 \quad \Delta = 9 + 72 = 81 = 9^2 \quad x_{1,2} = \frac{-3 \pm 9}{-4} < \begin{matrix} -\frac{3}{2} \\ 3 \end{matrix}$
 $= -2(x + \frac{3}{2})(x-3) = (-2x-3)(x-3)$

(σ) $6x^2 - x - 9 \quad \Delta = 1 + 48 = 49 = 7^2 \quad x_{1,2} = \frac{1 \pm 7}{12} < \begin{matrix} \frac{2}{3} \\ -\frac{1}{2} \end{matrix}$
 $= 6(x - \frac{2}{3})(x + \frac{1}{2}) = (3x-2)(2x+1)$

(ζ) $\frac{1}{3}x^2 - x - 6 \quad \Delta = 1 + 4 \cdot 6 \cdot \frac{1}{3} = 9 \quad x_{1,2} = \frac{1 \pm 3}{\frac{2}{3}} < \begin{matrix} \frac{4 \cdot 3}{2} = 6 \\ \frac{-2 \cdot 3}{2} = -3 \end{matrix}$
 $= \frac{1}{3}(x-6)(x+3)$

(η) $x^2 - (2-\sqrt{2})x - \sqrt{8} \quad \Delta = (2-\sqrt{2})^2 + 4\sqrt{8} = 4+2-4\sqrt{2}+8\sqrt{2} = 6+4\sqrt{2} = (\sqrt{2}+2)^2$
 $= (x-2)(x+\sqrt{2})$
 $x_{1,2} = \frac{2-\sqrt{2} \pm \sqrt{2}+2}{2} < \begin{matrix} 2 \\ -\sqrt{2} \end{matrix}$

(θ) $x^2 + (\sqrt{2}-\sqrt{3})x - \sqrt{6} \quad \Delta = 2+3-2\sqrt{6} = 5-2\sqrt{6} = (\sqrt{3}-\sqrt{2})^2$
 $= (x-\sqrt{3})(x+\sqrt{2})$
 $x_{1,2} = \frac{\sqrt{3}-\sqrt{2} \pm \sqrt{3}-\sqrt{2}}{2} < \begin{matrix} \sqrt{3} \\ -\sqrt{2} \end{matrix}$

Άσκηση 2

$$(a) x^3 - 3x^2 - 4x = x(x^2 - 3x - 4) = x(x-4)(x+1)$$

$$(b) 2x^3 - 12x^2 + 18x = 2x(x^2 - 6x + 9) = 2x(x-3)^2$$

$$(γ) 6x^3 + 7x^2 - 3x = x(6x^2 + 7x - 3) = x \cdot 6 \left(x - \frac{1}{3}\right) \left(x + \frac{3}{2}\right) = x(3x-1) \left(x + \frac{3}{2}\right)$$

$$(δ) \Delta = 49 + 72 = 121 \quad x_{1,2} = \frac{-7 \pm 11}{12} \begin{cases} \frac{1}{3} \\ -\frac{3}{2} \end{cases}$$

$$x^4 - 5x^2 + 4 \text{ θέτω } w = x^2$$

$$w^2 - 5w + 4 \quad \Delta = 25 - 16 = 9 \quad w_{1,2} = \frac{5 \pm 3}{2} \begin{cases} 4 \\ 1 \end{cases}$$

- Για $w = 4$, $x = \pm 2$
- Για $w = 1$, $x = \pm 1$

$$\text{άρα } x^4 - 5x^2 + 4 = (x-2)(x+2)(x-1)(x+1)$$

$$(ε) x^2 - 3|x| - 10 \text{ θέτω } w = |x|$$

$$w^2 - 3w - 10 \quad \Delta = 9 + 40 = 49 \quad w_{1,2} = \frac{3 \pm 7}{2} \begin{cases} 5 \\ -2 \end{cases}$$

$$\text{άρα } x^2 - 3|x| - 10 = (w-5)(w+2) = (|x|-5)(|x|+2)$$

$$(ς) x^6 + 7x^3 - 8 \text{ θέτω } w = x^3$$

$$w^2 + 7w - 8 \quad \Delta = 49 + 32 = 81 = 9^2$$

$$w_{1,2} = \frac{-7 \pm 9}{2} \begin{cases} 1 \\ -8 \end{cases}$$

$$x^6 + 7x^3 - 8 = (w-1)(w+8) = (x^3-1)(x^3+8)$$

$$= (x-1)(x^2+x+1)(x+2)(x^2-2x+4)$$

$$(ζ) (x^2+2)^2 - 9(x^2+2) + 18 \text{ θέτω } w = x^2+2$$

$$w^2 - 9w + 18 \quad \Delta = 81 - 72 = 9 \quad w_{1,2} = \frac{9 \pm 3}{2} \begin{cases} 6 \\ 3 \end{cases}$$

$$= (w-6)(w-3) = (x^2+2-6)(x^2+2-3) = (x^2-4)(x^2-1)$$

$$= (x-2)(x+2)(x-1)(x+1)$$

Aufgabe 3

(3)

(a) $(x-2y)^2 - 4(x-2y) - 5$ Setze $w = x-2y$

$$w^2 - 4w - 5 \quad w_1 = 5, w_2 = -1$$

$$= (w-5)(w+1) = (x-2y-5)(x-2y+1)$$

(b) $(x-y)^2 - 6(x-y) + 9 = (x-y-3)^2$

(c) $x^2 - 3yx + 2y^2 \quad \Delta = 9y^2 - 4 \cdot 1 \cdot 2y^2 = y^2 \geq 0$

$$x_{1,2} = \frac{3y \pm y}{2} \quad \begin{cases} \frac{3y+y}{2} = 2y \\ \frac{3y-y}{2} = y \end{cases}$$

$$= (x-2y)(x-y)$$

(d) $-x^2 + xy + 6y^2 \quad \Delta = y^2 - 4(-1) \cdot 6y^2 = 25y^2$

$$x_{1,2} = \frac{-y \pm 5y}{-2} \quad \begin{cases} \frac{-y+5y}{-2} = -2y \\ \frac{-y-5y}{-2} = 3y \end{cases}$$

$$= -(x-3y)(x+2y)$$

(e) $6x^2 + 7xy - 20y^2 \quad \Delta = 49y^2 - 4 \cdot 6 \cdot (-20y^2) = 49y^2 + 480y^2 = (23y)^2$

$$x_{1,2} = \frac{-7y \pm 23y}{12} = \begin{cases} \frac{-7y+23y}{12} = \frac{16y}{12} = \frac{4y}{3} \\ \frac{-7y-23y}{12} = \frac{-30y}{12} = -\frac{5y}{2} \end{cases}$$

$$= 6 \left(x - \frac{4y}{3} \right) \left(x + \frac{5y}{2} \right) = (3x-4y)(2x+5y)$$

(f) $x^2 - 2ax - 8a^2 \quad \Delta = 4a^2 - 4 \cdot 1 \cdot (-8a^2) = 4a^2 + 32a^2 = 36a^2$

$$x_{1,2} = \frac{2a \pm 6a}{2} \quad \begin{cases} 4a \\ -2a \end{cases}$$

$$= (x-4a)(x+2a)$$

(g) $x^2 + 2(a-1)x - 3a(a+2) \quad \Delta = 4(a-1)^2 + 4 \cdot 3a(a+2)$
 $= 4(a^2 - 2a + 1) + 12a(a+2)$
 $= 4a^2 - 8a + 4 + 12a^2 + 24a = 16a^2 + 16a + 4 = 4(4a^2 + 4a + 1)$
 $= 4(2a+1)^2$

$$x_{1,2} = \frac{-2(a-1) \pm 2(2a+1)}{2} = \frac{-2a+2 \pm 4a+2}{2} \quad \begin{cases} \frac{-2a+2+4a+2}{2} = a+2 \\ \frac{-2a+2-4a-2}{2} = -3a \end{cases}$$

$$= \begin{cases} a+2 \\ -3a \end{cases} \quad \text{d.h. } (x-a-2)(x+3a)$$

$$(4) \quad x^2 - (a+b)x + a(a+b)$$

(4)

$$\Delta = (a+b)^2 - 4 \cdot 1 \cdot a(a+b) = \cancel{4a^2} + \cancel{4ab} + b^2 - \cancel{4a^2} - \cancel{4ab} = b^2$$

$$x_{1,2} = \frac{a+b \pm b}{2} \begin{cases} \frac{a+b+b}{2} = a+b \\ \frac{a+b-b}{2} = a \end{cases}$$

$$= (x-a-b)(x-a)$$

Axonon 4

$$(a) \quad \frac{x^2 - 2x - 3}{x^2 - 9} = \frac{(x-3)(x+1)}{(x-3)(x+3)} = \frac{x+1}{x+3} \quad y.a \quad x \neq \pm 3$$

$$(b) \quad \frac{x^2 - 4x + 4}{x^2 + 3x - 10} = \frac{(x-2)^2}{(x+5)(x-2)} = \frac{x-2}{x+5} \quad y.a \quad x \neq 2 \text{ ca } x \neq -5$$

$$(r) \quad \frac{2x^2 + 3x - 2}{2x^2 + 7x + 6} \quad \Delta = 9 - 4 \cdot 2(-2) = 9 + 16 = 5^2$$

$$x_{1,2} = \frac{-3 \pm 5}{4} \begin{cases} \frac{1}{2} \\ -2 \end{cases}$$

$$\Delta = 49 - 48 = 1 \quad x_{1,2} = \frac{-7 \pm 1}{4} \begin{cases} -\frac{3}{2} \\ -2 \end{cases}$$

$$= \frac{2(x - \frac{1}{2})(x+2)}{2(x + \frac{3}{2})(x+2)} = \frac{2x-1}{2x+3}$$

$$(s) \quad \frac{2x^2 + x - 6}{4x^2 - 12x + 9} \quad \Delta = 1 + 48 = 7^2 \quad x_{1,2} = \frac{-1 \pm 7}{4} \begin{cases} \frac{3}{2} \\ -2 \end{cases}$$

$$\Delta = 144 - 144 \quad x = \frac{12}{8} = \frac{3}{2}$$

$$= \frac{2(x - \frac{3}{2})(x+2)}{4(x - \frac{3}{2})^2} = \frac{(x+2)}{2x-3}$$

$$(e) \quad \frac{2|x|^2 + 5|x| - 3}{|x|^2 - 2|x| - 15} \quad \Delta = 25 + 24 = 7^2 \quad x_{1,2} = \frac{-5 \pm 7}{4} \begin{cases} 1 \\ -3 \end{cases}$$

$$\Delta' = 4 + 60 = 8^2$$

$$x_{1,2} = \frac{2 \pm 8}{2} \begin{cases} 5 \\ -3 \end{cases}$$

$$= \frac{2(|x|-1)(|x|+3)}{(|x|-5)(|x|+3)} = \frac{2(|x|-1)}{|x|-5}$$

$$(α) \frac{x^6 - 7x^3 - 8}{x^2 - x - 2} \quad \text{Θέτω } w = x^3 \quad w^2 - 7w - 8, w_1 = \frac{7 \pm 9}{2} < \frac{8}{-1} \quad (5)$$

$$\Delta = 49 + 32 = 81$$

$$= \frac{(x^3 - 8)(x^3 + 1)}{(x-2)(x+1)} = \frac{(\cancel{x-2})(x^2 + 2x + 4)(\cancel{x+1})(x^3 - x + 1)}{(\cancel{x-2})(\cancel{x+1})}$$

$$= (x^2 + 2x + 4)(x^3 - x + 1)$$

Άσκηση 5

$$(α) \frac{1}{x^2 - 5x + 6} - \frac{x}{x-3} = \frac{1-x}{x-2} \quad \text{Πρέπει } x \neq 2 \text{ και } x \neq 3$$

$$\Leftrightarrow \frac{1}{(x-3)(x-2)} - \frac{x}{x-3} = \frac{1-x}{x-2} \quad \Leftrightarrow 1 - (x-2) \cdot x = (1-x)(x-3)$$

$$\Leftrightarrow 1 - x^2 + 2x = x - 3 - x^2 + 3x \quad \Leftrightarrow 2x = 4 \quad \Leftrightarrow \boxed{x=2} \text{ απαγορεύεται}$$

Άρα $\emptyset$ λύση

$$(β) \frac{x+3}{x-1} = \frac{x+4}{x-2} - \frac{2(x+1)}{x^2 - 3x + 2} \quad \Leftrightarrow \frac{x+3}{x-1} = \frac{x+4}{x-2} - \frac{2(x+1)}{(x-1)(x-2)}$$

Για $x \neq 1$ και $x \neq 2$

$$\Leftrightarrow (x-2)(x+3) = (x-1)(x+4) - 2(x+1)$$

$$\Leftrightarrow \cancel{x^2} + 3x - \cancel{2x} - 6 = \cancel{x^2} + 4x - x - 4 - \cancel{2x} - 2$$

$$\Leftrightarrow 0x = 0 \quad \text{οπότε } x \in \mathbb{R} - \{1, 2\}$$

$$(γ) \frac{2(x-1)}{x^2 - x - 2} - \frac{x-5}{x^2 - 3x + 2} = \frac{16}{x^2 - 1}$$

$$\Leftrightarrow \frac{2(x-1)}{(x+1)(x-2)} - \frac{x-5}{(x-1)(x-2)} = \frac{16}{(x-1)(x+1)}$$

$x \neq 1$

και $x \neq 2$ και $x \neq -1$

Εκπν = $(x+1)(x-1)(x-2)$

$$\Leftrightarrow 2(x-1)^2 - (x-5)(x+1) = 16(x-2)$$

$$\Leftrightarrow 2x^2 - 4x + 2 - x^2 - x + 5x + 5 = 16x - 32$$

$$\Leftrightarrow x^2 - 16x + 39 = 0 \quad \Delta = 16^2 - 4 \cdot 39 = 256 - 156 = 100$$

$$x_{1,2} = \frac{16 \pm 10}{2} < \frac{13}{3}$$

Aσκηση 6

(α) Πρέπει $|x|^2 + 4|x| + 3 \neq 0$ θεω $|x| = w$

$$w^2 + 4w + 3 \neq 0 \quad \Delta = 16 - 12 = 4 \quad w_{1,2} = \frac{-4 \pm 2}{2} \begin{matrix} -1 \\ -3 \end{matrix}$$

$\Rightarrow (w+1)(w+3) \neq 0 \Rightarrow w \neq -1$ και $w \neq -3$

$\Rightarrow |x| \neq -1$ και $|x| \neq -3$ ισχύει πάντα

άρα $x \in \mathbb{R}$

(β) $w^2 - 3w - 4 \quad \Delta = 9 + 16 = 25 \quad w_{1,2} = \frac{3 \pm 5}{2} \begin{matrix} 4 \\ -1 \end{matrix}$

$$w^2 + w - 6 \quad \Delta = 1 + 24 = 25 \quad w_{1,2} = \frac{-1 \pm 5}{2} \begin{matrix} 2 \\ -3 \end{matrix}$$

άρα $A = \frac{(|x|-4)(|x|-1)(|x|-2)(|x|+3)}{(|x|+1)(|x|+3)}$

$\Rightarrow A = (|x|-4)(|x|-2)$

(γ) $A=3 \Rightarrow (|x|-4)(|x|-2)=3$

$\Rightarrow |x|^2 - 2|x| - 4|x| + 8 = 3$

$\Rightarrow |x|^2 - 6|x| + 5 = 0 \quad |x| = w$

άρα $w^2 - 6w + 5 = 0 \quad \Delta = 36 - 20 = 16$

$$w_{1,2} = \frac{6 \pm 4}{2} \begin{matrix} 5 \\ 1 \end{matrix}$$

άρα $|x|=5 \Rightarrow x = \pm 5$

$|x|=1 \Rightarrow x = \pm 1$

Aσκηση 7

(α) $x^2 + 2x - 3 > 0 \quad \Delta = 4 + 12 = 16 \quad x_{1,2} = \frac{-2 \pm 4}{2} \begin{matrix} 1 \\ -3 \end{matrix}$

x	$-\infty$	-3	1	$+\infty$	
$x^2 + 2x - 3$	+	0	-	0	+

$x \in (-\infty, -3) \cup (1, +\infty)$

$$(B) -x^2 + 3x + 4 \leq 0 \quad \Delta = 9 + 16 = 25 \quad x_{1,2} = \frac{-3 \pm 5}{-2} < \begin{matrix} -1 \\ 4 \end{matrix} \quad (7)$$

x	$-\infty$	-1	4	$+\infty$
$-x^2 + 3x + 4$	$-$	0	$+$	0

$x \in (-\infty, -1] \cup [4, +\infty)$

$$(D) x^2 - 4x + 4 \geq 0 \Leftrightarrow (x - 2)^2 \geq 0$$

x	2
$x^2 - 4x + 4$	$+$

$x \in \mathbb{R}$

$$(E) -x^2 + 6x - 9 < 0 \Leftrightarrow -(x - 3)^2 < 0 \quad x \in \mathbb{R} - \{3\}$$

$$(E) 2x^2 - 4x + 2 < 0 \Leftrightarrow x^2 - 2x + 1 < 0 \Leftrightarrow (x - 1)^2 < 0 \text{ αδωρη}$$

$$(C) x^2 - x + 2 > 0 \quad \Delta = 1 - 8 = -7 < 0$$

ισχύει για κάθε $x \in \mathbb{R}$

$$(J) -x^2 + 2x - 3 \geq 0 \quad \Delta = 4 - 4(-1)(-3) = 4 - 12 = -8 < 0$$

x	$-\infty$	$+\infty$
$-x^2 + 2x - 3$	$-$	$-$

$\alpha\beta\alpha \text{ αδωρη}$

$A \propto \eta \propto \theta$

$$(a) -2x^2 - 4x \leq 0 \quad 2x(x + 2) = 0 \Leftrightarrow x = 0 \vee x = -2$$

x	$-\infty$	-2	0	$+\infty$
$-2x^2 - 4x$	$-$	0	$+$	0

$x \in (-\infty, -2] \cup [0, +\infty)$

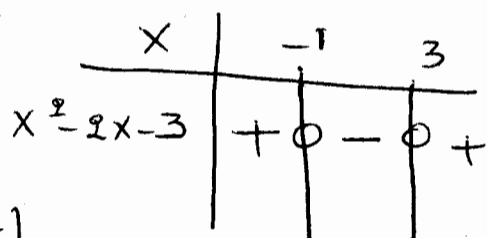
$$(B) -x > \frac{x^2}{4} \Leftrightarrow x^2 + 4x < 0 \quad \alpha\beta\alpha \quad x = 0 \vee x = -4$$

x	-4	0
$x^2 + 4x$	$+$	0

$x \in (-4, 0)$

$$(8) (x+3)^2 > 4(2x+3) \Leftrightarrow x^2 + 6x + 9 > 8x + 12$$

$$\Leftrightarrow x^2 - 2x - 3 > 0 \quad \Delta = 4 + 12 = 4^2 \quad x_{1,2} = \frac{2 \pm 4}{2} \begin{matrix} 3 \\ -1 \end{matrix}$$



$$x \in (-\infty, -1) \cup (3, +\infty)$$

$$(8) 4(x-5) - (x-4)(x+4) \geq 0$$

$$\Leftrightarrow 4x - 20 - x^2 + 16 \geq 0 \Leftrightarrow -x^2 + 4x - 4 \geq 0$$

$$\Leftrightarrow -(x-2)^2 \geq 0 \Leftrightarrow \boxed{x=2}$$

Άσκηση 9

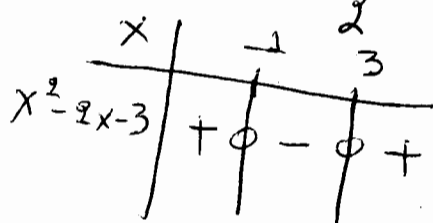
$$(a) x^2 - 2x - 3 < 0 \quad \text{και} \quad -x^2 + x + 2 \geq 0$$

$$\Delta = 4 + 12 = 4^2$$

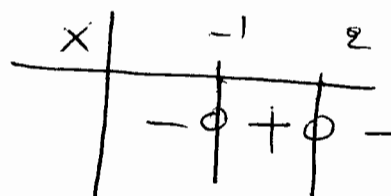
$$x_{1,2} = \frac{2 \pm 4}{2} \begin{matrix} 3 \\ -1 \end{matrix}$$

$$\Delta = 1 + 8 = 3^2$$

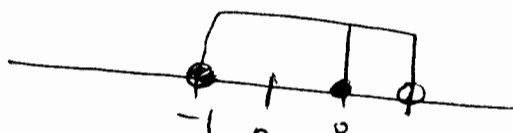
$$x_{1,2} = \frac{-1 \pm 3}{-2} \begin{matrix} -1 \\ 2 \end{matrix}$$



$$x \in (-1, 3)$$



$$x \in [-1, 2]$$



$$x \in [-1, 2]$$

$$(B) 3 \leq x^2 - 2x < 8$$

$$3 \leq x^2 - 2x$$

$$\Leftrightarrow x^2 - 2x - 3 \geq 0$$

$$\Delta = 4 + 12 = 4^2$$

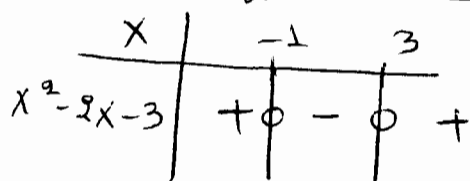
$$x_{1,2} = \frac{2 \pm 4}{2} \begin{matrix} 3 \\ -1 \end{matrix}$$

$$\text{και} \quad x^2 - 2x < 8$$

$$\Leftrightarrow x^2 - 2x - 8 < 0$$

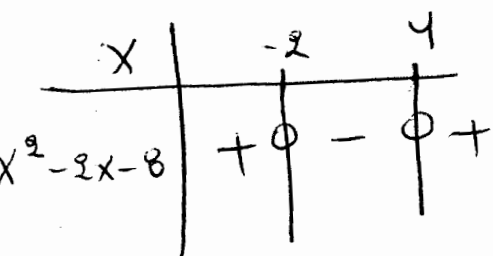
$$\Delta = 4 + 32 = 6^2$$

$$x_{1,2} = \frac{2 \pm 6}{2} \begin{matrix} 4 \\ -2 \end{matrix}$$



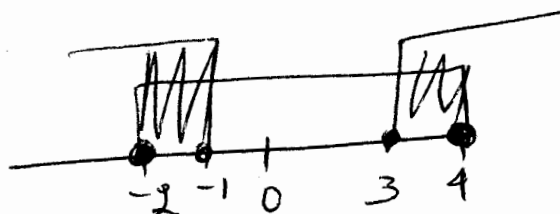
$$x \in (-\infty, -1] \cup [3, +\infty)$$

(9)



$$x \in [-2, 4]$$

$$\tau \varepsilon \lambda \text{ και } x \in [-2, -1] \cup [3, 4]$$

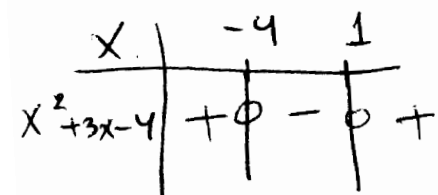


$$(8) \quad 4(x+1) < x(x+7) \quad \text{και} \quad x(x+7) < 2(2x+5)$$

$$\Leftrightarrow 4x+4 < x^2+7x \quad \text{και} \quad x^2+7x < 4x+10$$

$$\Leftrightarrow x^2+3x-4 > 0$$

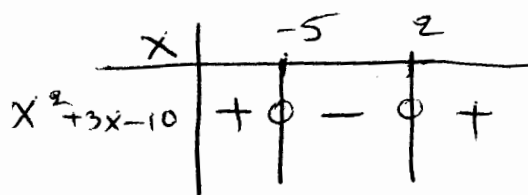
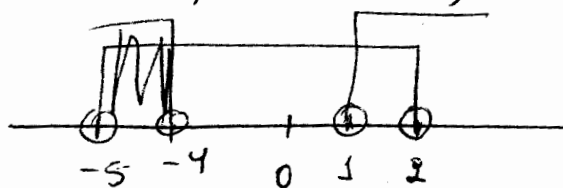
$$\text{και} \quad x^2+3x-10 < 0$$



$$\Delta = 9+16=25$$

$$x_{1,2} = \frac{-3 \pm 5}{2} < \begin{matrix} 1 \\ -4 \end{matrix}$$

$$x \in (-\infty, -4) \cup (1, +\infty)$$



$$\Delta = 9+40=49$$

$$x_{1,2} = \frac{-3 \pm 7}{2} < \begin{matrix} 2 \\ -5 \end{matrix}$$

$$x \in (-5, 2)$$

$$\tau \varepsilon \lambda \text{ και } x \in (-5, -4) \cup (1, 2)$$

Άσκηση 10

$$(a) \quad x^2+2x-4 < 3x(x-1)$$

$$\Leftrightarrow x^2+2x-4 < 3x^2-3x$$

$$\Leftrightarrow 2x^2-5x+4 > 0$$

$$\Delta = 25-32=-7 < 0 \quad \mu \varepsilon \quad a=2 > 0$$

άρα ισχύει

$$(B) (x-3)(x+3) > 2(2x-7)$$

$$\Leftrightarrow x^2 - 9 > 4x - 14 \Leftrightarrow x^2 - 4x + 5 > 0$$

$$\Delta = 16 - 20 = -4 < 0 \text{ και } a = 1 > 0 \text{ άρα ισχύει}$$

$$(δ) (3x-y)^2 - 2(3x-y) + 2 > 0$$

$$w = 3x - y$$

$$\text{Άρα } w^2 - 2w + 2 > 0$$

$$\Delta = 4 - 8 = -4 < 0 \text{ και } a > 0 \text{ άρα ισχύει για κάθε } x, y \in \mathbb{R}$$

$$(ε) -2(4x-5y)^2 + 3(4x-5y) - 3 < 0$$

$$w = 4x - 5y$$

$$-2w^2 + 3w - 3 < 0 \quad \Delta = 9 - 24 = -15 < 0 \text{ και } a < 0$$

άρα ισχύει για κάθε $x, y \in \mathbb{R}$

Άσκηση 11

$$x^2 - (3\lambda - 1)x + (\lambda - 1)(\lambda + 1) = 0$$

$$\begin{aligned} \Delta &= (3\lambda - 1)^2 - 4(\lambda - 1)(\lambda + 1) = 9\lambda^2 - 6\lambda + 1 - 4\lambda^2 + 4 \\ &= 5\lambda^2 - 6\lambda + 5 \end{aligned}$$

$$\Delta' = 36 - 4 \cdot 25 < 0 \text{ και } a > 0$$

άρα $\Delta > 0$ για κάθε $\lambda \in \mathbb{R}$

Άσκηση 12

$$\begin{aligned} \Delta &= (\lambda - 1)^2 - 4(\lambda^2 - \lambda + 1) = \lambda^2 - 2\lambda + 1 - 4\lambda^2 + 4\lambda - 4 \\ &= -3\lambda^2 + 2\lambda - 3 \end{aligned}$$

$$\Delta' = 4 - 36 = -32 < 0$$

τελικά $\Delta < 0$

Ασκηση 13

(a) $x^4 - 10x^2 + 9 \geq 0 \quad w = x^2$

$$w^2 - 10w + 9 \geq 0 \quad \Delta = 100 - 36 = 64$$

$$w_{1,2} = \frac{10 \pm 8}{2} < \begin{matrix} 9 \\ 1 \end{matrix}$$

$$\Leftrightarrow (w-9)(w-1) \geq 0 \Leftrightarrow (x^2-9)(x^2-1) \geq 0$$

$$\Leftrightarrow (x-3)(x+3)(x-1)(x+1) \geq 0$$

x	-3	-1	1	3
x^2-9	+ 0 -	-	-	- 0 +
x^2-1	+	+ 0 -	- 0 +	+
ΠΙΝ	+ 0 -	- 0 +	+ 0 -	- 0 +

$$x \in (-\infty, -3] \cup [-1, 1] \cup [3, +\infty)$$

(B) $|x|^2 - 7|x| + 6 < 0 \quad \text{θετ } w \quad w = |x|$

$$w^2 - 7w + 6 < 0 \quad \Delta = 49 - 24 = 25$$

$$w_{1,2} = \frac{7 \pm 5}{2} < \begin{matrix} 6 \\ 1 \end{matrix}$$

$$(w-6)(w-1) < 0 \Leftrightarrow (|x|-6)(|x|-1) < 0$$

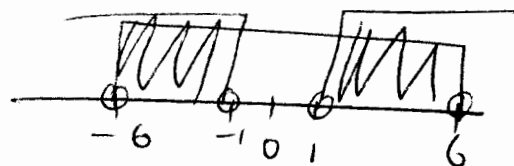
x	1	6
$w^2 - 7w + 6$	+ 0 -	- 0 +

$$w \in (1, 6) \quad \vee \quad 1 < w < 6$$

$$\Leftrightarrow 1 < |x| < 6$$

$$|x| > 1 \Leftrightarrow x > 1 \vee x < -1$$

$$|x| < 6 \Leftrightarrow -6 < x < 6$$



$$\text{Τελικά } x \in (-6, -1) \cup (1, 6)$$

$$(7) |x^2 + 3x - 1| < 3 \Leftrightarrow -3 < x^2 + 3x - 1 < 3$$

$$\Leftrightarrow -3 < x^2 + 3x - 1 \quad \text{ka}1 \quad x^2 + 3x - 1 < 3$$

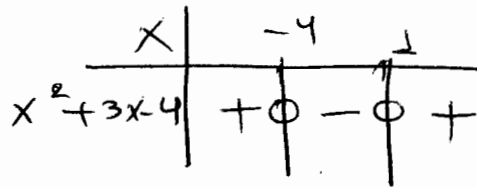
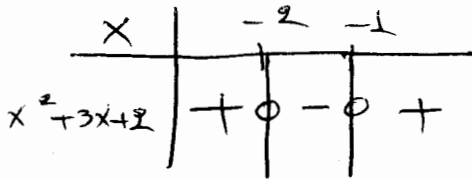
$$\Leftrightarrow x^2 + 3x + 2 > 0 \quad \text{ka}1 \quad x^2 + 3x - 4 < 0$$

$$\Delta = 9 - 8 = 1$$

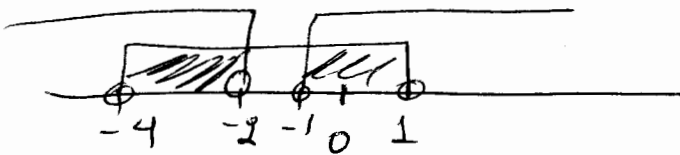
$$\text{ka}1 \quad \Delta = 9 + 16 = 25$$

$$x_{1,2} = \frac{-3 \pm 1}{2} \begin{cases} -1 \\ -2 \end{cases}$$

$$x_{1,2} = \frac{-3 \pm 5}{2} \begin{cases} 1 \\ -4 \end{cases}$$



$$x \in (-\infty, -2) \cup (-1, +\infty) \quad \text{ka}1 \quad x \in (-4, 1)$$



$$x \in (-4, -2) \cup (-1, 1)$$

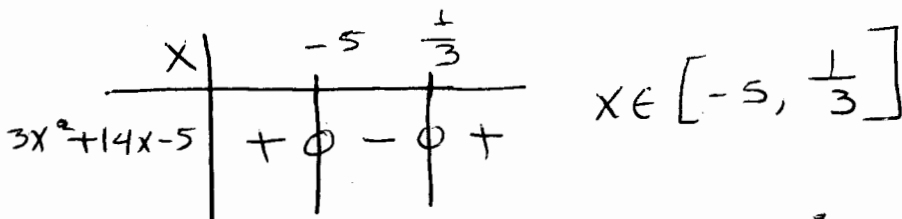
$$(8) |x-3| \geq 2|x+1|$$

$$\Leftrightarrow (x-3)^2 \geq 4(x+1)^2 \Leftrightarrow x^2 - 6x + 9 \geq 4x^2 + 8x + 4$$

$$\Leftrightarrow 3x^2 + 14x - 5 \leq 0$$

$$\Delta = 196 + 60 = 256$$

$$x_{1,2} = \frac{-14 \pm 16}{6} \begin{cases} \frac{2}{6} = \frac{1}{3} \\ -\frac{30}{6} = -5 \end{cases}$$

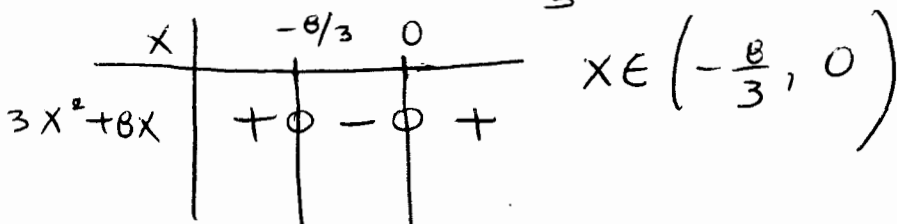


$$x \in [-5, \frac{1}{3}]$$

$$(9) 2|x+2| < |x+4| \Leftrightarrow 4(x+2)^2 < (x+4)^2$$

$$\Leftrightarrow 4x^2 + 16x + 16 < x^2 + 8x + 16 \Leftrightarrow 3x^2 + 8x < 0$$

$$\Leftrightarrow x = 0 \vee x = -\frac{8}{3}$$



$$x \in (-\frac{8}{3}, 0)$$

Axonon 14

$$x^2 + (\lambda - 3)x + 6 - \lambda = 0$$

$$\text{Θελωτε } \Delta > 0 \Leftrightarrow (\lambda - 3)^2 - 4 \cdot 1 \cdot (6 - \lambda) > 0$$

$$\Leftrightarrow \lambda^2 - 6\lambda + 9 - 24 + 4\lambda > 0$$

$$\Leftrightarrow \lambda^2 - 2\lambda - 15 > 0 \quad \Delta' = 4 + 60 = 8^2$$

$$\lambda_{1,2} = \frac{2 \pm 8}{2} \begin{matrix} 5 \\ -3 \end{matrix}$$

x	$-\infty$	-3	5	$+\infty$
		+	-	+

$$\lambda \in (-\infty, -3) \cup (5, +\infty)$$

Axonon 15

$$-x^2 + (\lambda + 5)x - 3\lambda - 7 = 0$$

$$\Delta < 0 \Leftrightarrow (\lambda + 5)^2 - 4(-1)(-3\lambda - 7) < 0$$

$$\Leftrightarrow \lambda^2 + 10\lambda + 25 - 12\lambda - 28 < 0$$

$$\Leftrightarrow \lambda^2 - 2\lambda - 3 < 0 \Leftrightarrow \lambda \in (-1, 3)$$

Axonon 16

$$\text{Περνυ } P < 0 \Leftrightarrow \frac{P}{a} < 0 \Leftrightarrow \frac{2\lambda^2 - 5\lambda - 12}{-2} < 0$$

$$\Leftrightarrow 2\lambda^2 - 5\lambda - 12 > 0 \quad \Delta = 25 + 96 = 121$$

$$\lambda_{1,2} = \frac{5 \pm 11}{4} \begin{matrix} 4 \\ -\frac{3}{2} \end{matrix}$$

λ	$-\infty$	$-\frac{3}{2}$	4	$+\infty$
$2\lambda^2 - 5\lambda - 12$		+	-	+

$$\lambda \in (-\infty, -\frac{3}{2}) \cup (4, +\infty)$$

Άσκηση 17

$$(a) x^2 + (2\mu - 1)x + (\mu - 1)(\mu + 1)$$

$$\text{Πρέπει } \Delta < 0 \Leftrightarrow (2\mu - 1)^2 - 4 \cdot 1 \cdot (\mu^2 - 1) < 0$$

$$\Leftrightarrow (4\mu^2 - 4\mu + 1) - 4\mu^2 + 4 < 0$$

$$\Leftrightarrow \cancel{4\mu^2} - 4\mu + 1 - \cancel{4\mu^2} + 4 < 0$$

$$\Leftrightarrow -4\mu < -5 \Leftrightarrow \mu > \frac{5}{4} \quad \mu \in \left(\frac{5}{4}, +\infty\right)$$

$$(b) (\mu + 5)x^2 + (\mu + 2)x + 1$$

$$\bullet \text{ Αν } \mu + 5 = 0 \Leftrightarrow \mu = -5$$

$$0x^2 - 3x + 1 = -3x + 1 \quad 1^{\text{ος}} \text{ Βαθμιαίος}$$

$$\bullet \text{ Αν } \mu \neq -5$$

$$\Delta < 0 \Leftrightarrow (\mu + 2)^2 - 4(\mu + 5) \cdot 1 < 0$$

$$\Leftrightarrow \mu^2 + \cancel{4\mu} + 4 - \cancel{4\mu} - 20 < 0$$

$$\Leftrightarrow \mu^2 - 16 < 0 \Leftrightarrow |\mu|^2 < 16 \Leftrightarrow |\mu| < 4 \Leftrightarrow -4 < \mu < 4$$

$$\mu \in (-4, 4)$$

Άσκηση 18

$$(\lambda - 1)x^2 + 4x + \lambda + 2 = 0$$

$$\text{Πρέπει } a > 0 \text{ και } \Delta < 0$$

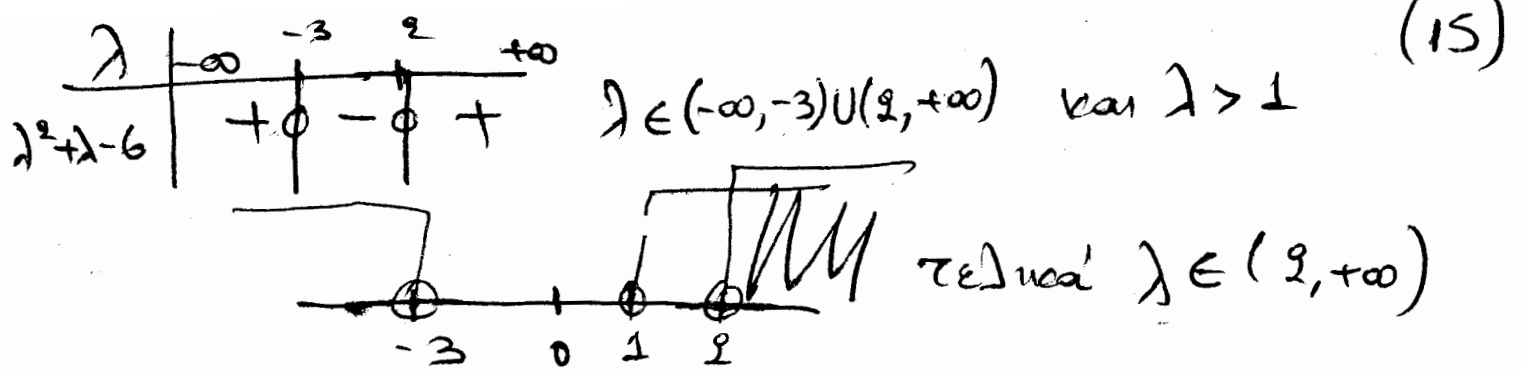
$$\lambda - 1 > 0 \Leftrightarrow \lambda > 1$$

$$\Delta = 16 - 4(\lambda - 1)(\lambda + 2) < 0$$

$$\Leftrightarrow 16 - 4(\lambda^2 + 2\lambda - \lambda - 2) < 0$$

$$\Leftrightarrow 16 - 4\lambda^2 - 4\lambda + 8 < 0 \Leftrightarrow -4\lambda^2 - 4\lambda + 24 < 0$$

$$\Leftrightarrow \lambda^2 + \lambda - 6 > 0 \quad \Delta' = 1 + 24 = 25 \quad \lambda_{1,2} = \frac{-1 \pm 5}{2} \begin{matrix} 2 \\ -3 \end{matrix}$$



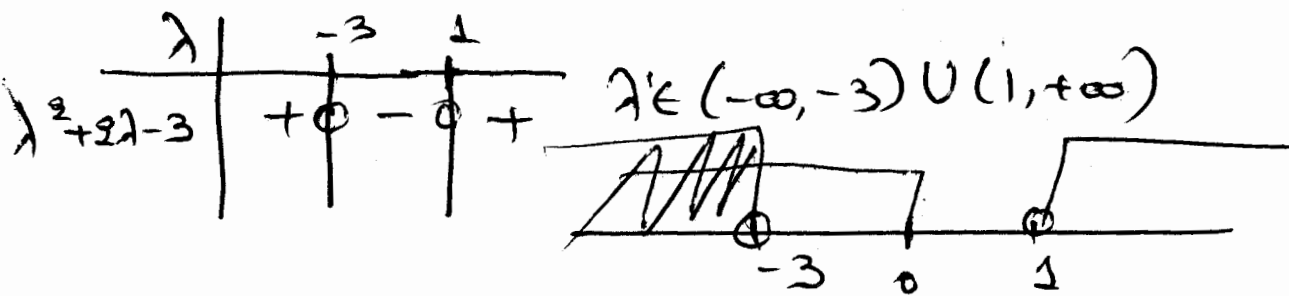
Άσκηση 19

Πρέπει $\lambda < 0$ και $\Delta < 0$

$$\Delta < 0 \Leftrightarrow (\lambda - 3)^2 - 4 \cdot \lambda \cdot 1 < 0 \Leftrightarrow \lambda^2 - 6\lambda + 9 - 4\lambda < 0$$

$$\Leftrightarrow -3\lambda^2 - 6\lambda + 9 < 0 \Leftrightarrow \lambda^2 + 2\lambda - 3 > 0$$

$$\Delta' = 4 + 12 = 16 \quad \lambda_{1,2} = \frac{-2 \pm 4}{2} < -3$$



$\tau \in \Delta$ και $\lambda \in (-\infty, -3)$

Άσκηση 20

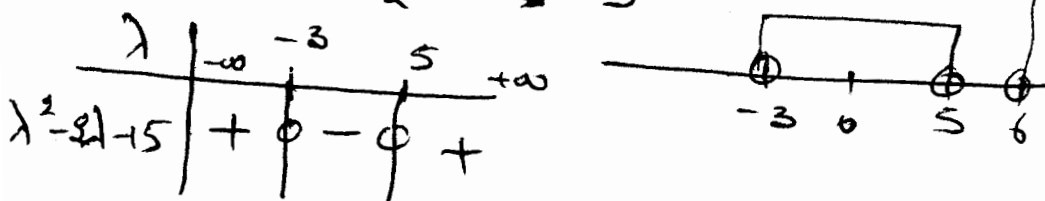
Πρέπει $\lambda - 6 < 0$ και $\Delta < 0$

$$\Leftrightarrow \boxed{\lambda < 6} \text{ και } \Delta = (\lambda - 3)^2 - 4(\lambda - 6)(-1) < 0$$

$$\Leftrightarrow \lambda^2 - 6\lambda + 9 + 4\lambda - 24 < 0$$

$$\Leftrightarrow \lambda^2 - 2\lambda - 15 < 0 \quad \Delta' = 4 + 60 = 64$$

$$\lambda_{1,2} = \frac{2 \pm 8}{2} \begin{matrix} 5 \\ -3 \end{matrix}$$



$\tau \in \Delta$ και $\lambda \in (-3, 5)$

Άσκηση 21

Πρέπει $\mu - 2 > 0$ και $\Delta < 0$

$$\Rightarrow \boxed{\mu > 2} \quad \text{και} \quad \Delta = 4\mu^2 - 4(\mu - 2)(2\mu - 3) < 0$$

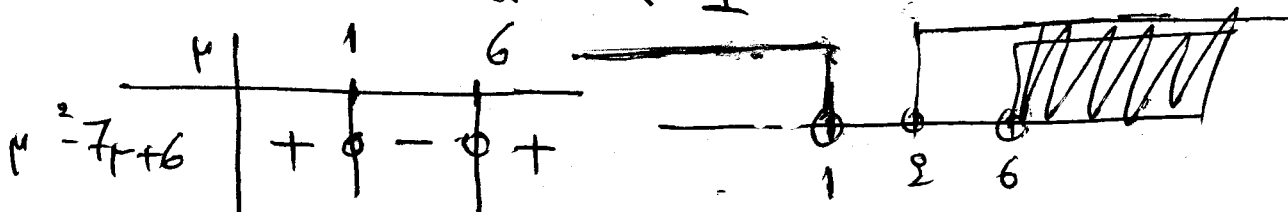
$$\Rightarrow \Delta = 4\mu^2 - 4(2\mu^2 - 3\mu - 4\mu + 6) < 0$$

$$\Rightarrow \Delta = 4\mu^2 - 8\mu^2 + 28\mu - 24 < 0$$

$$\Rightarrow \Delta = -4\mu^2 + 28\mu - 24 < 0$$

$$\Rightarrow \Delta = \mu^2 - 7\mu + 6 > 0 \quad \Delta' = 49 - 24 = 25$$

$$\mu_{1,2} = \frac{7 \pm 5}{2} \quad \begin{matrix} 6 \\ 1 \end{matrix}$$



τε) και $\mu \in (6, +\infty)$