

This is the odd–even rounding rule. For example, consider the number 3.250 000 0... where the zeros continue indefinitely. How does this number round to 2 s.f.? Because the digit before the 5 is **even** we do not round up, so 3.250 000 0... becomes 3.2. But 3.350 000 0... rounds up to 3.4 because the digit before the 5 is **odd**.

Nature of science

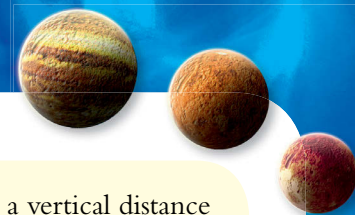
Early work on electricity and magnetism was hampered by the use of different systems of units in different parts of the world. Scientists realised they needed to have a common system of units in order to learn from each other's work and reproduce experimental results described by others. Following an international review of units that began in 1948, the SI system was introduced in 1960. At that time there were six base units. In 1971 the mole was added, bringing the number of base units to the seven in use today.

As the instruments used to measure quantities have developed, the definitions of standard units have been refined to reflect the greater precision possible. Using the transition of the caesium-133 atom to measure time has meant that smaller intervals of time can be measured accurately. The SI system continues to evolve to meet the demands of scientists across the world. Increasing precision in measurement allows scientists to notice smaller differences between results, but there is always uncertainty in any experimental result. There are no 'exact' answers.



Test yourself

- 1 How long does light take to travel across a proton?
- 2 How many hydrogen atoms does it take to make up the mass of the Earth?
- 3 What is the age of the universe expressed in units of the Planck time?
- 4 How many heartbeats are there in the lifetime of a person (75 years)?
- 5 What is the mass of our galaxy in terms of a solar mass?
- 6 What is the diameter of our galaxy in terms of the astronomical unit, i.e. the distance between the Earth and the Sun ($1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$)?
- 7 The molar mass of water is 18 g mol^{-1} . How many molecules of water are there in a glass of water (mass of water 300 g)?
- 8 Assuming that the mass of a person is made up entirely of water, how many molecules are there in a human body (of mass 60 kg)?
- 9 Give an order-of-magnitude estimate of the density of a proton.
- 10 How long does light take to traverse the diameter of the solar system?
- 11 An electron volt (eV) is a unit of energy equal to $1.6 \times 10^{-19} \text{ J}$. An electron has a kinetic energy of 2.5 eV.
 - a How many joules is that?
 - b What is the energy in eV of an electron that has an energy of $8.6 \times 10^{-18} \text{ J}$?
- 12 What is the volume in cubic metres of a cube of side 2.8 cm?
- 13 What is the side in metres of a cube that has a volume of 588 cubic millimetres?
- 14 Give an order-of-magnitude estimate for the mass of:
 - a an apple
 - b this physics book
 - c a soccer ball.



- 15 A white dwarf star has a mass about that of the Sun and a radius about that of the Earth. Give an order-of-magnitude estimate of the density of a white dwarf.
- 16 A sports car accelerates from rest to 100 km per hour in 4.0 s. What fraction of the acceleration due to gravity is the car's acceleration?
- 17 Give an order-of-magnitude estimate for the number of electrons in your body.
- 18 Give an order-of-magnitude estimate for the ratio of the electric force between two electrons 1 m apart to the gravitational force between the electrons.
- 19 The frequency f of oscillation (a quantity with units of inverse seconds) of a mass m attached to a spring of spring constant k (a quantity with units of force per length) is related to m and k . By writing $f = cm^x k^y$ and matching units on both sides, show that $f = c\sqrt{\frac{k}{m}}$, where c is a dimensionless constant.
- 20 A block of mass 1.2 kg is raised a vertical distance of 5.55 m in 2.450 s. Calculate the power delivered. ($P = \frac{mgh}{t}$ and $g = 9.81 \text{ m s}^{-2}$)
- 21 Find the kinetic energy ($E_K = \frac{1}{2}mv^2$) of a block of mass 5.00 kg moving at a speed of 12.5 m s^{-1} .
- 22 Without using a calculator, **estimate** the value of the following expressions. Then compare your estimate with the exact value found using a calculator.
- a $\frac{243}{43}$
- b 2.80×1.90
- c $312 \times \frac{480}{160}$
- d $\frac{8.99 \times 10^9 \times 7 \times 10^{-16} \times 7 \times 10^{-6}}{(8 \times 10^2)^2}$
- e $\frac{6.6 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2}$

1.2 Uncertainties and errors

This section introduces the basic methods of dealing with experimental error and uncertainty in measured physical quantities. Physics is an experimental science and often the experimenter will perform an experiment to test the prediction of a given theory. No measurement will ever be completely accurate, however, and so the result of the experiment will be presented with an experimental error.

Types of uncertainty

There are two main types of uncertainty or error in a measurement. They can be grouped into **systematic** and **random**, although in many cases it is not possible to distinguish clearly between the two. We may say that random uncertainties are almost always the fault of the observer, whereas systematic errors are due to both the observer and the instrument being used. In practice, all uncertainties are a combination of the two.

Systematic errors

A **systematic error** biases measurements in the same direction; the measurements are always too large or too small. If you use a metal ruler to measure length on a very hot day, all your length measurements will be too small because the metre ruler expanded in the hot weather. If you use an ammeter that shows a current of 0.1 A even before it is connected to

Learning objectives

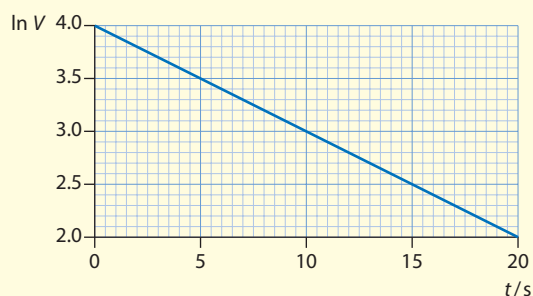
- Distinguish between random and systematic uncertainties.
- Work with absolute, fractional and percentage uncertainties.
- Use error bars in graphs.
- Calculate the uncertainty in a gradient or an intercept.



Test yourself

- 23 The magnitudes of two forces are measured to be 120 ± 5 N and 60 ± 3 N. Find the sum and difference of the two magnitudes, giving the uncertainty in each case.
- 24 The quantity Q depends on the measured values a and b in the following ways:
- $Q = \frac{a}{b}$, $a = 20 \pm 1$, $b = 10 \pm 1$
 - $Q = 2a + 3b$, $a = 20 \pm 2$, $b = 15 \pm 3$
 - $Q = a - 2b$, $a = 50 \pm 1$, $b = 24 \pm 1$
 - $Q = a^2$, $a = 10.0 \pm 0.3$
 - $Q = \frac{a^2}{b^2}$, $a = 100 \pm 5$, $b = 20 \pm 2$
- In each case, find the value of Q and its uncertainty.
- 25 The centripetal force is given by $F = \frac{mv^2}{r}$. The mass is measured to be 2.8 ± 0.1 kg, the velocity 14 ± 2 m s⁻¹ and the radius 8.0 ± 0.2 m; find the force on the mass, including the uncertainty.
- 26 The radius r of a circle is measured to be 2.4 cm $\pm$ 0.1 cm. Find the uncertainty in:
- the area of the circle
 - the circumference of the circle.
- 27 The sides of a rectangle are measured as 4.4 ± 0.2 cm and 8.5 ± 0.3 cm. Find the area and perimeter of the rectangle.
- 28 The length L of a pendulum is increased by 2%. Find the percentage increase in the period T .
- $$\left(T = 2\pi\sqrt{\frac{L}{g}}\right)$$
- 29 The volume of a cone of base radius R and height h is given by $V = \frac{\pi R^2 h}{3}$. The uncertainty in the radius and in the height is 4%. Find the percentage uncertainty in the volume.
- 30 In an experiment to measure current and voltage across a device, the following data was collected: $(V, I) = \{(0.1, 26), (0.2, 48), (0.3, 65), (0.4, 90)\}$. The current was measured in mA and the voltage in mV. The uncertainty in the current was ± 4 mA. Plot the current versus the voltage and draw the best-fit line through the points. Suggest whether the current is proportional to the voltage.

- 31 In a similar experiment to that in question 30, the following data was collected for current and voltage: $(V, I) = \{(0.1, 27), (0.2, 44), (0.3, 60), (0.4, 78)\}$ with an uncertainty of ± 4 mA in the current. Plot the current versus the voltage and draw the best-fit line. Suggest whether the current is proportional to the voltage.
- 32 A circle and a square have the same perimeter. Which shape has the larger area?
- 33 The graph shows the natural logarithm of the voltage across a capacitor of capacitance $C = 5.0$ μ F as a function of time. The voltage is given by the equation $V = V_0 e^{-t/RC}$, where R is the resistance of the circuit. Find:
- the initial voltage
 - the time for the voltage to be reduced to half its initial value
 - the resistance of the circuit.



- 34 The table shows the mass M of several stars and their corresponding luminosity L (power emitted).
- Plot L against M and draw the best-fit line.
 - Plot the logarithm of L against the logarithm of M . Use your graph to find the relationship between these quantities, assuming a power law of the kind $L = kM^\alpha$. Give the numerical value of the parameter α .

Mass M (in solar masses)	Luminosity L (in terms of the Sun's luminosity)
1.0 ± 0.1	1 ± 0
3.0 ± 0.3	42 ± 4
5.0 ± 0.5	230 ± 20
12 ± 1	4700 ± 50
20 ± 2	$26\,500 \pm 300$

Nature of science

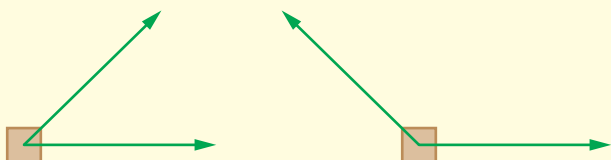
For thousands of years, people across the world have used maps to navigate from one place to another, making use of the ideas of distance and direction to show the relative positions of places. The concept of vectors and the algebra used to manipulate them were introduced in the first half of the 19th century to represent real and complex numbers in a geometrical way. Mathematicians developed the model and realised that there were two distinct parts to their directed lines – scalars and vectors. Scientists and mathematicians saw that this model could be applied to theoretical physics, and by the middle of the 19th century vectors were being used to model problems in electricity and magnetism.

Resolving a vector into components and reconstructing the vector from its components are useful mathematical techniques for dealing with measurements in three-dimensional space. These mathematical techniques are invaluable when dealing with physical quantities that have both magnitude and direction, such as calculating the effect of multiple forces on an object. In this section you have done this in two dimensions, but vector algebra can be applied to three dimensions and more.



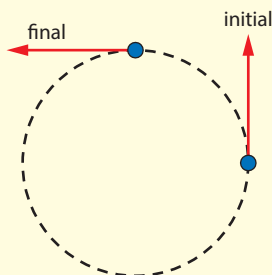
Test yourself

- 35 A body is acted upon by the two forces shown in the diagram. In each case draw the one force whose effect on the body is the same as the two together.

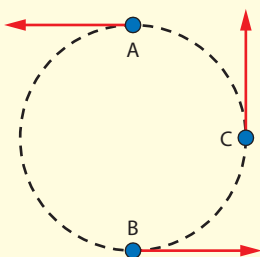


- 36 Vector A has a magnitude of 12.0 units and makes an angle of 30° with the positive x -axis. Vector B has a magnitude of 8.00 units and makes an angle of 80° with the positive x -axis. Using a graphical method, find the magnitude and direction of the vectors:
- a** $A + B$ **b** $A - B$ **c** $A - 2B$
- 37 Repeat the previous problem, this time using components.
- 38 Find the magnitude and direction of the vectors with components:
- a** $A_x = -4.0$ cm, $A_y = -4.0$ cm
b $A_x = 124$ km, $A_y = -158$ km
c $A_x = 0$, $A_y = -5.0$ m
d $A_x = 8.0$ N, $A_y = 0$
- 39 The components of vectors A and B are as follows: $(A_x = 2.00, A_y = 3.00)$, $(B_x = -2.00, B_y = 5.00)$. Find the magnitude and direction of the vectors:
- a** A **b** B **c** $A + B$
d $A - B$ **e** $2A - B$
- 40 The position vector of a moving object has components $(r_x = 2, r_y = 2)$ initially. After a certain time the position vector has components $(r_x = 4, r_y = 8)$. Find the displacement vector.

- 41 The diagram shows the velocity vector of a particle moving in a circle with speed 10 ms^{-1} at two separate points. The velocity vector is tangential to the circle. Find the vector representing the **change** in the velocity vector.



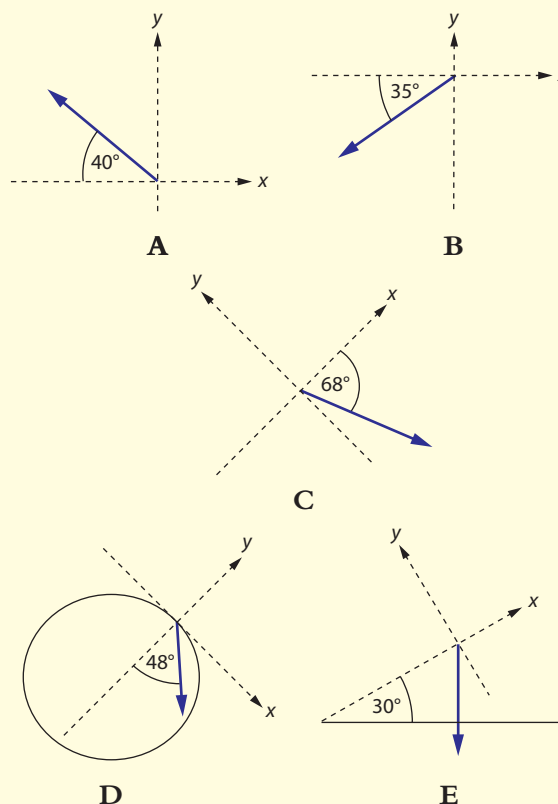
- 42 In a certain collision, the momentum vector of a particle changes direction but not magnitude. Let $\mathbf{p}$ be the momentum vector of a particle suffering an elastic collision and changing direction by 30° . Find, in terms of p ($= |\mathbf{p}|$), the magnitude of the vector representing the change in the momentum vector.
- 43 The velocity vector of an object moving on a circular path has a direction that is tangential to the path (see diagram).



If the speed (magnitude of velocity) is constant at 4.0 ms^{-1} , find the change in the velocity vector as the object moves:

- from A to B
- from B to C.
- What is the change in the velocity vector from A to C? How is this related to your answers to **a** and **b**?

- 44 For each diagram, find the components of the vectors along the axes shown. Take the magnitude of each vector to be 10.0 units.



- 45 Vector **A** has a magnitude of 6.00 units and is directed at 60° to the positive x -axis. Vector **B** has a magnitude of 6.00 units and is directed at 120° to the positive x -axis. Find the magnitude and direction of vector **C** such that $\mathbf{A} + \mathbf{B} + \mathbf{C} = 0$. Place the three vectors so that one begins where the previous ends. What do you observe?
- 46 Plot the following pairs of vectors on a set of x - and y -axes. The angles given are measured counter-clockwise from the positive x -axis. Then, using the algebraic component method, find their sum in magnitude and direction.
- 12.0 N at 20° and 14.0 N at 50°
 - 15.0 N at 15° and 18.0 N at 105°
 - 20.0 N at 40° and 15.0 N at 310° (i.e. -50°)

Exam-style questions

1 What is the equivalent of 80 years in seconds?

- A** 10^7 **B** 10^9 **C** 10^{11} **D** 10^{13}

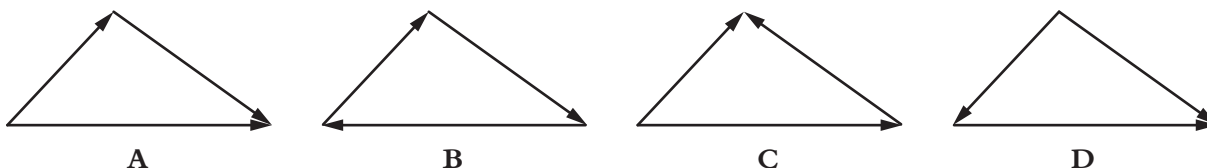
2 A book has 500 pages (printed on both sides). The width of the book excluding the covers is 2.5 cm. What is the approximate width in mm of one sheet of paper?

- A** 0.01 **B** 0.1 **C** 0.5 **D** 1.0

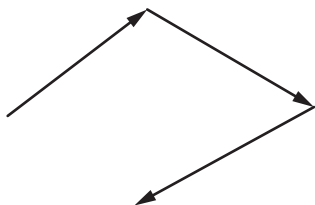
3 The speed of sound is approximately 330 m s^{-1} . A storm is 3 km away. Approximately how much later after seeing lightning will thunder be heard?

- A** 0.1 s **B** 1 s **C** 3 s **D** 10 s

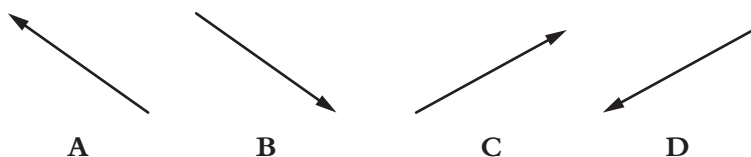
4 In which of the following diagrams do the three forces add up to zero?



5 Three forces act on a body as shown.



Which fourth force is required so that the four forces add up to zero?

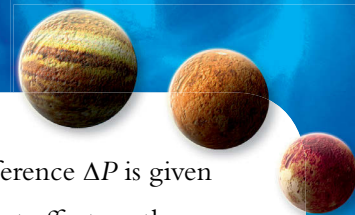


6 A force of 25 N acts normally on a surface of area 5.0 cm^2 . What is the pressure on the surface in N m^{-2} ?

- A** 5 **B** 5×10^4 **C** 5.0 **D** 5.0×10^4

7 The side of a cube is measured with an uncertainty of 2%. What is the uncertainty in the volume of the cube?

- A** 2% **B** 4% **C** 6% **D** 8%



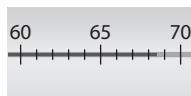
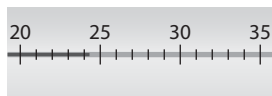
- 8 The flow rate Q through a tube of length L and radius r whose ends are kept at a pressure difference ΔP is given by $Q = \frac{cr^4\Delta P}{L}$, where c is a constant. The percentage uncertainty of which quantity has the largest effect on the percentage uncertainty in Q ?

- A r
- B ΔP
- C L
- D r, L and ΔP each give the same contribution

- 9 The force of air resistance F on a car depends on speed v through the formula $F = av^2 + bv$, where a and b are constants. Which of the following graphs will result in a straight-line graph?

- A F against v
- B F against v^2
- C $\frac{F}{v}$ against v
- D $\frac{F}{v}$ against $\frac{1}{v}$

- 10 The diagram shows the temperature of a liquid before and after heating.



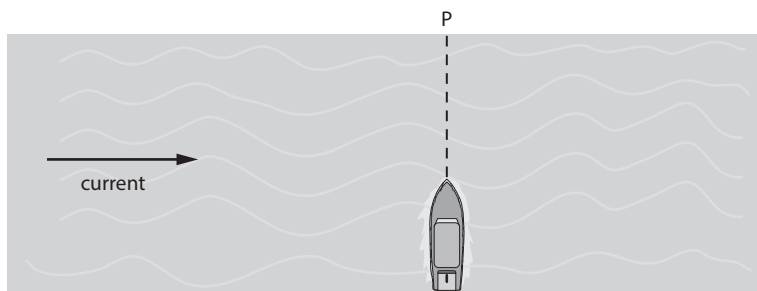
What is the best estimate for the temperature increase of the liquid?

- A (44.0 ± 0.5) degrees
 - B (44 ± 1.0) degrees
 - C (44 ± 1) degrees
 - D (44.0 ± 2.0) degrees
- 11 A student wishes to measure the acceleration of free fall by letting a ping pong ball drop from one fixed height from the floor. He measures the height. Using a stopwatch, he measures the time for the ball to drop to the floor. He then uses the equation $h = \frac{1}{2}gt^2$ to calculate g .

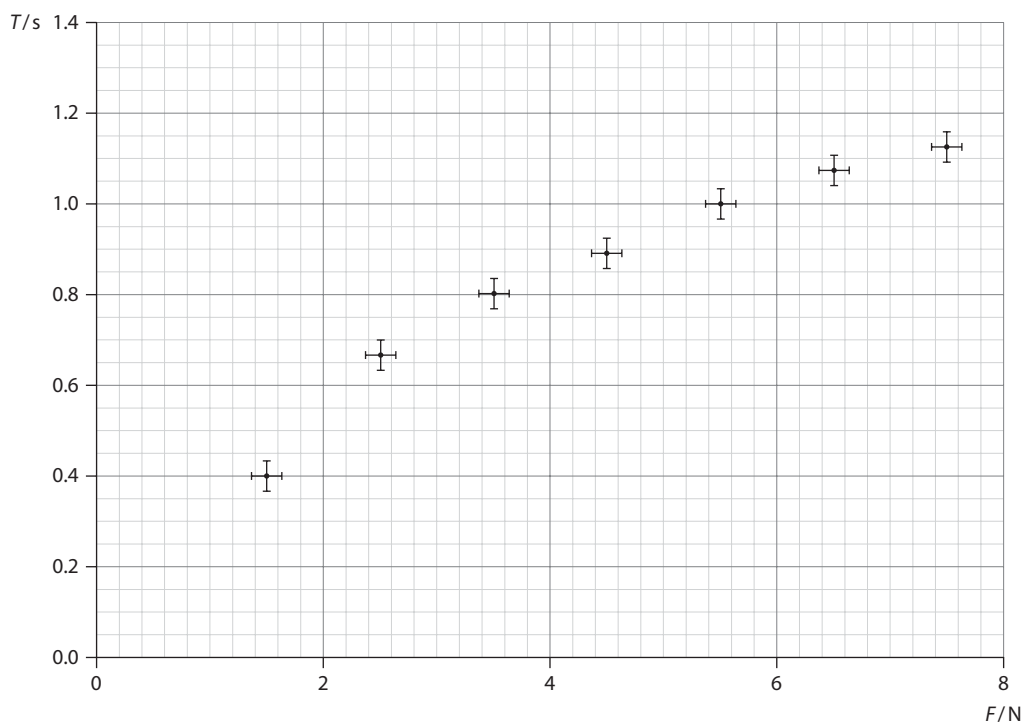
State and discuss **three** improvements to the student's lab experiment.

[6]

- 12 A man wants to cross a river with a motorboat. The speed of the motorboat in still water is 4.0 m s^{-1} . The river is 30 m wide. There is a current in the river whose speed with respect to the shore is 3.0 m s^{-1} .



- The man aims the boat towards P. Determine the distance from P at which he will reach the shore. [2]
 - A woman in an identical boat leaves from the same spot as the man but wants to land at P. Determine the direction in which she has to turn her boat to do this. [3]
 - Determine which person reaches the shore in the least time. [2]
- 13 A student investigated the oscillation period, T , of a clamped rod for various loads F applied to the rod. She graphed the following results.



- Copy the graph and draw the best-fit line for these data. [2]
- Predict the period of oscillation of the rod when no load is applied to it. [1]
- The student claims that T is proportional to F . Explain to the student how the results show she is not correct. [2]
- Determine the absolute uncertainty in T^2 for the data point corresponding to $F = 5.5 \text{ N}$. [2]
- Another student suspects that T^2 is proportional to F . By drawing a graph of T^2 against F discuss whether this student's claim is correct. [4]
- Calculate the slope of the graph drawn in e, including its uncertainty. [3]

Nature of science

The simple and the complex

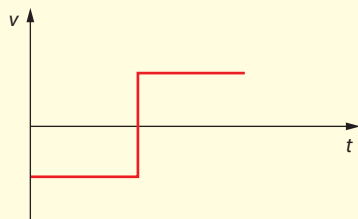
Careful observation of motion in the natural world led to the equations for motion with uniform acceleration along a straight line that we have used in this section. Thinking about what causes an object to move links to the idea of forces. However, although the material in this section is perhaps some of the 'easiest' material in your physics course, it does not enable one to understand the falling of a leaf off a tree. The falling leaf is complicated because it is acted upon by several forces: its weight, but also by air resistance forces that constantly vary as the orientation and speed of the leaf change. In addition, there is wind to consider as well as the fact that turbulence in air greatly affects the motion of the leaf. So the physics of the falling leaf is far away from the physics of motion along a straight line at constant acceleration. But learning the principles of physics in a simpler context allows its application in more involved situations.



Test yourself

Uniform motion

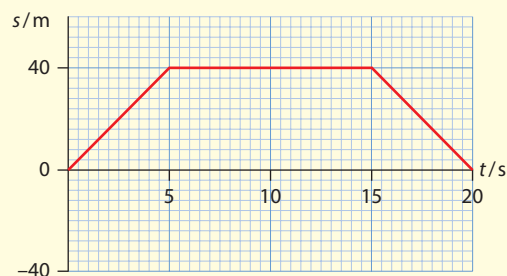
- 1 A car must be driven a distance of 120 km in 2.5 h. During the first 1.5 h the average speed was 70 km h^{-1} . Calculate the average speed for the remainder of the journey.
- 2 Draw the position–time graph for an object moving in a straight line with a velocity–time graph as shown below. The initial position is zero. You do not have to put any numbers on the axes.



- 3 Two cyclists, **A** and **B**, have displacements 0 km and 70 km, respectively. At $t = 0$ they begin to cycle towards each other with velocities 15 km h^{-1} and 20 km h^{-1} , respectively. At the same time, a fly that was sitting on **A** starts flying towards **B** with a velocity of 30 km h^{-1} . As soon as the fly reaches **B** it immediately turns around and flies towards **A**, and so on until **A** and **B** meet.
 - a Find the position of the two cyclists and the fly when all three meet.
 - b Determine the distance travelled by the fly.

- 4 An object moving in a straight line has the displacement–time graph shown.

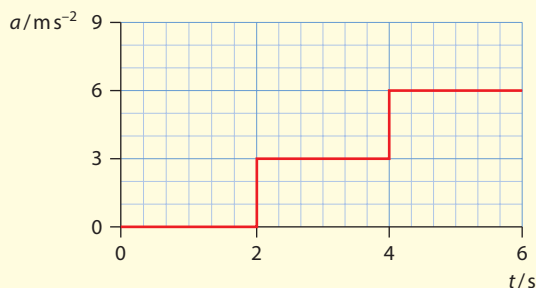
- a Find the average speed for the trip.
- b Find the average velocity for the trip.



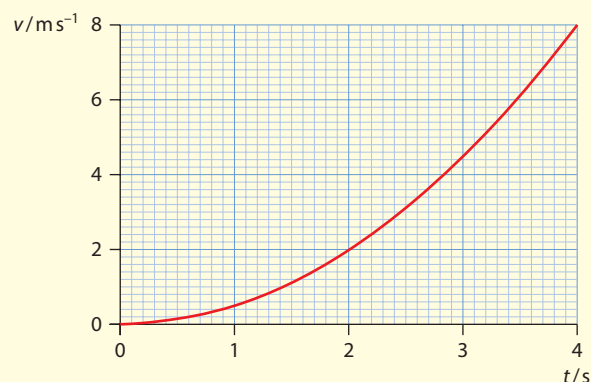
Accelerated motion

- 5 The initial velocity of a car moving on a straight road is 2.0 m s^{-1} . It becomes 8.0 m s^{-1} after travelling for 2.0 s under constant acceleration. Find the acceleration.
- 6 A car accelerates from rest to 28 m s^{-1} in 9.0 s. Find the distance it travels.
- 7 A particle has an initial velocity of 12 m s^{-1} and is brought to rest over a distance of 45 m. Find the acceleration of the particle.
- 8 A particle at the origin has an initial velocity of -6.0 m s^{-1} and moves with an acceleration of 2.0 m s^{-2} . Determine when its position will become 16 m.

- 9 A plane starting from rest takes 15.0 s to take off after speeding over a distance of 450 m on the runway with constant acceleration. Find the take-off velocity.
- 10 A car is travelling at 40.0 m s^{-1} . The driver sees an emergency ahead and 0.50 s later slams on the brakes. The deceleration of the car is 4.0 m s^{-2} .
- Find the distance travelled before the car stops.
 - Calculate the stopping distance if the driver could apply the brakes instantaneously without a reaction time.
 - Calculate the difference in your answers to **a** and **b**.
 - Assume now that the car was travelling at 30.0 m s^{-1} instead. Without performing any calculations, state whether the answer to **c** would now be less than, equal to or larger than before. Explain your answer.
- 11 Two balls are dropped from rest from the same height. One of the balls is dropped 1.00 s after the other.
- Find the distance that separates the two balls 2.00 s after the second ball is dropped.
 - State what happens to the distance separating the balls as time goes on.
- 12 A particle moves in a straight line with an acceleration that varies with time as shown in the diagram. Initially the velocity of the object is 2.00 m s^{-1} .
- Find the maximum velocity reached in the first 6.00 s of this motion.
 - Draw a graph of the velocity versus time.



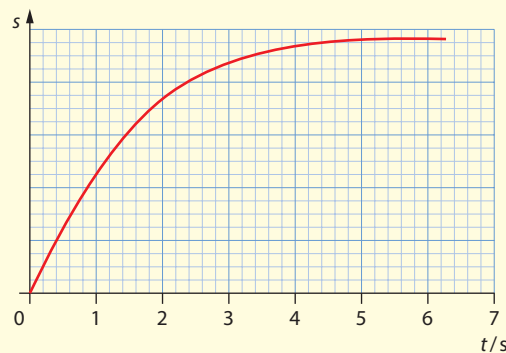
- 13 The graph shows the variation of velocity with time of an object. Find the acceleration at 2.0 s.



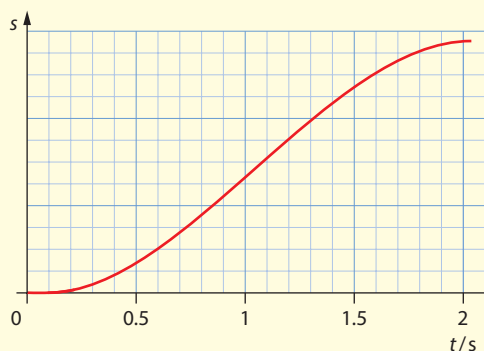
- 14 The graph shows the variation of the position of a moving object with time. Draw the graph showing the variation of the velocity of the object with time.



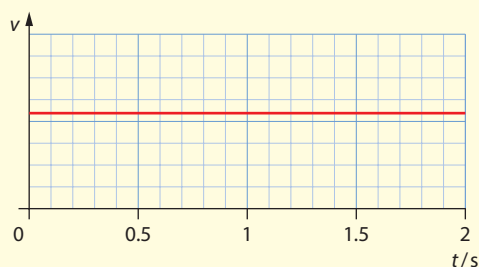
- 15 The graph shows the variation of the position of a moving object with time. Draw the graph showing the variation of the velocity of the object with time.



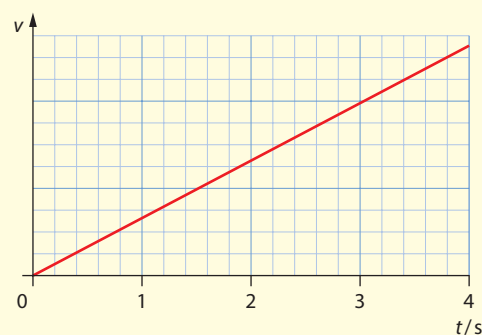
- 16 The graph shows the variation of the position of a moving object with time. Draw the graph showing the variation of the velocity of the object with time.



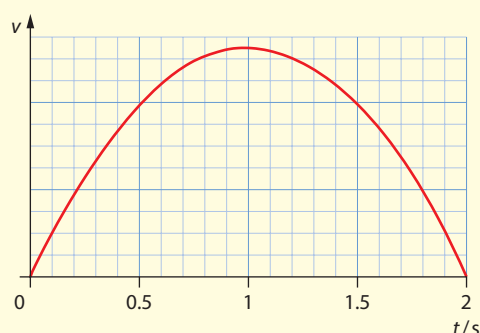
- 17 The graph shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the position of the object with time.



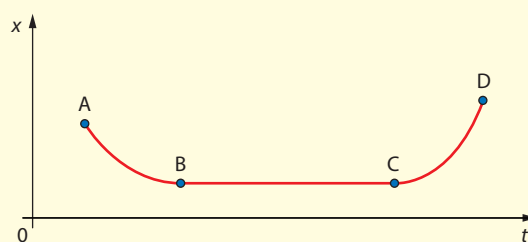
- 18 The graph shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the position of the object with time (assuming a zero initial position).



- 19 The graph shows the variation of the velocity of a moving object with time. Draw the graph showing the variation of the acceleration of the object with time.



- 20 Your brand new convertible Ferrari is parked 15 m from its garage when it begins to rain. You do not have time to get the keys, so you begin to push the car towards the garage. The maximum acceleration you can give the car is 2.0 m s^{-2} by pushing and 3.0 m s^{-2} by pulling back on the car. Find the least time it takes to put the car in the garage. (Assume that the car, as well as the garage, are point objects.)
- 21 The graph shows the displacement versus time of an object moving in a straight line. Four points on this graph have been selected.

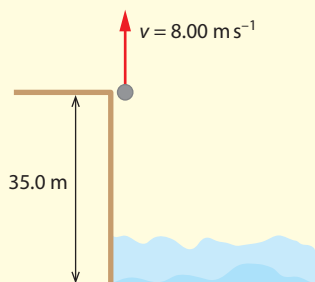


- Is the velocity between **A** and **B** positive, zero or negative?
- What can you say about the velocity between **B** and **C**?
- Is the acceleration between **A** and **B** positive, zero or negative?
- Is the acceleration between **C** and **D** positive, zero or negative?

22 Sketch velocity–time sketches (no numbers are necessary on the axes) for the following motions.

- A ball is dropped from a certain height and bounces off a hard floor. The speed just before each impact with the floor is the same as the speed just after impact. Assume that the time of contact with the floor is negligibly small.
- A cart slides with negligible friction along a horizontal air track. When the cart hits the ends of the air track it reverses direction with the same speed it had right before impact. Assume the time of contact of the cart and the ends of the air track is negligibly small.
- A person jumps from a hovering helicopter. After a few seconds she opens a parachute. Eventually she will reach a terminal speed and will then land.

23 A stone is thrown vertically up from the edge of a cliff 35.0 m from the sea. The initial velocity of the stone is 8.00 m s^{-1} .



Determine:

- the maximum height of the stone
 - the time when it hits the sea
 - the velocity just before hitting the sea
 - the distance the stone covers
 - the average speed and the average velocity for this motion.
- 24 A ball is thrown upward from the edge of a cliff with velocity 20.0 m s^{-1} . It reaches the bottom of the cliff 6.0 s later.
- Determine the height of the cliff.
 - Calculate the speed of the ball as it hits the ground.

Projectile motion

25 A ball rolls off a table with a horizontal speed of 2.0 m s^{-1} . The table is 1.3 m high. Calculate how far from the table the ball will land.

26 Two particles are on the same vertical line. They are thrown horizontally with the same speed, 4.0 m s^{-1} , from heights of 4.0 m and 8.0 m.

- Calculate the distance that will separate the two objects when both land on the ground.
- The particle at the 4.0 m height is now launched with horizontal speed u such that it lands at the same place as the particle launched from 8.0 m. Calculate u .

27 For an object thrown at an angle of 40° to the horizontal at a speed of 20 m s^{-1} , draw graphs of:

- horizontal velocity against time
- vertical velocity against time
- acceleration against time.

28 Determine the maximum height reached by an object thrown with speed 24 m s^{-1} at 40° to the horizontal.

29 An object is thrown with speed 20.0 m s^{-1} at an angle of 50° to the horizontal. Draw graphs to show the variation with time of:

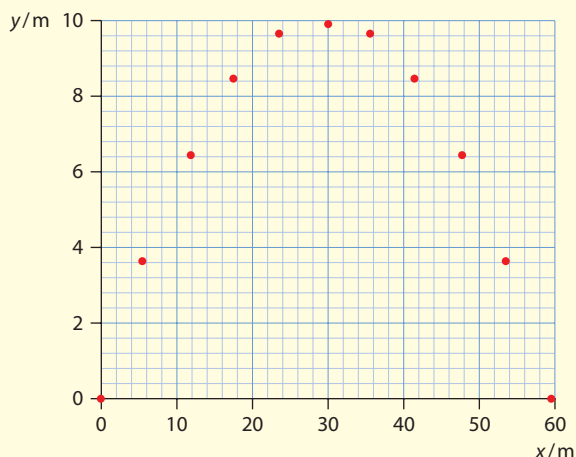
- the horizontal position
- the vertical position.

30 A cruel hunter takes aim horizontally at a chimp that is hanging from the branch of a tree, as shown in the diagram. The chimp lets go of the branch as soon as the hunter pulls the trigger. Treating the chimp and the bullet as point particles, determine if the bullet will hit the chimp.





- 31 A ball is launched from the surface of a planet. Air resistance and other frictional forces are neglected. The graph shows the position of the ball every 0.20 s.



- a Use this graph to determine:
- the components of the initial velocity of the ball
 - the angle to the horizontal the ball was launched at
 - the acceleration of free fall on this planet.

- b Make a copy of the graph and draw two arrows to represent the velocity and the acceleration vectors of the ball at $t = 1.0$ s.
- c The ball is now launched under identical conditions from the surface of a **different** planet where the acceleration due to gravity is twice as large. Draw the path of the ball on your graph.

- 32 A stone is thrown with a speed of 20.0 m s^{-1} at an angle of 48° to the horizontal from the edge of a cliff 60.0 m above the surface of the sea.
- Calculate the velocity with which the stone hits the sea.
 - Discuss qualitatively the effect of air resistance on your answer to a.
- 33 a State what is meant by **terminal speed**.
- b A ball is dropped from rest. The force of air resistance in the ball is proportional to the ball's speed. Explain why the ball will reach terminal speed.

2.2 Forces

This section is an introduction to Newton's laws of motion. Classical physics is based to a great extent on these laws. It was once thought that knowledge of the present state of a system and all forces acting on it would enable the complete prediction of the state of that system in the future. This classical version of determinism has been modified partly due to quantum theory and partly due to chaos theory.

Forces and their direction

A **force** is a vector quantity. It is important that we are able to correctly identify the **direction** of forces. In this section we will deal with the following forces.

Learning objectives

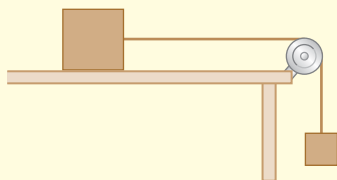
- Treat bodies as point particles.
- Construct and interpret free-body force diagrams.
- Apply the equilibrium condition, $\Sigma F = 0$.
- Understand and apply Newton's three laws of motion.
- Solve problems involving solid friction.



? Test yourself

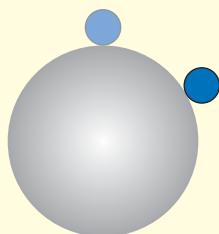
Equilibrium

- 34 A block rests on a rough table and is connected by a string that goes over a pulley to a second hanging block, as shown in the diagram. Draw the forces on each body.

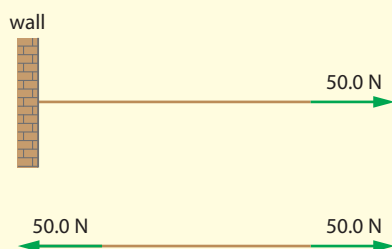


- 35 A bead rolls on the surface of a sphere, having started from the top, as shown in the diagram. On a copy of the diagram, draw the forces on the bead:

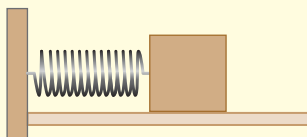
- a at the top
- b at the point where it is about to leave the surface of the sphere.



- 36 Look at the diagram. State in which case the tension in the string is largest.

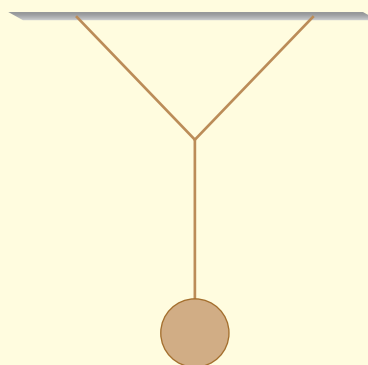


- 37 A spring is compressed by a certain distance and a mass is attached to its right end, as shown in the diagram. The mass rests on a rough table. On a copy of the diagram, draw the forces acting on the mass.

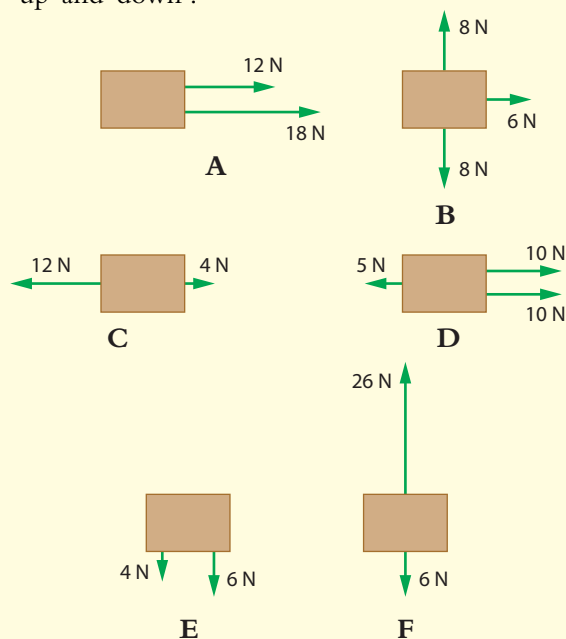


- 38 A mass hangs attached to three strings, as shown in the diagram. On a copy of the diagram, draw the forces on:

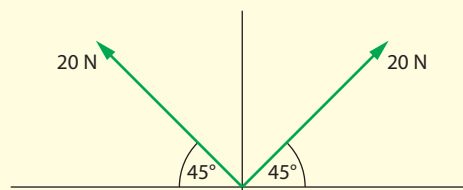
- a the hanging mass
- b the point where the strings join.



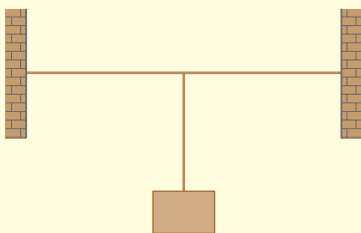
- 39 Find the net force on each of the bodies shown in the diagrams. The only forces acting are the ones shown. Indicate direction by 'right', 'left', 'up' and 'down'.



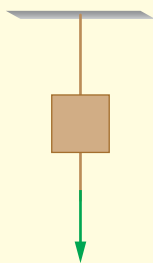
- 40 Find the magnitude and direction of the net force in the diagram.



- 41 Explain why it is impossible for a mass to hang attached to two horizontal strings as shown in the diagram.

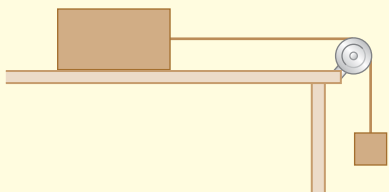


- 42 A mass is hanging from a string that is attached to the ceiling. A second piece of string (identical to the first) hangs from the lower end of the mass (see diagram).



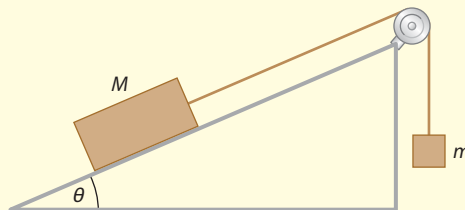
State and explain which string will break if:

- a the bottom string is slowly pulled with ever increasing force
 - b the bottom string is very abruptly pulled down.
- 43 A mass of 2.00 kg rests on a rough horizontal table. The coefficient of static friction between the block and the table is 0.60 . The block is attached to a hanging mass by a string that goes over a smooth pulley, as shown in the diagram. Determine the largest mass that can hang in this way without forcing the block to slide.



- 44 A girl tries to lift a suitcase of weight 220 N by pulling upwards on it with a force of 140 N . The suitcase does not move. Calculate the reaction force from the floor on the suitcase.

- 45 A block of mass 15.0 kg rests on a horizontal table. A force of 50.0 N is applied vertically downward on the block. Calculate the force that the block exerts on the table.
- 46 A block of mass M is connected with a string to a smaller block of mass m . The big block is resting on a smooth inclined plane as shown in the diagram. Determine the angle of the plane in terms of M and m in order to have equilibrium.



Accelerated motion

- 47 Describe under what circumstances a constant force would result in **a** an increasing and **b** a decreasing acceleration on a body.
- 48 A car of mass 1400 kg is on a muddy road. If the force from the engine pushing the car forward exceeds 600 N , the wheels slip (i.e. they rotate without rolling). Estimate the car's maximum acceleration on this road.
- 49 A man of mass m stands in an elevator.
- a Find the reaction force from the elevator floor on the man when:
 - i the elevator is standing still
 - ii the elevator moves up at constant speed v
 - iii the elevator accelerates down with acceleration a
 - iv the elevator accelerates down with acceleration $a = g$.
 - b What happens when $a > g$?
- 50 Get in an elevator and stretch out your arm holding your heavy physics book. Press the button to go up. Describe and explain what is happening to your stretched arm. Repeat as the elevator comes to a stop at the top floor. What happens when you press the button to go down and what happens when the elevator again stops? Explain your observations carefully using the second law of motion.

- 51 The diagram shows a person in an elevator pulling on a rope that goes over a pulley and is attached to the top of the elevator. The mass of the elevator is 30.0 kg and that of the person is 70 kg.

- On a copy of the diagram, draw the forces on the person.
- Draw the forces on the elevator.
- The elevator accelerates upwards at 0.50 m s^{-2} . Find the reaction force on the person from the elevator floor.
- The force the person exerts on the elevator floor is 300 N. Find the acceleration of the elevator ($g = 10 \text{ m s}^{-2}$).



- 52 A massless string has the same tension throughout its length. Suggest why.
- 53 a Calculate the tension in the string joining the two masses in the diagram.
b If the position of the masses is interchanged, will the tension change?



- 54 A mass of 3.0 kg is acted upon by three forces of 4.0 N, 6.0 N and 9.0 N and is in equilibrium. Convince yourself that these forces can indeed be in equilibrium. The 9.0 N force is suddenly removed. Determine the acceleration of the mass.

Learning objectives

- Understand the concepts of kinetic, gravitational potential and elastic potential energy.
- Understand work done as energy transferred.
- Understand power as the rate of energy transfer.
- Understand and apply the principle of energy conservation.
- Calculate the efficiency in energy transfers.

2.3 Work, energy and power

This section deals with energy, one of the most basic concepts in physics. We introduce the principle of energy conservation and learn how to apply it to various situations. We define kinetic and potential energy, work done and power developed.

Energy

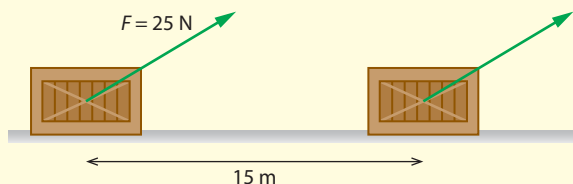
Energy is a concept that we all have an intuitive understanding of. Chemical energy derived from food keeps us alive. Chemical energy from gasoline powers our cars. Electrical energy keeps our computers going. Nuclear fusion energy produces light and heat in the Sun that sustains life on Earth. And so on. Very many experiments, from the subatomic to the cosmic scale, appear to be consistent with the principle of **conservation of energy** that states that energy is not created or destroyed but is only transformed from one form into another. This means that any change in the energy of a system must be accompanied by a change in the energy of the surroundings of the system such that:

$$\Delta E_{\text{system}} + \Delta E_{\text{surroundings}} = 0$$

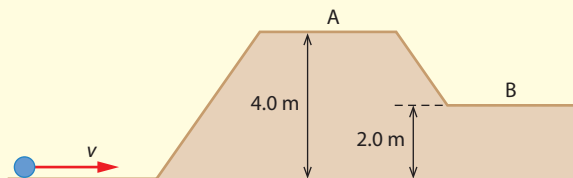


? Test yourself

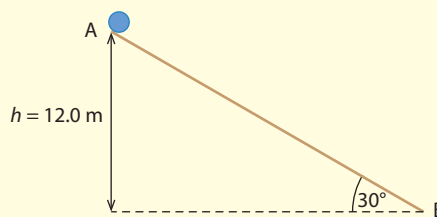
- 55 A horizontal force of 24 N pulls a body a distance of 5.0 m along its direction. Calculate the work done by the force.
- 56 A block slides along a rough table and is brought to rest after travelling a distance of 2.4 m. A force of 3.2 N opposes the motion. Calculate the work done by the opposing force.
- 57 A block is pulled as shown in the diagram by a force making an angle of 20° to the horizontal. Find the work done by the pulling force when its point of application has moved 15 m.



- 58 A block of mass 2.0 kg and an initial speed of 5.4 m s^{-1} slides on a rough horizontal surface and is eventually brought to rest after travelling a distance of 4.0 m. Calculate the frictional force between the block and the surface.
- 59 A spring of spring constant $k = 200 \text{ N m}^{-1}$ is slowly extended from an extension of 3.0 cm to an extension of 5.0 cm. Calculate the work done by the extending force.
- 60 Look at the diagram.
- Calculate the minimum speed v the ball must have in order to make it to position B.
 - What speed will the mass have at B?
 - Given that $v = 12.0 \text{ m s}^{-1}$, calculate the speed at A and B.

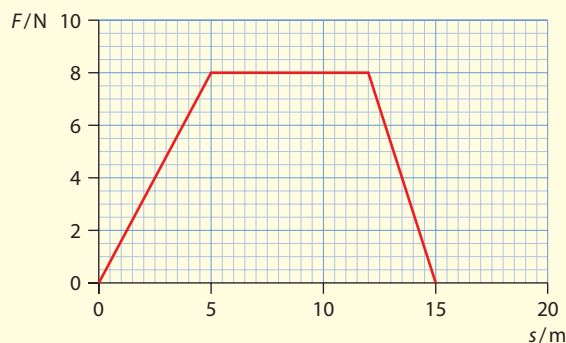


- 61 The speed of the 8.0 kg mass in position A in the diagram is 6.0 m s^{-1} . By the time it gets to B the speed is measured to be 12.0 m s^{-1} .



Estimate the frictional force opposing the motion. (The frictional force is acting along the plane.)

- 62 A force F acts on a body of mass $m = 2.0 \text{ kg}$ initially at rest. The graph shows how the force varies with distance travelled (along a straight line).



- Find the work done by this force.
 - Calculate the final speed of the body.
- 63 A body of mass 12 kg is dropped vertically from rest from a height of 80 m.
- Ignoring any resistance forces during the motion of this body, draw graphs to represent the variation with distance fallen of:
 - the potential energy
 - the kinetic energy.
 - For the same motion draw graphs to represent the variation with time of:
 - the potential energy
 - the kinetic energy.
 - Describe qualitatively the effect of a constant resistance force on each of the four graphs you drew.
- 64 The engine of a car is developing a power of 90 kW when it is moving on a horizontal road at a constant speed of 100 km h^{-1} . Estimate the total horizontal force opposing the motion of the car.



- 65** The motor of an elevator develops power at a rate of 2500 W.
- Calculate the speed that a 1200 kg load is being raised at.
 - In practice it is found that the load is lifted more slowly than indicated by your answer to **a**. Suggest reasons why this is so.
- 66** A load of 50 kg is raised a vertical distance of 15 m in 125 s by a motor.
- Estimate the power necessary for this.
 - The power supplied by the motor is in fact 80 W. Calculate the efficiency of the motor.
 - The same motor is now used to raise a load of 100 kg the same distance. The efficiency remains the same. Estimate how long this would take.
- 67** The top speed of a car whose engine is delivering 250 kW of power is 240 km h⁻¹. Calculate the value of the resistance force on the car when it is travelling at its top speed on a level road.
- 68** An elevator starts on the ground floor and stops on the 10th floor of a high-rise building. The elevator reaches a constant speed by the time it reaches the 1st floor and decelerates to rest between the 9th and 10th floors. Describe the energy transformations taking place between the 1st and 9th floors.
- 69** A mass m of 4.0 kg slides down a frictionless incline of $\theta = 30^\circ$ to the horizontal. The mass starts from rest from a height of 20 m.
- Sketch a graph of the kinetic and potential energies of the mass as a function of time.
 - Sketch a graph of the kinetic and potential energies of the mass as a function of distance travelled along the incline.
 - On each graph, sketch the sum of the potential and kinetic energies.
- 70** A mass m is being pulled up an inclined plane of angle θ by a rope along the plane.
- Find is the tension in the rope if the mass moves up at constant speed v .
 - Calculate is the work done by the tension when the mass moves up a distance of d m along the plane.
 - Find is the work done by the weight of the mass.
 - Find is the work done by the normal reaction force on the mass.
 - What is the net work done on the mass?
- 71** A battery toy car of mass 0.250 kg is made to move up an inclined plane that makes an angle of 30° with the horizontal. The car starts from rest and its motor provides a constant acceleration of 4.0 m s^{-2} for 5.0 s. The motor is then turned off.
- Find the distance travelled in the first 5 s.
 - Find the furthest the car gets on the inclined plane.
 - Calculate when the car returns to its starting position.
 - Sketch a graph of the velocity as a function of time.
 - On the same axes, sketch a graph of the kinetic energy and potential energy of the car as a function of the distance travelled.
 - State the periods in the car's motion in which its mechanical energy is conserved.
 - Estimate the average power developed by the car's motor.
 - Determine the maximum power developed by the motor.

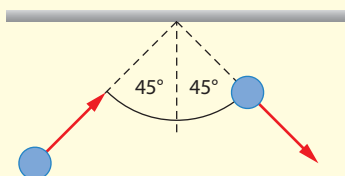


Test yourself

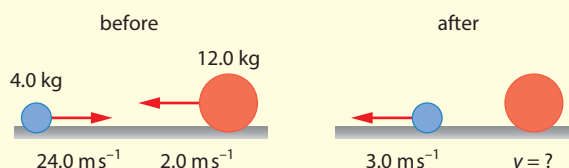
- 72 The momentum of a ball increased by 12.0 N s as a result of a force that acted on the ball for 2.00 s . Find the average force on the ball.
- 73 A 0.150 kg ball moving horizontally at 3.00 m s^{-1} collides normally with a vertical wall and bounces back with the same speed.
- Calculate the impulse delivered to the ball.
 - The ball was in contact with the wall for 0.125 s . Find the average force exerted by the ball on the wall.
- 74 The bodies in the diagram suffer a head-on collision and stick to each other afterwards. Find their common velocity.



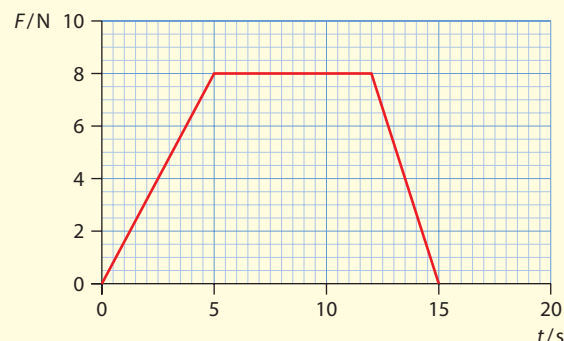
- 75 A ball of mass 250 g rolling on a horizontal floor with a speed 4.00 m s^{-1} hits a wall and bounces with the same speed, as shown in the diagram.



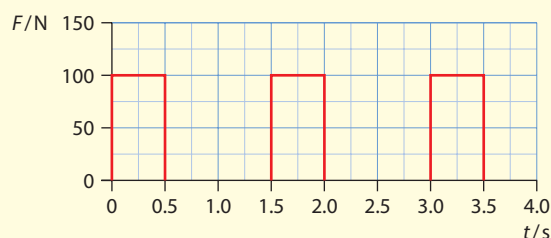
- What is the magnitude and direction of the momentum change of the ball?
 - Is momentum conserved here? Why or why not?
- 76 Two masses moving in a straight line towards each other collide as shown in the diagram. Find the velocity (magnitude and direction) of the heavier mass after the collision.



- 77 A time-varying force varies with time as shown in the graph. The force acts on a body of mass 4.0 kg .
- Find the impulse of the force from $t = 0$ to $t = 15 \text{ s}$.
 - Find the speed of the mass at 15 s , assuming the initial velocity was zero.
 - State the initial velocity of the body such it is brought to rest at 15 s .



- 78 A boy rides on a scooter pushing on the road with one foot with a horizontal force that depends on time, as shown in the graph. While the scooter rolls, a constant force of 25 N opposes the motion. The combined mass of the boy and scooter is 25 kg .
- Find the speed of the boy after 4.0 s , assuming he started from rest.
 - Draw a graph to represent the variation of the boy's speed with time.



- 79 A ball of mass m is dropped from a height of h_1 and rebounds to a height of h_2 . The ball is in contact with the floor for a time interval of t .
- Show that the average net force on the ball is given by:

$$F = m \frac{\sqrt{2gh_1} + \sqrt{2gh_2}}{2}$$
 - If $h_1 = 8.0 \text{ m}$, $h_2 = 6.0 \text{ m}$, $t = 0.125 \text{ s}$ and $m = 0.250 \text{ kg}$, calculate the average force exerted by the ball on the floor.

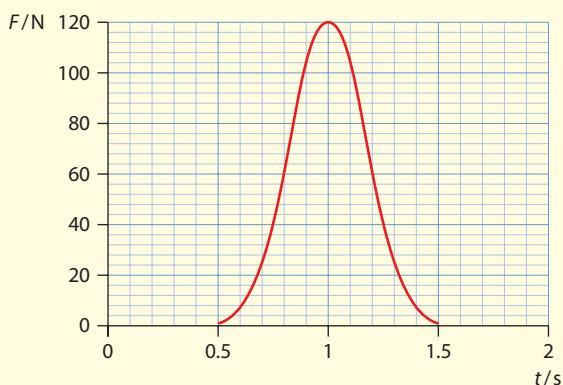
80 A ball of mass m moving vertically, hits a horizontal floor normally with speed v_1 and rebounds with speed v_2 . The ball was in contact with the floor for a time t .

a Show that the average force F on the ball from the floor during the collision is given by:

$$F = \frac{m(v_1 + v_2)}{t} + mg$$

b Find an expression for the average net force on the ball.

81 The diagram shows the variation with time of the force exerted on a ball as the ball came into contact with a spring.



a For how long was the spring in contact with the ball?

b Estimate the magnitude of the change in momentum of the ball.

c What was the average force that was exerted on the ball?

82 Two masses of 2.0 kg and 4.0 kg are held in place, compressing a spring between them. When they are released, the 2.0 kg moves away with a speed of 3.0 ms^{-1} . What was the energy stored in the spring?

83 A rocket in space where gravity is negligible has a mass (including fuel) of 5000 kg. It is desired to give the rocket an average acceleration of 15.0 ms^{-2} during the first second of firing the engine. The gases leave the rocket at a speed of 1500 ms^{-1} (relative to the rocket). Estimate how much fuel must be burnt in that second.

Exam-style questions

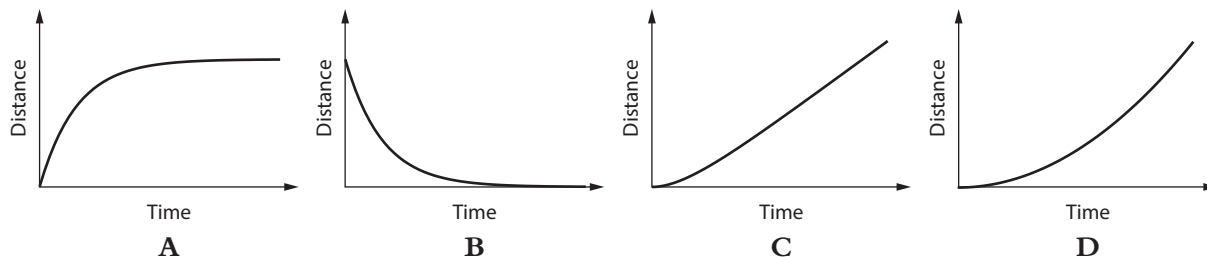
1 Four cars race along a given race track starting at the same time. The car that will reach the finishing line first is the one with the largest

- A** maximum speed
- B** acceleration
- C** power
- D** average speed

- 2 A body that started from rest moves with constant acceleration in a straight line. After travelling a distance d the speed of the car is v . What is the distance travelled when the speed of the car was $\frac{v}{2}$?

A $\frac{d}{2}$ **B** $\frac{d}{\sqrt{2}}$ **C** $\frac{d}{4}$ **D** $\frac{d}{2\sqrt{2}}$

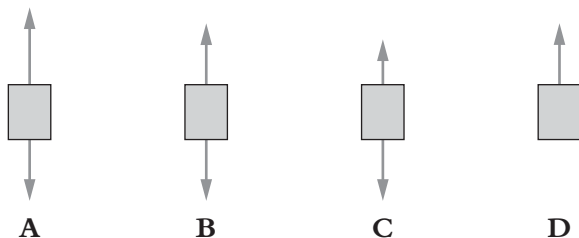
- 3 A sphere falls through a liquid and eventually reaches terminal speed. Which graph shows the variation with time of the distance travelled by the sphere?



- 4 A steel ball of mass m is thrown vertically downwards with initial speed u near the Earth's surface. The rate of change of the momentum of the ball as it falls is:

A 0 **B** mu **C** $m(u + gt)$ **D** mg

- 5 A lunar module is descending vertically above the lunar surface. The speed of the module is decreasing. Which is a free-body diagram of the forces on the landing module?



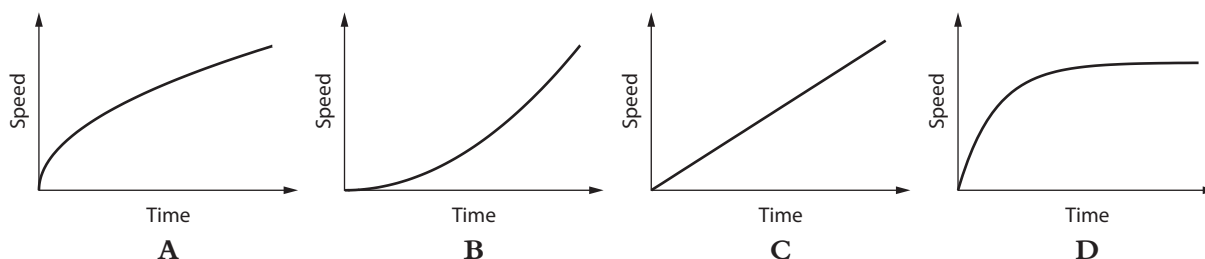
- 6 A person of mass m stands on weighing scales in an elevator. The elevator is accelerating upwards with acceleration a . The reaction force from the scales on the person is R . What is the reading on the scales?

A mg **B** $R + ma$ **C** $R - ma$ **D** R

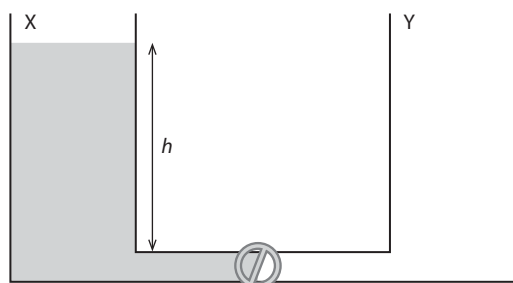
- 7 A body of mass $3M$ at rest explodes into two pieces of mass M and $2M$. What is the ratio of the kinetic energy of M to that of $2M$?

A $\frac{1}{4}$ **B** $\frac{1}{2}$ **C** 4 **D** 2

- 8 The power delivered by a car engine is constant. A car starts from rest. Resistance forces are negligible. Which graph shows the variation with time of the speed of the car?

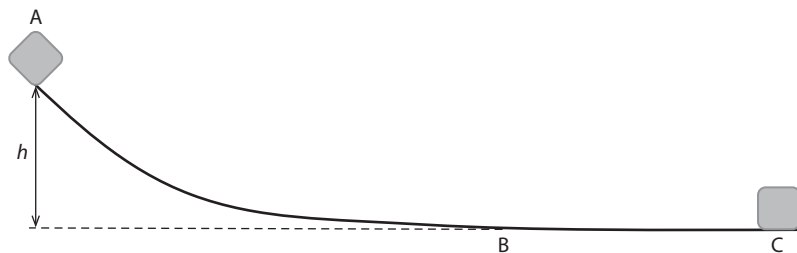


- 9 The diagram shows two identical containers, X and Y, that are connected by a thin tube of negligible volume. Initially container X is filled with water of mass m up to a height h and Y is empty.



The valve is then opened and both containers contain equal quantities of water. The loss of gravitational potential energy of the water is:

- A** 0 **B** $\frac{mgh}{8}$ **C** $\frac{mgh}{4}$ **D** $\frac{mgh}{2}$
- 10 A person of mass m stands on roller skates facing a wall. After pushing against the wall with a constant force F he moves away, reaching speed v after a distance d . What is the work done by F ?
- A** zero **B** mv^2 **C** $\frac{1}{2}mv^2$ **D** Fd
- 11 In a factory blocks of ice slide down a smooth curved path AB and then on to a rough horizontal path starting at B.



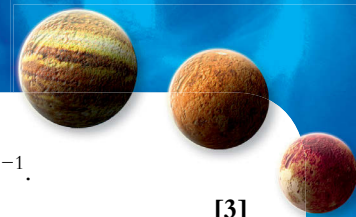
The length of the curved path AB is s ; the block of ice takes time t to move from A to B.

- a** Explain why, for the motion of the block from A to B:

- i** the formula $s = \frac{1}{2}gt^2$ does not apply.
- ii** the formula $v = \sqrt{2gh}$ does apply.

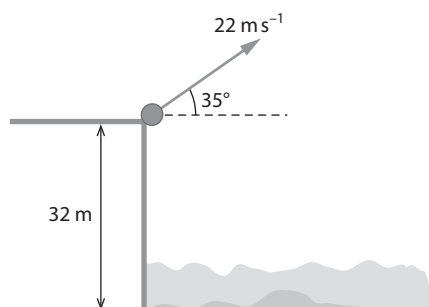
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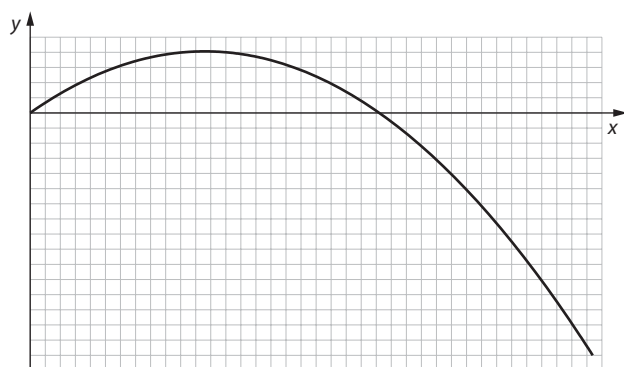
- b** A block of ice of mass 25 kg slides from A to B. The speed of the block at B is $v_B = 4.8 \text{ m s}^{-1}$. Calculate the height h . [3]
- c i** The coefficient of dynamic friction between the block of ice and the rough surface BC is 0.45. Show that the distance BC at which the block of ice is brought to rest is 2.7 m. [2]
- ii** Calculate the time it takes the block of ice to cover the distance BC. [2]
- d** The factory also produces blocks of ice of mass 50 kg that slide down the same path starting at A. Predict, for this heavier block of ice, the speed at B and the stopping distance BC. (The coefficient of friction stays the same.) [3]

- 12** A stone of mass 0.20 kg is thrown with speed 22 m s^{-1} from the edge of a cliff that is 32 m above the sea. The initial velocity of the stone makes an angle of 35° with the horizontal. Air resistance is neglected.



- a i** Determine the horizontal and vertical components of the initial velocity. [2]
- ii** Sketch graphs showing the variation with time of the horizontal and vertical components of velocity. [2]
- b i** Calculate the maximum height above the cliff reached by the stone. [3]
- ii** State the net force on the stone at the highest point in its path. [1]
- c i** Using conservation of energy, determine the speed of the stone as it hits the sea. [2]
- ii** Hence or otherwise, determine the time it took the stone to reach the surface of the sea. [2]

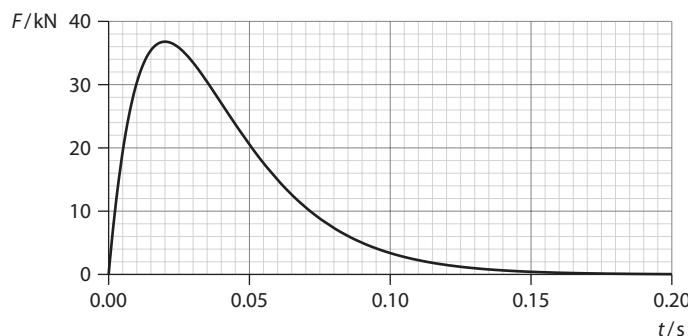
The graph shows the path followed by this stone, until just before hitting the sea, in the absence of air resistance.



- d i** On a copy of the axes above, draw the path of the stone in the presence of an air resistance force opposite to the velocity and proportional to the speed. [3]
- ii** State and explain one difference between your graph and the graph above. [2]

- 13 A toy helicopter has mass $m = 0.30 \text{ kg}$ and blade rotors of radius $R = 0.25 \text{ m}$. It may be assumed that as the blades turn, the air exactly under the blades is pushed downwards with speed v . The density of air is $\rho = 1.2 \text{ kg m}^{-3}$.
- Show that the force that the rotor blades exert on the air is $\rho\pi R^2 v^2$. [3]
 - Hence estimate the speed v when the helicopter just hovers. [2]
 - Determine the power generated by the helicopter's motor when it just hovers as in **a**. [2]
 - The rotor blades now move faster pushing air downwards at a speed double that found in **a**. The helicopter is raised vertically a distance of 12 m .
Estimate:
 - the time needed to raise the helicopter. [2]
 - the speed of the helicopter after it is raised 12 m . [2]
 - the work done by the rotor in raising the helicopter. [1]

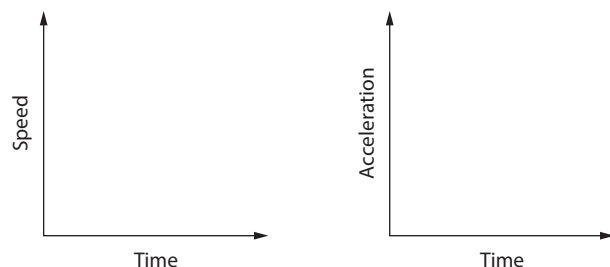
- 14 It is proposed to launch projectiles of mass 8.0 kg from satellites in space in order to destroy incoming ballistic missiles. The launcher exerts a force on the projectile that varies with time according to the graph.



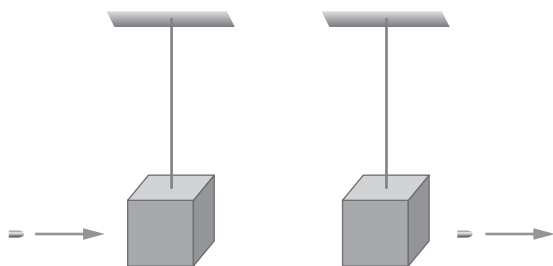
The impulse delivered to the projectile is $2.0 \times 10^3 \text{ N s}$. The projectile leaves the launcher in 0.20 s .

- Estimate:
 - the area under the curve [1]
 - the average acceleration of the projectile [3]
 - the average speed of the projectile [2]
 - the length of the launcher. [2]
 - Calculate, for the projectile as it leaves the launcher:
 - the speed [2]
 - the kinetic energy. [2]
 - Estimate the power delivered to the projectile by the launcher. [2]
- 15 A car of weight $1.4 \times 10^4 \text{ N}$ is moving up an incline at a constant speed of 6.2 m s^{-1} . The incline makes an angle of 5.0° to the horizontal. A frictional force of 600 N acts on the car in a direction opposite to the velocity.
- State the net force on the car. [1]
 - Calculate the force F pushing the car up the incline. [3]
 - The power supplied by the car is 15 kW . Determine the efficiency of the car engine in pushing the car uphill. [3]

- c The car is now allowed to roll down the incline from rest with the engine off. The only resistance force on the car is assumed to be proportional to speed. On a copy of the axes below, draw sketch graphs to show the variation with time of:
- i the speed of the car [2]
 - ii the acceleration of the car. [2]



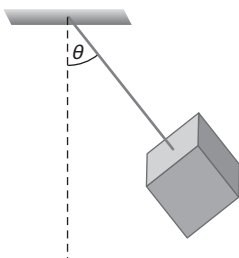
- 16 A bullet of mass 0.090 kg is shot at a wooden block of mass 1.20 kg that is hanging vertically at the end of a string.



The bullet enters the block with speed 130 m s^{-1} and leaves it with speed 90 m s^{-1} . The mass of the block does not change appreciably as a result of the hole made by the bullet.

- a i Calculate the change in the momentum of the bullet. [2]
- ii Show that the initial velocity of the block is 3.0 m s^{-1} . [1]
- iii Estimate the loss of kinetic energy in the bullet–block system. [2]

As a result of the impact, the block is displaced. The maximum angle that the string makes with the vertical is θ . The length of the string is 0.80 m .



- b Show that $\theta \approx 65^\circ$. [3]
- c i State and explain whether the block in **b** is in equilibrium. [2]
- ii Calculate the tension in the string in **b**. [3]

- b** The temperature is constant at melting, shown by the flat part of the graph, so the melting temperature is 48°C .
- c** The sample is melting from 120 s to 560 s, i.e. for 440 s. The heat supplied during this time is therefore:

$$Q = Pt = 70 \times 440 = 30\,800\text{ J}$$

So the specific latent heat of fusion is:

$$L_F = \frac{Q}{m} = \frac{30\,800}{0.120} = 2.6 \times 10^5 \text{ J kg}^{-1}$$

- d** The liquid increases its temperature from 48°C to 56°C in 40 s. In these 40 s the heat provided is:

$$Q = Pt = 70 \times 40 = 2800\text{ J}$$

Using $Q = mc\Delta T$:

$$0.120 \times c \times (56 - 48) = 2800\text{ J}$$

$$\Rightarrow c = 2.9 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$$

The method of mixtures

The electrical method described in Worked example 3.5 is one method for measuring specific heat capacity and latent heat. Another method, the **method of mixtures**, measures the specific heat capacity of a solid as follows. A solid is put in a container of hot water and allowed time to reach a constant temperature. The temperature of the solid is thus that of the water and is recorded. The solid is then transferred into a calorimeter of known specific heat capacity and initial temperature, which contains a liquid such as water (Figure 3.7). The calorimeter is insulated. The final temperature of the water is recorded after thermal equilibrium has been reached.

For example, consider a mass of 0.400 kg of a solid at 80°C that is put in a 100 g copper calorimeter containing 800 g of water at 20°C . The final temperature of the water is measured to be 22°C . From these values, we may deduce the specific heat capacity of the solid as follows.

Using $Q = mc\Delta T$, the amount of thermal energy (in joules) lost by the solid is:

$$0.400 \times c \times (80 - 22) = 23.2c$$

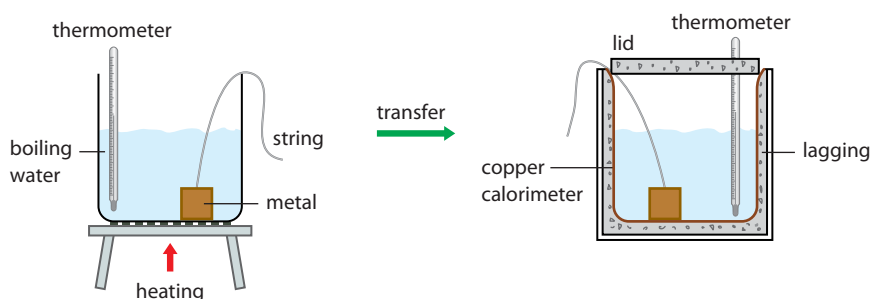


Figure 3.7 The hot metal is placed in the cold water in the calorimeter. The hot metal is removed from the container of boiling water and is quickly placed inside an insulated calorimeter containing cold water.

Exam tip

It is likely that the solid lost heat to the surrounding air while it was being transferred. This means that the actual temperature of the solid is less than we supposed. The actual specific heat capacity is therefore larger than the calculated value.

The amount of thermal energy gained by the calorimeter (see Table 3.1 for the value of c for copper) and the water is:

$$0.100 \times 385 \times (22 - 20) + 0.800 \times 4200 \times (22 - 20) = 6797 \text{ J}$$

calorimeter water

Equating the two we find that $c = 293 \text{ J kg}^{-1} \text{ K}^{-1}$.

The same method can be applied to measure the specific latent heat of fusion of ice. To do this, place a quantity of ice at 0°C (the ice must therefore come from a mixture with water at 0°C) into a calorimeter containing water at a few degrees above room temperature. Blot the ice dry before putting it into the calorimeter. The mass of the ice can be determined by weighing the calorimeter at the end of the experiment.

For example, suppose that 25.0 g of ice at 0.00°C is placed in an aluminium calorimeter of mass 250 g containing 300 g of water at 24.0°C . The temperature of the water is measured at regular intervals of time until the temperature reaches a minimum value of 17.0°C . The calorimeter and water lost heat, which the ice received.

Heat lost by calorimeter and water:

$$0.250 \times 900 \times (24 - 17) + 0.300 \times 4200 \times (24 - 17) = 10\,395 \text{ J}$$

Heat received by ice:

$$0.025 \times L_F + 0.025 \times 4200 \times 17 = 0.025 \times L_F + 1785$$

Equating the two gives:

$$1785 + 0.025 \times L_F = 10\,395 \Rightarrow L \approx 344 \text{ kJ kg}^{-1}$$

Nature of science

Models change

As already mentioned, heat was once thought to be a fluid (caloric). Conservation of energy was a natural consequence of this model of heat: a body lost a certain amount of fluid and another gained it. Energy was conserved. So the concept of heat as a fluid seemed natural. But there are phenomena that cannot be explained with this simple picture. For one thing, if heat is a fluid it must have mass. So when heat leaves a body, the body must lose mass. This is not observed, so the caloric theory must be wrong. The theory has many other failings and was abandoned in the 19th century. A major problem is that it does not take account the atomic theory of matter. The theory we use now is based on statistical mechanics, which uses probability theory to predict the average behaviour of very large numbers of particles.



? Test yourself

- 1 A hot body is brought into contact with a colder body until their temperatures are the same. Assume that no other bodies are nearby.
 - a Discuss whether the energy lost by one body is equal to the energy gained by the other.
 - b Discuss whether the temperature drop of one body is equal to the temperature rise of the other.
- 2
 - a A body of mass 0.150 kg has its temperature increased by 5.00°C when 385 J of energy is provided to it. Calculate the body's specific heat capacity.
 - b Another body of mass 0.150 kg has its temperature increased by 5.00 K when 385 J of energy is provided to it. Calculate this body's specific heat capacity.
- 3 A calorimeter of mass 90 g and specific heat capacity $420\text{ J kg}^{-1}\text{ K}^{-1}$ contains 310 g of a liquid at 15.0°C . An electric heater rated at 20.0 W warms the liquid to 19.0°C in 3.0 min . Assuming there are no energy losses to the surroundings, estimate the specific heat capacity of the liquid.
- 4 A calorimeter for which $mc = 25\text{ J K}^{-1}$ contains 140 g of a liquid. An immersion heater is used to provide energy at a rate of 40 W for a total time of 4.0 min . The temperature of the liquid increases by 15.8°C . Calculate the specific heat capacity of the liquid. State an assumption made in reaching this result.
- 5 A car of mass 1360 kg descends from a hill of height 86 m at a constant speed. Assuming that all of the gravitational potential energy lost by the car goes into heating the brakes, estimate the rise in the temperature of the brakes. (It takes 16 kJ of energy to increase the temperature of the brake drums by 1 K ; ignore any energy losses to the surroundings.)
- 6 A radiator made out of iron of specific heat capacity $450\text{ J kg}^{-1}\text{ K}^{-1}$ has a mass of 45.0 kg and is filled with 23.0 kg of water of specific heat capacity $4200\text{ J kg}^{-1}\text{ K}^{-1}$.
 - a Determine the energy required to raise the temperature of the radiator–water system by 1 K .
 - b If energy is provided to the radiator at the rate of 450 W , calculate how long it will take for the temperature to increase by 20.0°C .
- 7 How much ice at -10°C must be dropped into a cup containing 300 g of water at 20°C in order for the temperature of the water to be reduced to 10°C ? The cup itself has a mass of 150 g and is made out of aluminium. Assume that no energy is lost to the surroundings.
- 8 The surface of a pond of area 20 m^2 is covered by ice of uniform thickness 6 cm . The temperature of the ice is -5°C . Calculate how much energy is required to melt this amount of ice into water at 0°C . (Take the density of ice to be 900 kg m^{-3} .)
- 9 Radiation from the Sun falls on the frozen surface of a pond at a rate of 600 W m^{-2} . The ice temperature is 0°C .
 - a Calculate how long it will take to melt a 1.0 cm thick layer of ice. (Take the density of ice to be 900 kg m^{-3} .)
 - b Suggest why the actual mass of ices that melts is less than your answer to a.
- 10
 - a Calculate how much energy is required to warm 1.0 kg ice initially at -10°C to ice at 0°C .
 - b Calculate how much energy is required to melt the ice at 0°C .
 - c Calculate how much energy is required to further increase the temperature of the water from 0°C to 10°C .
 - d State in which stage (warming the ice, melting the ice, warming the water) the energy requirement is largest.
- 11 Ice at 0°C is added to 1.0 kg of water at 20°C , cooling it down to 10°C . Determine how much ice was added.
- 12 A quantity of 100 g of ice at 0°C and 50 g steam at 100°C are added to a container that has 150 g water at 30°C . Determine the final temperature in the container. Ignore the container itself in your calculations.

Nature of science

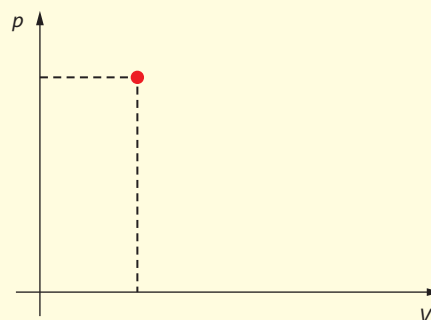
Models must be correct but also simple

Boyle thought that a gas consists of particles joined by springs. Newton thought that a gas consists of particles that exert repulsive forces on each other. Bernoulli thought that a gas is a collection of a very large number of particles that exert forces on each other only when they collide. All three could explain why a gas exerts a pressure on its container but it is Bernoulli's picture that is the simplest. We assume that the ordinary laws of mechanics apply to the individual particles making up the gas. Even though the laws apply to each individual particle we cannot observe or analyse each particle individually since there are so many of them. By concentrating on average behaviours of the whole gas and using probability and statistics, physicists developed a new field of physics known as statistical mechanics. This has had enormous success in advancing our understanding of gases and other systems, including where the approximation to an ideal gas breaks down.

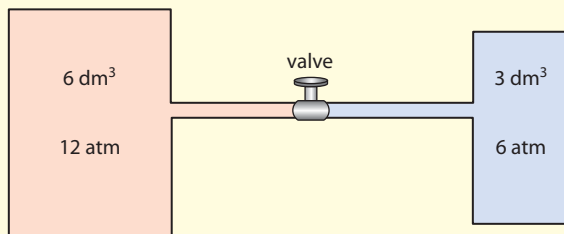


Test yourself

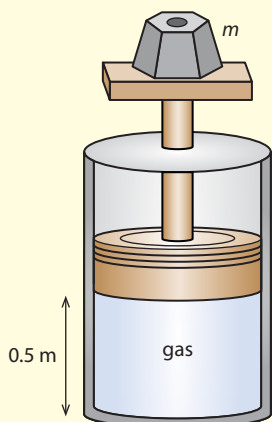
- 13 Calculate the number of molecules in 28 g of hydrogen gas (molar mass 2 g mol^{-1}).
- 14 Calculate the number of moles in 6.0 g of helium gas (molar mass 4 g mol^{-1}).
- 15 Determine the number of moles in a sample of a gas that contains 2.0×10^{24} molecules.
- 16 Determine the mass in grams of carbon (molar mass 12 g mol^{-1}) that contains as many molecules as 21 g of krypton (molar mass 84 g mol^{-1}).
- 17 A sealed bottle contains air at 22.0°C and a pressure of $12.0 \times 10^5\text{ Pa}$. The temperature is raised to 120.0°C . Calculate the new pressure.
- 18 A gas has pressure $8.2 \times 10^6\text{ Pa}$ and volume $2.3 \times 10^{-3}\text{ m}^3$. The pressure is reduced to $4.5 \times 10^6\text{ Pa}$ at constant temperature. Calculate the new volume of the gas.
- 19 A mass of 12.0 kg of helium is required to fill a bottle of volume $5.00 \times 10^{-3}\text{ m}^3$ at a temperature of 20.0°C . Determine the pressure in helium.
- 20 Determine the mass of carbon dioxide required to fill a tank of volume $12.0 \times 10^{-3}\text{ m}^3$ at a temperature of 20.0°C and a pressure of 4.00 atm.
- 21 A flask of volume $300.0 \times 10^{-6}\text{ m}^3$ contains air at a pressure of $5.00 \times 10^5\text{ Pa}$ and a temperature of 27.0°C . The flask loses molecules at a rate of 3.00×10^{19} per second. Estimate how long it takes for the pressure in the flask to fall to half its original value. (Assume that the temperature of the air remains constant during this time.)
- 22 The point marked in the diagram represents the state of a fixed quantity of ideal gas in a container with a movable piston. The temperature of the gas in the state shown is 600 K. Copy the diagram. Indicate on the diagram the point representing the new state of the gas after the following separate changes.
 - a The volume doubles at constant temperature.
 - b The volume doubles at constant pressure.
 - c The pressure halves at constant volume.



- 23 Two ideal gases are kept at the same temperature in two containers separated by a valve, as shown in the diagram. Estimate the pressure when the valve is opened. (The temperature stays the same.)

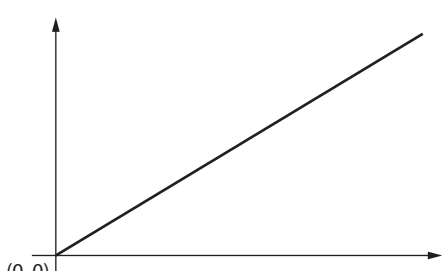


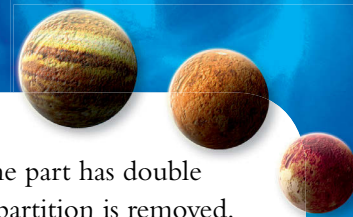
- 24 The diagram shows a cylinder in a vacuum, which has a movable, frictionless piston at the top. An ideal gas is kept in the cylinder. The piston is at a distance of 0.500 m from the bottom of the cylinder and the volume of the cylinder is 0.050 m³. The weight on top of the cylinder has a mass of 10.0 kg. The temperature of the gas is 19.0 °C.



- a Calculate the pressure of the gas.
 - b Determine how many molecules there are in the gas.
 - c The temperature is increased to 152 °C. Calculate the new volume of the gas.
- 25 The molar mass of a gas is 28 g mol⁻¹. A container holds 2.00 mol of this gas at 0.00 °C and a pressure of 1.00 × 10⁵ Pa. Determine the mass and volume of the gas.
- 26 A balloon has a volume of 404 m³ and is filled with helium of mass 70.0 kg. The temperature inside the balloon is 17.0 °C. Determine the pressure inside the balloon.
- 27 A flask has a volume of 5.0 × 10⁻⁴ m³ and contains air at a temperature of 300 K and a pressure of 150 kPa.
- a Calculate the number of moles of air in the flask.
 - b Determine the number of molecules in the flask.
 - c Estimate the mass of air in the flask. You may take the molar mass of air to be 29 g mol⁻¹.
- 28 The molar mass of helium is 4.00 g mol⁻¹.
- a Calculate the volume of 1.0 mol of helium at standard temperature and pressure (stp) i.e. at $T = 273 \text{ K}$, $p = 1.0 \times 10^5 \text{ Pa}$.
 - b Determine the density of helium at stp.
 - c Estimate the density of oxygen gas at stp (the molar mass of, oxygen gas is 32 g mol⁻¹).
- 29 The density of an ideal gas is 1.35 kg m⁻³. The temperature in kelvin and the pressure are both doubled. Calculate the new density of the gas.
- 30 Calculate the average speed (r.m.s.) of helium atoms at a temperature of 850 K. The molar mass of helium is 4.0 g mol⁻¹.
- 31 Show that the average (r.m.s.) speed of molecules of a gas of molar mass M (in kg mol⁻¹) kept at a temperature T is given by $c = \sqrt{\frac{3RT}{M}}$.
- 32
- a Calculate the average random kinetic energy of a gas kept at a temperature of 300 K.
 - b Determine the ratio of the average speeds (r.m.s. speeds) of two ideal gases of molar mass 4.0 g mol⁻¹ and 32 g mol⁻¹, which are kept at the same temperature.

Exam-style questions

- 1 Body X whose temperature is 0°C is brought into thermal contact with body Y of equal mass and temperature 100°C . The only exchanges of heat that take place are between X and Y. The specific heat capacity of X is greater than that of Y. Which statement about the final equilibrium temperature T of the two bodies is correct?
- A $T = 50^{\circ}\text{C}$
B $0 < T < 50^{\circ}\text{C}$
C $100^{\circ}\text{C} > T > 50^{\circ}\text{C}$
D Answer depends on value of mass
- 2 Energy is provided to a liquid at its boiling point at a rate of P joules per second. The rate at which mass is boiling away is μ kg per second. The specific latent heat of vaporisation of the liquid is
- A μP B $\frac{P}{\mu}$ C $\frac{\mu}{P}$ D $\frac{1}{\mu P}$
- 3 The following are all assumptions of the kinetic theory of gases, except which one?
- A The duration of a collision is very small compared to the time in between collisions.
B The collisions are elastic.
C The average kinetic energy of molecules is proportional to temperature.
D The volume of molecules is negligible compared to the volume of the gas.
- 4 In the context of a fixed mass of an ideal gas, the graph could represent the variation of:
- 
- A pressure with volume at constant temperature
B volume with Celsius temperature at constant pressure
C pressure with Celsius temperature at constant volume
D pressure with inverse volume at constant temperature
- 5 The temperature of an ideal gas of pressure 200 kPa is increased from 27°C to 54°C at constant volume. Which is the best estimate for the new pressure of the gas?
- A 400 kPa B 220 kPa C 180 kPa D 100 kPa



- 6 A container of an ideal gas that is isolated from its surroundings is divided into two parts. One part has double the volume of the other. The pressure in each part is p and the temperature is the same. The partition is removed. What is the pressure in the container now?

A p B $2p$ C $\frac{3p}{2}$ D $4p$

- 7 Different quantities of two ideal gases X and Y are kept at the same temperature. Which of the following is a correct comparison of the average kinetic energy and internal energy of the two gases?

	Average kinetic energy	Internal energy
A	same	same
B	same	different
C	different	same
D	different	different

- 8 The temperature of an ideal gas is doubled. The average speed of the molecules increases by a factor of

A $\sqrt{2}$ B 2 C $2\sqrt{2}$ D 4

- 9 Two ideal gases X and Y are kept at the same temperature. Gas X has molar mass m_X and gas Y has molar mass μ_Y . The ratio of average speeds of the molecules of gas X to that of gas Y is

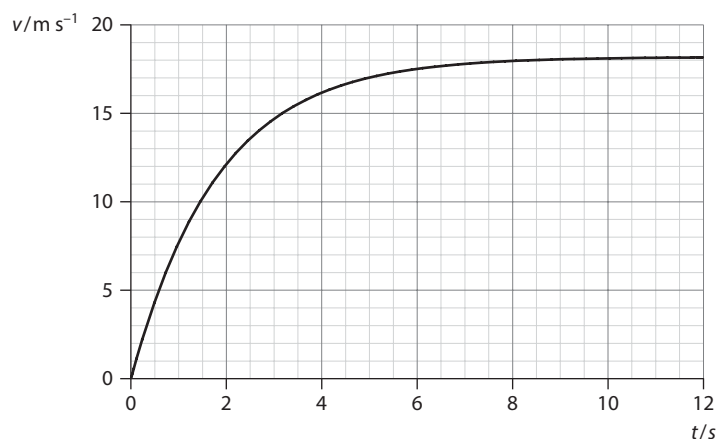
A $\frac{\mu_X}{\mu_Y}$ B $\frac{\mu_Y}{\mu_X}$ C $\sqrt{\frac{\mu_X}{\mu_Y}}$ D $\sqrt{\frac{\mu_Y}{\mu_X}}$

- 10 The pressure of a fixed quantity of ideal gas is doubled. The average speed of the molecules is also doubled. The original density of the gas is ρ . Which is the new density of the gas?

A $\frac{\rho}{2}$ B ρ C 2ρ D 4ρ

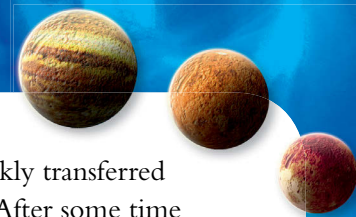
- 11 a Calculate the volume of 1 mol of helium gas (molar mass 4 g mol^{-1}) at temperature 273 K and pressure $1.0 \times 10^5 \text{ Pa}$. [2]
- b i Find out how much volume corresponds to each molecule of helium. [2]
- ii The diameter of an atom of helium is about 31 pm . Discuss whether or not the ideal gas is a good approximation to the helium gas in a. [2]
- c Consider now 1 mol of lead (molar mass 207 g mol^{-1} , density $11.3 \times 10^3 \text{ kg m}^{-3}$). How much volume corresponds to each atom of lead? [3]
- d Find the ratio of these volumes (helium to lead) and hence determine the order of magnitude of the ratio: separation of helium atoms to separation of lead atoms. [2]

- 12 a Define what is meant by **specific heat capacity** of a substance. [1]
- b Consider two metals that have different specific heat capacities. The energies required to increase the temperature of 1 mol of aluminium and 1 mol of copper by the same amount are about the same. Yet the specific heat capacities of the two metals are very different. Suggest a reason for this. [2]
- A hair dryer consists of a coil that warms air and a fan that blows the warm air out. The coil generates thermal energy at a rate of 600 W. Take the density of air to be 1.25 kg m^{-3} and its specific heat capacity to be $990 \text{ J kg}^{-1} \text{ K}^{-1}$. The dryer takes air from a room at 20°C and delivers it at a temperature of 40°C .
- c What mass of air flows through the dryer per second? [2]
- d What volume of air flows per second? [1]
- e The warm air makes water in the hair evaporate. If the mass of water in the air is 180 g, calculate how long it will take to dry the hair. (The heat required to evaporate 1 g of water at 40°C is 2200 J.) [2]
- 13 The graph shows the variation with time of the speed of an object of mass 8.0 kg that has been dropped (from rest) from a certain height.



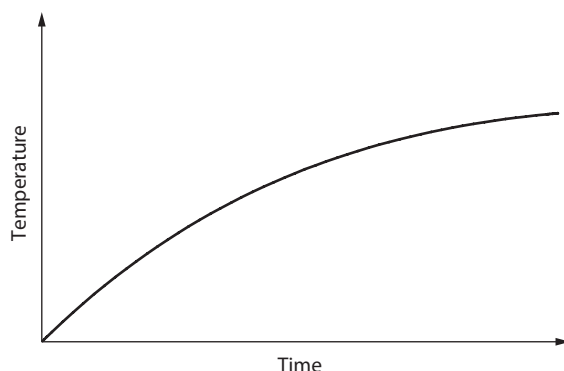
The body hits the ground 12 seconds later. The specific heat capacity of the object is $320 \text{ J kg}^{-1} \text{ K}^{-1}$.

- a i Explain how we may deduce that there must be air resistance forces acting on the object. [2]
- ii Estimate the height from which the object was dropped. [2]
- iii Calculate the speed the object would have had if there were no air resistance forces. [2]
- b Estimate the change in temperature of the body from the instant it was dropped to just before impact. List any assumptions you make. [4]



- 14 A piece of tungsten of mass 50 g is placed over a flame for some time. The metal is then quickly transferred to a well-insulated aluminium calorimeter of mass 120 g containing 300 g of water at 22 °C. After some time the temperature of the water reaches a maximum value of 31 °C.
- State what is meant by the internal energy of a piece of tungsten. [1]
 - Calculate the temperature of the flame. You may use these specific heat capacities: water $4.2 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$, tungsten $1.3 \times 10^2 \text{ J kg}^{-1} \text{ K}^{-1}$ and aluminum $9.0 \times 10^2 \text{ J kg}^{-1} \text{ K}^{-1}$. [3]
 - State and explain whether the actual flame temperature is higher or lower than your answer to b. [2]

- 15
- Describe what is meant by the internal energy of a substance. [1]
 - A student claims that the kelvin temperature of a body is a measure of its internal energy. Explain why this statement is not correct by reference to a solid melting. [2]
 - In an experiment, a heater of power 35 W is used to warm 0.240 kg of a liquid in an uninsulated container. The graph shows the variation with time of the temperature of the liquid.



The liquid never reaches its boiling point.

Suggest why the temperature of the liquid approaches a constant value. [2]

- After the liquid reaches a constant temperature the heater is switched off. The temperature of the liquid decreases at a rate of 3.1 K min^{-1} . Use this information to estimate the specific heat capacity of the liquid. [3]

- 16 The volume of air in a car tyre is about $1.50 \times 10^{-2} \text{ m}^3$ at a temperature of 0.0 °C and pressure 250 kPa.
- Calculate the number of molecules in the tyre. [2]
 - Explain why, after the car is driven for a while, the pressure of the air in the tyre will increase. [3]
 - Calculate the new pressure of the tyre when the temperature increases to 35 °C and the volume expands to $1.60 \times 10^{-2} \text{ m}^3$.
 - The car is parked for the night and the volume, pressure and temperature of the air in the tyre return to their initial values. A small leak in the tyre reduces the pressure from 250 kPa to 230 kPa in the course of 8 h. Estimate (stating any assumptions you make):
 - the average rate of loss of molecules (in molecules per second) [2]
 - the total mass of air lost (take the molar mass of air to be 29 g mol^{-1}). [3]

For a system to oscillate it is necessary to have a **restoring force**, i.e. a force that brings the system back towards its equilibrium position when the system is displaced away from equilibrium.

A very special periodic oscillation is called **simple harmonic motion (SHM)** and is the main topic of this section. The defining property of all simple harmonic oscillations is that the magnitude of the acceleration of the body that has been displaced away from equilibrium is proportional to the displacement and the direction of the acceleration is towards the equilibrium position. Mathematically these two conditions can be stated as:

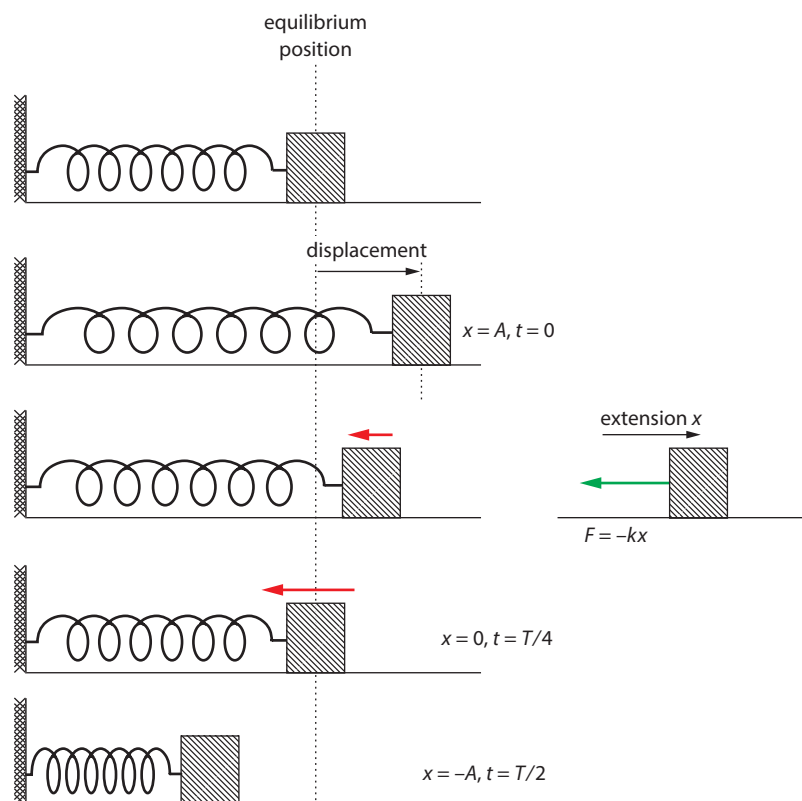
$$a \propto -x$$

(Since $F = ma$, this is equivalent to saying that the restoring force is proportional to and opposite to the displacement.)

The main characteristics of SHM are:

- the period and amplitude are constant
- the period is independent of the amplitude
- the displacement, velocity and acceleration are sine or cosine functions of time.

Consider a block of mass m placed at the end of a horizontal spring. If we displace the block to the right and then release it, it will perform oscillations about its equilibrium position (the vertical dotted line) between the extreme positions of the second and last diagrams in Figure 4.2. The oscillations have amplitude A .



Oscillations in which the period is independent

of the amplitude are called **isochronous**. Such oscillations are essential for accurate time-keeping. The oscillations of a simple pendulum are approximately isochronous, which means that the period is independent of the amplitude as long as the amplitude is small. While he was attempting to solve the non-isochronous aspect of the simple pendulum, the great Dutch scientist Christiaan Huygens (1629–1695) discovered important mathematical and physical aspects of the cycloid curve. The cycloid is the curve that is traced by a point on the rim of a wheel as the wheel turns.

Figure 4.2 The mass–spring system. The net force on the body is proportional to the displacement and opposite in direction.

Consider the block when it is in an arbitrary position, as in the third diagram in Figure 4.2 on the previous page. At that position the extension of the spring is x . The magnitude of the tension F in the spring is therefore (by Hooke's law) equal to $F=kx$, where k is the spring constant. The tension force is directed to the left.

Taking **displacement** to the right of the equilibrium position as positive, then $ma = -kx$ since the tension force is directed to the left, and so is taken as negative. We can rewrite this equation as:

$$a = -\frac{k}{m}x$$

This tells us that the acceleration has a direction which is opposite to and a magnitude that is proportional to the displacement from equilibrium, so the oscillations will be simple harmonic (assuming there are no frictional forces). A graph of acceleration versus displacement gives a straight line through the origin with a negative slope, as shown in Figure 4.3.

Therefore, in general, to check whether SHM will take place, we must check that:

- 1 there is a fixed equilibrium position
- 2 when the particle is moved away from equilibrium, the acceleration of the particle is both proportional to the amount of displacement and in the opposite direction to it.

Let us have a look at Figure 4.4. A body has been displaced from equilibrium and is then released. The figure shows how the displacement x varies with time t . We can extract lots of information from this graph. To begin with we see that the maximum displacement is 5.0 cm and so this is the amplitude of the motion: $x_0 = 5.0$ cm.

Next we see that the period is 2.0 s: we obtain this by looking at the time from one peak to the next. What about the velocity in this oscillation? We know that velocity is the gradient of a displacement–time graph. So we can say something about velocity by looking at the gradient of this graph at each point. At $t=0$ the gradient is zero, so the velocity is also zero. As t increases the gradient becomes negative and at $t=1.0$ s it becomes zero again. The gradient has its largest magnitude at $t=0.5$ s. From $t=1.0$ s to $t=2.0$ s the gradient is positive. Its magnitude is largest at $t=1.5$ s. Figure 4.5 shows the actual velocity plotted against time, which agrees with our qualitative analysis.

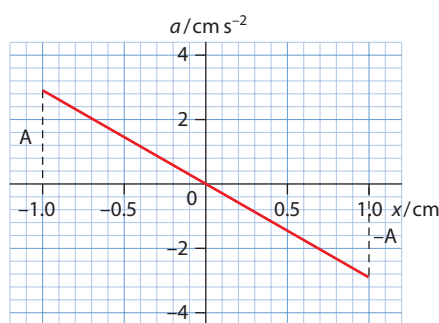


Figure 4.3 The graph of acceleration a versus displacement x is a straight line through the origin with a negative slope.

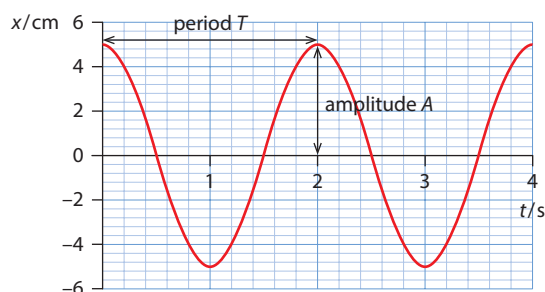


Figure 4.4 The variation of displacement x with time t in SHM.

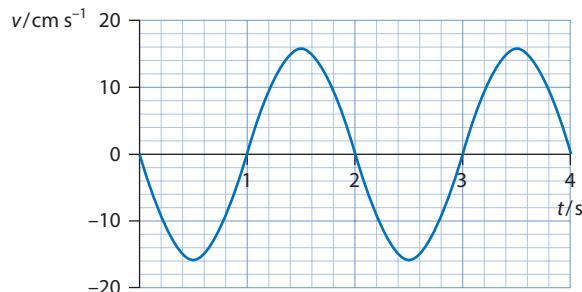


Figure 4.5 The variation of velocity v with time t in SHM.

In the same way, we can get information about acceleration by looking at the gradient of the velocity–time graph. But it is much simpler to recall that $a \propto -x$. This means that the acceleration–time graph has the opposite shape to the displacement–time graph – the peaks on the displacement–time graph become troughs on the acceleration–time graph (of course the scale on the vertical axis will be different). The actual acceleration–time graph is shown in **Figure 4.6**. **Figure 4.7** shows all three graphs together.

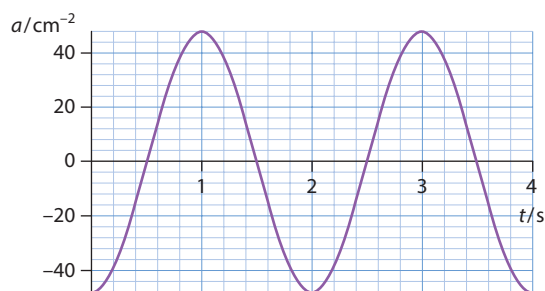


Figure 4.6 The variation of acceleration a with time t in SHM.

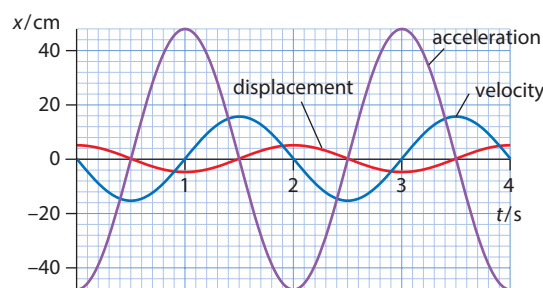


Figure 4.7 The variation of displacement, velocity and acceleration in SHM on the same axes.

We define the **frequency** f of the oscillations as the number of full oscillations per second. Since we have one oscillation in a time equal to the period T , the number of oscillations per second is $\frac{1}{T}$ and so:

$$f = \frac{1}{T}$$

The three curves in **Figure 4.7** all have the same period, so all three curves also have the same frequency.

In **Figure 4.4** the maximum displacement is at $t=0$, so the displacement is a cosine function of time. However, the graph of displacement versus time does not have to be a cosine function. The three graphs of **Figure 4.8** all show simple harmonic oscillations with the same amplitude and period (and hence frequency). There is however a **phase difference** between them. The blue curve is the red curve shifted forward by some amount. And the purple curve is the red curve shifted forward by an even greater amount. The amount by which one curve is shifted forward relative to another curve is called the phase difference between the two curves. Technically, the phase difference is described in terms of an angle ϕ , where:

$$\phi = \frac{\text{shift}}{T} \times 360^\circ$$

Relative to the red curve, the blue curve is shifted by 0.125 s and the period is 1.00 s, so the phase difference is:

$$\phi = \frac{0.125}{1.00} \times 360^\circ = 45^\circ \text{ (or } \frac{\pi}{4} \text{ radians)}$$

Relative to the purple curve, the purple curve is shifted by 0.250 s and the period is again 1.00 s, so the phase difference is:

$$\phi = \frac{0.250}{1.00} \times 360^\circ = 90^\circ \text{ (or } \frac{\pi}{2} \text{ radians)}$$

The unit of frequency is the inverse second, s^{-1} , which is called the hertz (Hz).

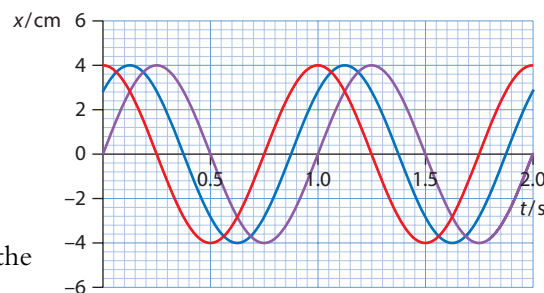


Figure 4.8 Three graphs of simple harmonic oscillations with a phase difference between them.

Worked examples

4.1 State and explain whether graphs I, II and III in Figure 4.9 represent simple harmonic oscillations.

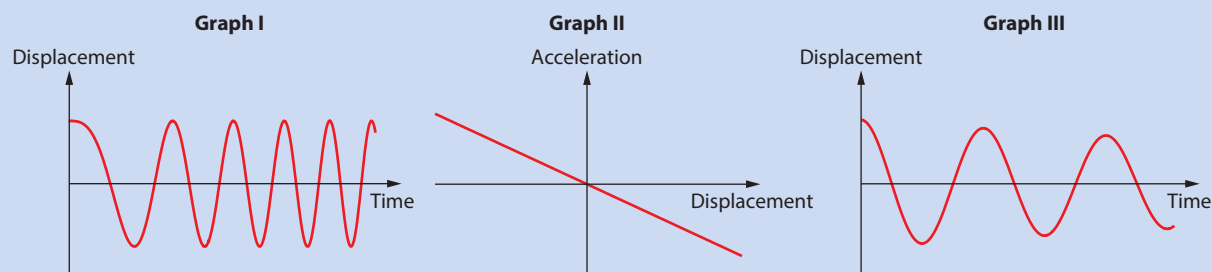


Figure 4.9

Graph I does not show SHM, since the period does not stay constant as time goes on. Graph II does, since the acceleration is proportional and opposite to displacement (straight-line graph through origin with negative slope). Graph III does not, since the amplitude does not stay constant.

4.2 The graph in Figure 4.10 shows the displacement of a particle from a fixed equilibrium position.

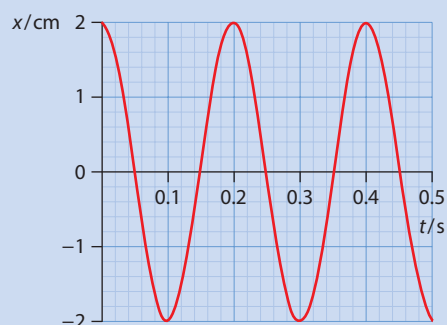


Figure 4.10 Graph showing the variation with time of the displacement of a particle performing SHM.

- a Use the graph to determine the period of the motion.
- b On a copy of the graph, mark:
 - i a point where the velocity is zero (label this with the letter Z)
 - ii a point where the velocity is positive and has the largest magnitude (label this with the letter V)
 - iii a point where the acceleration is positive and has the largest magnitude (label this with the letter A).

- a The period is read off the graph as $T = 0.20$ s.
- b
 - i The velocity is zero at any point where the displacement is at a maximum or a minimum.
 - ii For example at $t = 0.15$ s.
 - iii For example at $t = 0.10$ s or $t = 0.30$ s.

Energy in simple harmonic motion

Consider again a particle at the end of a horizontal spring. Let the extension of the spring be x at a particular instant of time, and let the velocity of the particle be v at that time. The system has elastic potential energy and kinetic energy, **Figure 4.11**.

The total energy of the system is then:

$$E = E_p + E_k$$

In the absence of frictional and other resistance forces, this total energy is conserved, and so $E = \text{constant}$.

The maximum velocity is achieved when $x = 0$, i.e. as the mass moves past its equilibrium position. Here there is no extension, so the elastic potential energy is zero. At the extremes of the motion, $x = \pm A$ and $v = 0$, so the kinetic energy is zero. Thus at $x = \pm A$ the system has elastic potential energy only, and at $x = 0$ it has kinetic energy only. At intermediate points the system has both forms of energy: elastic potential energy and kinetic energy. During an oscillation, we therefore have **transformations** from one form of energy into another. This allows us to write:

$$E = E_p + E_k = (E_k)_{\max} = (E_p)_{\max}$$

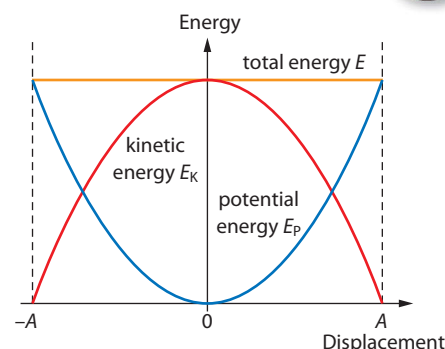


Figure 4.11 Graphs showing the variation with displacement of the potential energy and kinetic energy of a particle on a spring. The total energy is a horizontal straight line.

Worked example

4.3 The graph in **Figure 4.12** shows the variation with displacement of the kinetic energy of a particle of mass 0.40 kg performing SHM at the end of a spring.

a Use the graph to determine:

- i** the total energy of the particle
- ii** the maximum speed of the particle
- iii** the amplitude of the motion
- iv** the potential energy when the displacement is 2.0 cm.

b On a copy of the axes, draw the variation with displacement of the potential energy of the particle.

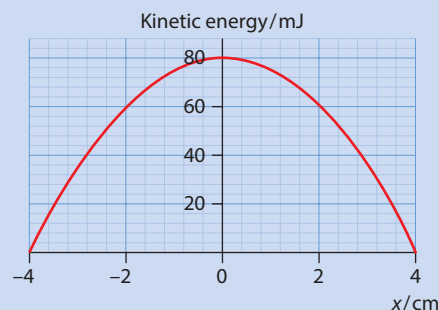


Figure 4.12 Graph showing the variation with displacement of the kinetic energy of a particle.

- a**
- i** The total energy is equal to the maximum kinetic energy, i.e. 80 mJ.
 - ii** The maximum speed is found from:

$$\frac{1}{2}mv_{\max}^2 = E_{\max}$$

$$v_{\max}^2 = \frac{2E_{\max}}{m}$$

$$v_{\max} = \sqrt{\frac{2 \times 80 \times 10^{-3}}{0.40}}$$

$$v_{\max} = 0.63 \text{ m s}^{-1}$$

- iii** The amplitude is 4.0 cm.
- iv** When $x = 2.0$ cm, the kinetic energy is 60 mJ and so the potential energy is 20 mJ.

b The graph is an inverted parabola as the blue curve in **Figure 4.11**.



Oscillations and time

The measurement of time depends on regular oscillations. In early time-keeping devices the oscillations were mechanical, for example the swinging of a simple pendulum in a clock. Now they are electrical oscillations in electronic circuits. The need for internationally accepted measures of time is essential for communications, travel, electricity supply and practically all other aspects of modern life.

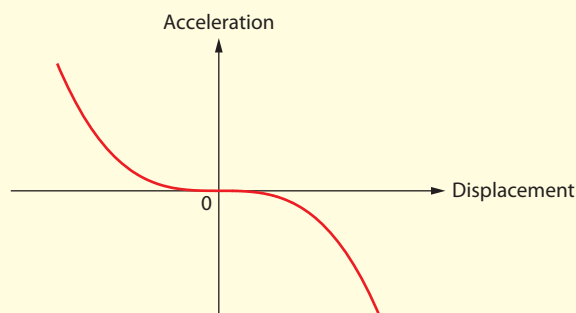
Nature of science

In real life we observe many oscillations for which the period is not independent of amplitude, for example the waving of a branch in the wind or the bouncing of a ball released above the ground. These do not obey the simple SHM equations in this section. But the general principles of physics we have met in this section govern many oscillations in the world about us, from water waves in the deep ocean to the vibration of a car's suspension system. The idea of the simple harmonic oscillator and the mathematics of SHM give physicists powerful tools to describe all periodic oscillations.



Test yourself

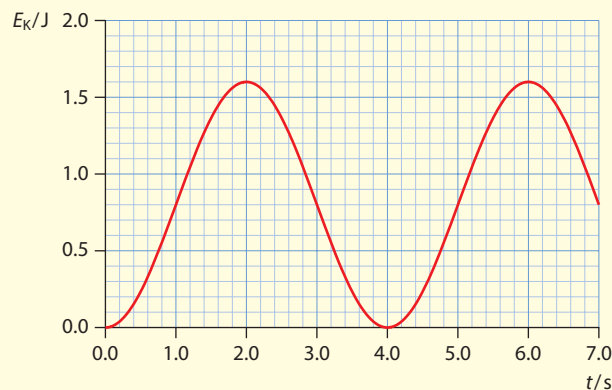
- 1 State what is meant by: **a** oscillation and **b** simple harmonic oscillation.
- 2 A ball goes back and forth along a horizontal floor, bouncing off two vertical walls. Suggest whether this motion is an example of an oscillation. If yes, state if the oscillation is simple harmonic.
- 3 A ball bounces vertically off the floor. Suggest whether this motion is an example of an oscillation. If yes, state if the oscillation is simple harmonic.
- 4 The graph shows the variation with displacement of the acceleration of a particle that is performing oscillations.



Explain:

- a** how it is known that the particle is performing oscillations
- b** why the oscillations are not simple harmonic.

- 5 The graph shows the variation with time of kinetic energy of a particle that is undergoing simple harmonic oscillations.



- a** Use the graph to calculate:
 - i** the mass of the particle
 - ii** the period of oscillations.
- b** Draw a graph to show the variation with time of the potential energy.

Nature of science

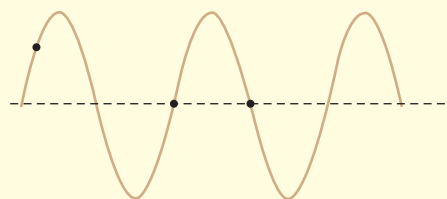
Careful observations of the vibrations of a plucked violin string led the Swiss mathematician Bernoulli to find a way to describe the oscillation using mathematics. This was a simple setting – a single string fixed at both ends – and led to a simple solution. Scientists found similar patterns in more complex oscillations and waves in the natural world, but also differences in the way that waves propagated. By looking for trends and discrepancies in their models for different waves produced under different conditions, scientists developed wave equations that apply across many areas of physics.



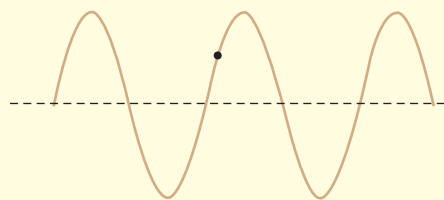
Test yourself

- 6 In football stadiums fans often create a 'wave' by standing up and sitting down again. Suggest factors that determine the speed of the 'wave'.
- 7 A number of dominoes are stood next to each other along a straight line. A small push is given to the first domino, and one by one the dominoes fall over.
 - a Outline how this is an example of wave motion.
 - b Suggest how the speed of the wave pulse could be increased.
 - c Design an experiment in which this problem can be investigated.
- 8 By making suitably labelled diagrams explain the terms:
 - a **wavelength**
 - b **period**
 - c **amplitude**
 - d **crest**
 - e **trough**.
- 9
 - a Explain, in the context of wave motion, what you understand by the term **displacement**.
 - b Using your answer in **a**, explain the difference between longitudinal and transverse waves.
 - c A rock thrown onto the still surface of a pond creates circular ripples moving away from the point of impact. Suggest why more than one ripple is created.
 - d Why does the amplitude decrease as the ripple moves away from the centre?

- 10 The diagram shows three points on a string on which a transverse wave propagates to the right.

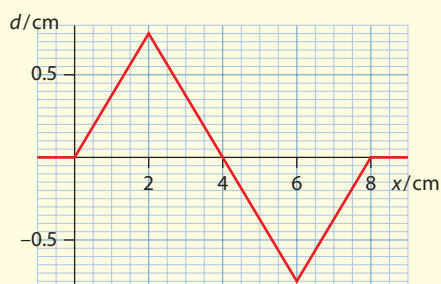


- a Indicate how these three points will move in the next instant of time.
 - b How would your answers change if the wave were moving to the left?
- 11 The diagram shows a piece of cork floating on the surface of water when a wave travels through the water to the right. Copy the diagram, and add to it the position of the cork half a wave period later.

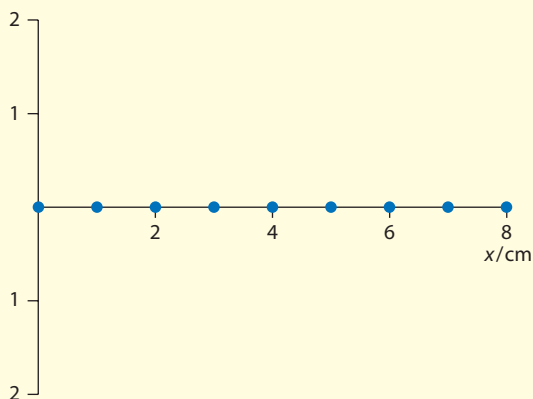


- 12 Calculate the wavelength that corresponds to a sound frequency of:
 - a 256 Hz
 - b 25 kHz.(Take the speed of sound to be 330 m s^{-1} .)

- 13 The graph shows the displacement y of the particles in a medium against position x when a longitudinal wave pulse travels through the medium from left to right with speed 1.0 cm s^{-1} . This is the displacement at $t = 0$.



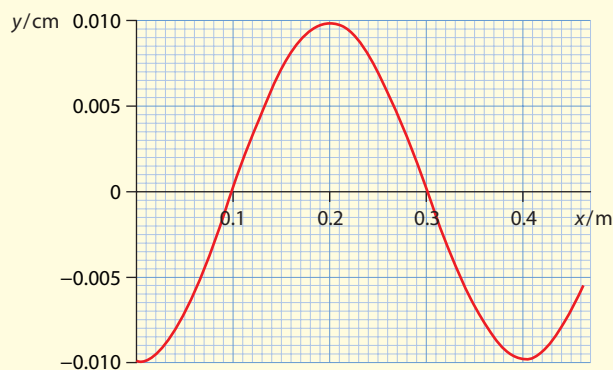
- a State what is meant by a **longitudinal** wave pulse. The diagram shows a line of nine molecules separated by 1.0 cm . The positions shown are the equilibrium positions of the molecules when no wave travels in the medium.



- b i Copy the diagram. Immediately below the copied line draw another line to show the position of these molecules when the pulse travels through the medium at $t = 0$.
 ii Indicate on the diagram the position of a compression.
 c i Repeat b i to show the position of these molecules at $t = 1.0 \text{ s}$.
 ii Comment on the position of the compression at 1.0 s .

- 14 The graph shows the variation with distance x of the displacement y of air molecules as a sound wave travels to the right through air. Positive displacement means motion to the right. The speed of sound in air is 340 m s^{-1} .

- a Determine the frequency of the sound wave.
 b State a distance x at which i a compression and ii a rarefaction occurs.



Learning objectives

- Describe waves in terms of wavefronts and rays.
- Solve problems using the concepts of intensity and amplitude and the inverse square law.
- Apply the principle of superposition to pulses and waves.
- Interpret diagrams of incident, reflected and transmitted beams in terms of polarisation.
- Solve problems with Malus' law.

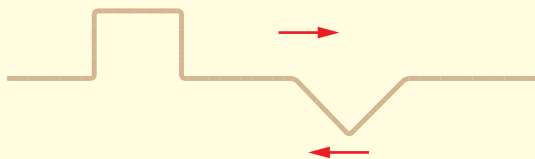
4.3 Wave characteristics

This section deals with ways to describe waves and the important principle of superposition. When two tennis balls collide they bounce off each other, but waves are different: they can go through each other without any 'memory' of a collision. Polarisation is a phenomenon that applies to transverse waves only. Light can be polarised, and so light is a transverse wave.

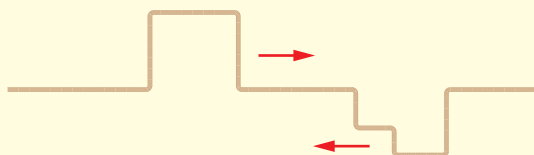


Test yourself

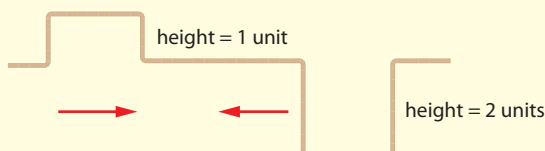
- 15 The diagram shows two pulses of equal width and height travelling in opposite directions on the same string. Draw the shape of the string when the pulses completely overlap.



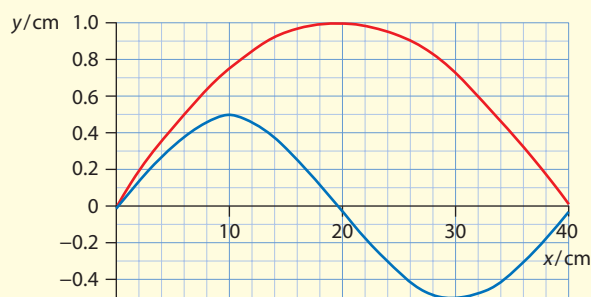
- 16 The diagram shows two pulses of equal width and height travelling in opposite directions on the same string. Draw the shape of the string when the pulses completely overlap.



- 17 The wave pulses shown in the diagram travel at 1 cm s^{-1} and both have width 2 cm . The heights are indicated on the diagram. In each case, draw the shape of the resulting pulse according to the principle of superposition at times $t = 0.5 \text{ s}$, $t = 1.0 \text{ s}$ and $t = 1.5 \text{ s}$. Take $t = 0 \text{ s}$ to be the time when the pulses are about to meet each other.



- 18 Two waves are simultaneously generated on a string. The graph shows the variation of displacement y with distance x . Draw the actual shape of the string.



- 19 In the context of wave motion explain, with the aid of a diagram, the terms:

a **wavefront**

b **ray**.

- 20 a State what is meant by **polarised light**.

b State **two** methods by which light can be polarised.

- 21 Suggest why only transverse waves can be polarised.

- 22 Light is incident on a polariser. The transmitted intensity is measured as the orientation of the polariser is changed. In each of the following **three** outcomes, determine whether the incident light is polarised, partially polarised or completely unpolarised, explaining your answers.

a The intensity of the transmitted light is the same no matter what the orientation of the polariser.

b The intensity of the transmitted light varies depending on the orientation of the polariser. At a particular orientation, the transmitted intensity is zero.

c The transmitted intensity varies as the orientation varies, but it never becomes zero.

- 23 a State Malus's law.

b Polarised light is incident on a polariser whose transmission axis makes an angle of 25° with the direction of the electric field of the incident light. Calculate the fraction of the incident light intensity that is transmitted through the polariser.

- 24 Two polarisers have their transmission axes at right angles to each other.

a Explain why no light will get transmitted through the second polariser.

b A third polariser is inserted in between the first two. Its transmission axis is at 45° to the other two. Determine whether any light will be transmitted by this arrangement of three polarisers.

c If the third polariser were placed in front of the first rather than in between the two, would your answer to **b** change?

Nature of science

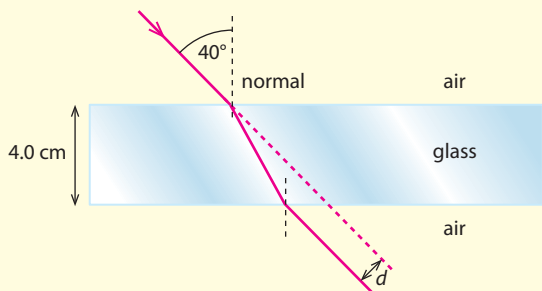
Competing theories and progress in science

At the start of this section we mentioned the conflict between the Newton and Huygens over the nature of light. In 1817 Augustin-Jean Fresnel published a new wave theory of light. The mathematician Siméon Poisson favoured the particle theory of light, and worked out that Fresnel's theory predicted the presence of a bright spot in the shadow of a circular object, which he believed was impossible. François Arago, a supporter of Fresnel, was able to show there was indeed a bright spot in the centre of the shadow. In further support of his theory, Fresnel was able to show that the polarisation of light could only be explained if light was a transverse wave. The wave theory then took precedence, until new evidence showed that light could behave as both a wave and a particle.



Test yourself

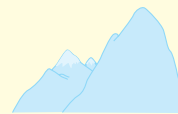
- 25 Red light of wavelength $6.8 \times 10^{-7} \text{ m}$ in air enters glass with a refractive index of 1.583, with an angle of incidence of 38° . Calculate:
- the angle of refraction
 - the speed of light in the glass
 - the wavelength of light in the glass.
- 26 Light of frequency $6.0 \times 10^{14} \text{ Hz}$ is emitted from point **A** and is directed toward point **B** a distance of 3.0 m away.
- Determine how long will it take light to get to **B**.
 - Calculate how many waves fit in the space between **A** and **B**.
- 27 A ray of light is incident on a rectangular block of glass of refractive index 1.450 at an angle of 40° , as shown in the diagram. The thickness of the block is 4.00 cm. Calculate the amount d by which the ray is deviated.
- 28 The speed of sound in air is 340 m s^{-1} and in water it is 1500 m s^{-1} . Determine the angle at which a beam of sound waves must hit the air–water boundary so that no sound is transmitted into the water.
- 29 Planar waves of wavelength 1.0 cm approach an aperture whose opening is also 1.0 cm. Draw the wavefronts of this wave as they emerge through the aperture.
- 30 Repeat question 52 for waves of wavelength 1 mm approaching an aperture of size 20 cm.
- 31 A radio station, R, emits radio waves of wavelength 1600 m which reach a house, H, directly and after reflecting from a mountain, M, behind the house (see diagram). The reception at the house is very poor. Estimate the shortest possible distance between the house and the mountain. (Pay attention to phase changes.)



R



H



M



? Test yourself

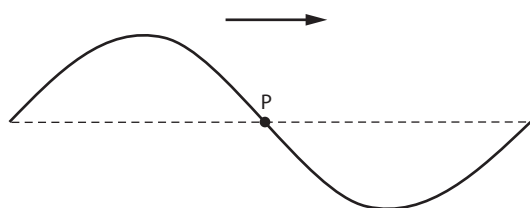
- 32 Describe what is meant by a standing wave. List the ways in which a standing wave differs from a travelling wave.
- 33 Outline how a standing wave is formed.
- 34 In the context of standing waves describe what is meant by:
 - a node
 - b antinode
 - c wave speed.
- 35 a Describe how you would arrange for a string that is kept under tension, with both ends fixed, to vibrate in its second harmonic mode.
 b Draw the shape of the string when it is vibrating in its second harmonic mode.
- 36 A string is held under tension, with both ends fixed, and has a first harmonic frequency of 250 Hz. The tension in the string is changed so that the speed increases by $\sqrt{2}$. Predict the new frequency of the first harmonic.
- 37 A string has both ends fixed. Determine the ratio of the frequency of the second to that of the first harmonic.
- 38 The wave velocity of a transverse wave on a string of length 0.500 m is 225 m s^{-1} .
 - a Determine the frequency of the first harmonic of a standing wave on this string when both ends are kept fixed.
 - b Calculate the wavelength of the sound produced in air by the oscillating string in a. (Take the speed of sound in air to be 340 m s^{-1} .)
- 39 Draw the standing wave representing the third harmonic standing wave in a tube with one closed and one open end.
- 40 A glass tube is closed at one end. The air column it contains has a length that can be varied between 0.50 m and 1.50 m. A tuning fork of frequency 306 Hz is sounded at the top of the tube. Predict the lengths of the air column at which loud sounds would be heard from the tube. (Take the speed of sound to be 340 m s^{-1} .)
- 41 A glass tube with one end open and the other closed is used in an experiment to determine the speed of sound. A tuning fork of frequency 427 Hz is used and a loud sound is heard when the air column has length equal to 20.0 cm.
 - a Calculate the speed of sound.
 - b Predict the next length of air column when a loud sound will again be heard.
- 42 A pipe with both ends open has two consecutive harmonics of frequency 300 Hz and 360 Hz.
 - a Suggest which harmonics are excited in the pipe.
 - b Determine the length of the pipe. (Take the speed of sound to be 340 m s^{-1} .)
- 43 A pipe X with both ends open and a pipe Y with one open and one closed end have the same frequency in the first harmonic. Calculate the ratio of the length of pipe X to that of pipe Y.
- 44 If you walk at one step a second holding a cup of water (diameter 8 cm) the water will spill out of the cup. Use this information to **estimate** the speed of the waves in water.
- 45 Consider a string with both ends fixed. A standing wave in the second harmonic mode is established on the string, as shown in the diagram. The speed of the wave is 180 m s^{-1} .

 - a Explain the meaning of **wave speed** in the context of standing waves.
 - b Consider the vibrations of two points on the string, P and Q. The displacement of point P is given by the equation $y = 5.0 \cos(45\pi t)$, where y is in mm and t is in seconds. Calculate the length of the string.
 - c State the phase difference between the oscillation of point P and that of point Q. Hence write down the equation giving the displacement of point Q.

- 46 A horizontal aluminium rod of length 1.2 m is hit sharply with a hammer. The hammer rebounds from the rod 0.18 ms later.
- Explain why the hammer rebounds.
 - Calculate the speed of sound in aluminium.
 - The hammer created a longitudinal standing wave in the rod. Estimate the frequency of the sound wave by assuming that the rod vibrates in the first harmonic.

Exam-style questions

- 1 The diagram shows a point P on a string at a particular instant of time. A transverse wave is travelling along the string from left to right.



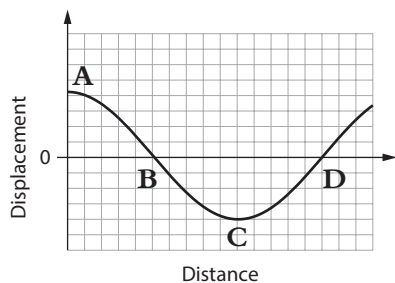
Which is correct about the direction and the magnitude of the velocity of point P at this instant?

	Direction	Magnitude
A	up	maximum
B	up	minimum
C	down	maximum
D	down	minimum

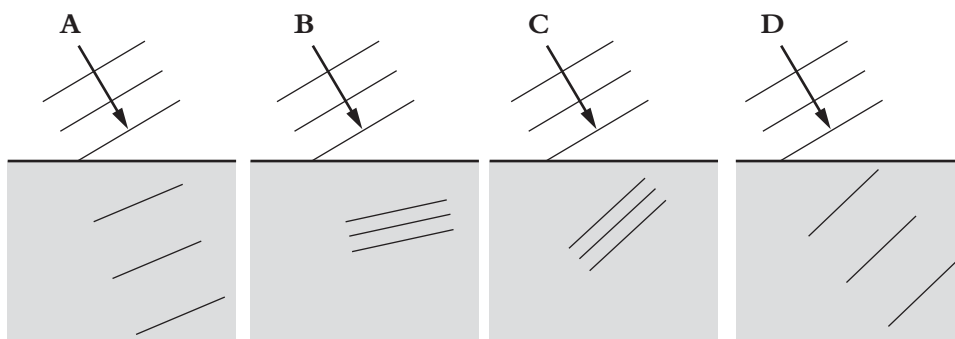
- 2 A tight horizontal rope with one end tied to a vertical wall is shaken with frequency f so that a travelling wave of wavelength λ is created on the rope. The rope is now shaken with a frequency $2f$. Which gives the new wavelength and speed of the wave?

	Wavelength	Speed
A	λ	$f\lambda$
B	λ	$2f\lambda$
C	$\frac{\lambda}{2}$	$f\lambda$
D	$\frac{\lambda}{2}$	$2f\lambda$

- 3 The graph shows the displacement of a medium when a longitudinal wave travels through the medium from left to right. Positive displacements correspond to motion to the right. Which point corresponds to the centre of a compression?

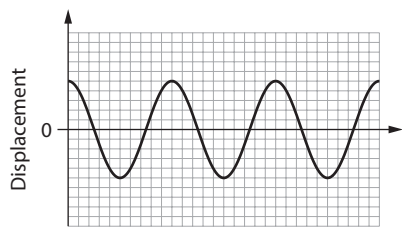
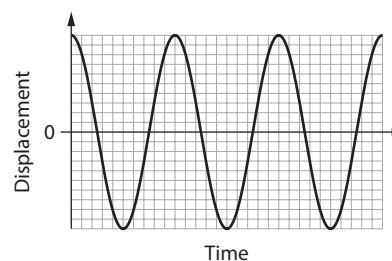


- 4 The diagram shows wavefronts of a wave entering a medium in which the wave speed decreases. Which diagram is correct?

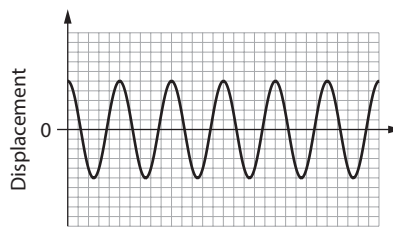


- 5 The graph shows the variation with time of the displacement of a particle in a medium when a wave of intensity I travels through the medium.

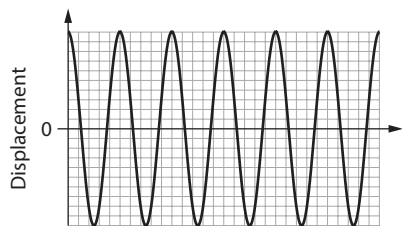
The intensity of the wave is halved. Which graph now represents the variation of displacement with time? (The scale on all graphs is the same.)



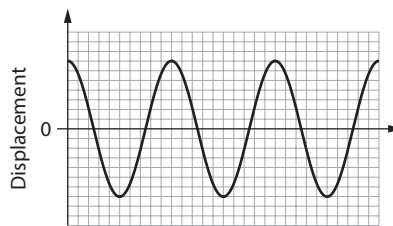
Time
A



Time
B



Time
C



Time
D

6 Which of the following does **not** apply to longitudinal waves?

- A superposition
- B formation of standing waves
- C interference
- D polarisation

7 Interference is observed with two identical coherent sources. The intensity of the waves at a point of constructive interference is I . What is the intensity when one source is removed?

- A 0 B I C $\frac{I}{2}$ D $\frac{I}{4}$

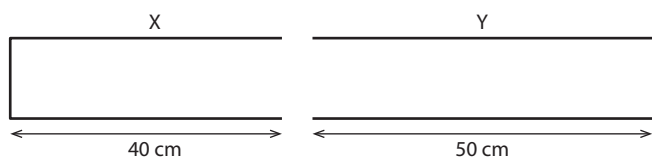
8 Unpolarised light of intensity I_0 is incident on two polarisers, one behind the other, with parallel transmission axes. The first polariser is rotated by 30° clockwise and the second 30° counter-clockwise. What is the intensity transmitted?

- A $\frac{I_0}{2}$ B $\frac{I_0}{4}$ C $\frac{I_0}{8}$ D $\frac{I_0}{16}$

9 A pipe of length 8.0 m is open at one end and closed at the other. The speed of sound is 320 m s^{-1} . Which is the lowest frequency of a standing wave that can be established within this pipe?

- A 5.0 Hz B 10 Hz C 15 Hz D 30 Hz

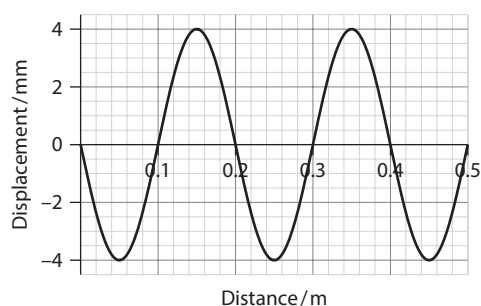
10 Travelling waves of wavelength 32 cm are created in a closed–open pipe X of length 40 cm and an open–open pipe Y of length 50 cm.



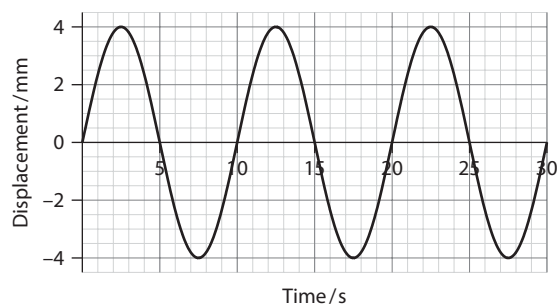
In which pipe or pipes will a standing wave be formed?

- A X only B Y only C neither X nor Y D both X and Y

11 A longitudinal wave is travelling through a medium. The displacement of the wave at $t = 10 \text{ s}$ is shown below. Positive displacements are directed to the right.

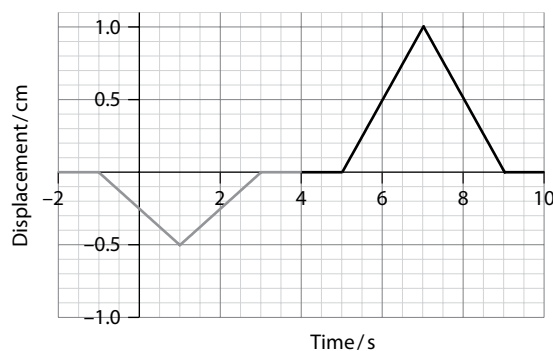
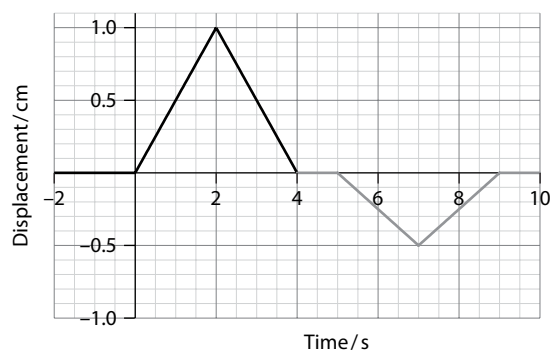


Point P is at a distance of 20 cm from the origin. The graph shows the variation with time of the displacement of point P.



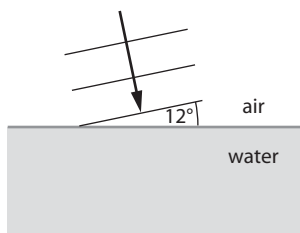
- a Distinguish a longitudinal from a transverse wave. [2]
- b State, for this wave:
 - i the amplitude [1]
 - ii the wavelength [1]
 - iii the frequency. [1]
- c Calculate the speed of the wave. [2]
- d Suggest whether the wave is travelling to the right or to the left. [3]
- e Point Q is a distance of 15 cm to the right of P. Draw the variation of the displacement of Q with time. [2]
- f The travelling wave in parts **a–d** is directed towards a pipe that has one end closed and the other open.
 - i State and explain the length of the pipe so that a standing wave in its first harmonic is established within the pipe. [2]
 - ii State **two** differences between a standing wave and a travelling wave. [2]
 - iii In the context of a standing wave state the meaning of the term **wave speed**. [2]

- 12 Two pulses travel towards each other on the same taut rope. The two graphs show the pulses before and after the collision. The speed of the black pulse is 15 ms^{-1} . The left diagram shows the pulses at $t = 0$.

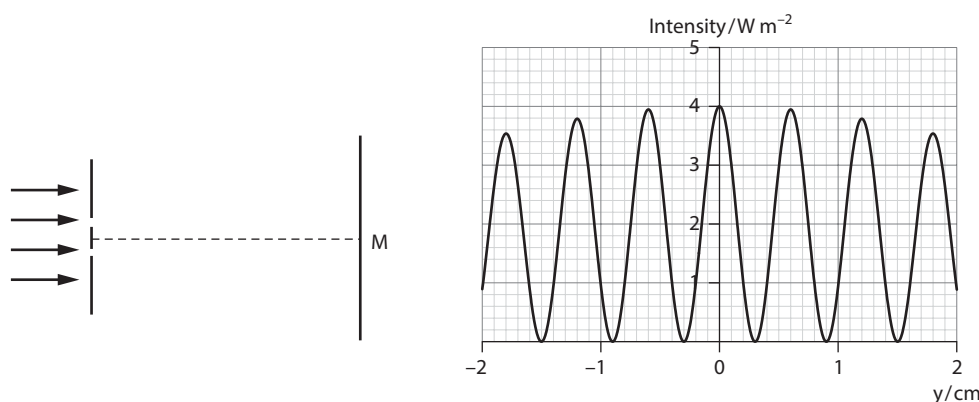


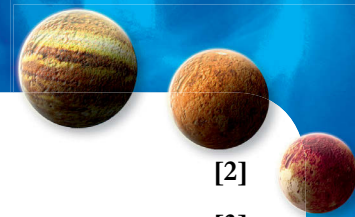
- a State the principle of superposition. [2]
- b State the speed of the grey pulse. [1]
- c
 - i Determine the time after which the two pulses completely overlap. [1]
 - ii Draw the shape of the rope at the time of complete overlap. [2]
- d
 - i Suggest whether any energy was lost during the collision of the two pulses. [2]
 - ii Comment on the shape of the rope in **c ii** by reference to your answer in **d i**. [3]

- 13 A man is swimming underwater at a depth of 2.0 m. The man looks upwards.
- Explain why he can see the world outside the water only through a circle on the surface of the water. [2]
 - Calculate the radius of this circle given that the refractive index of water is 1.33. [2]
 - Discuss how the answer to **b** changes (if at all) if he looks up from a greater depth. [2]
 - Sound waves travelling in air approach an air–water boundary. The speed of sound in air is 340 m s^{-1} and in water it is 1500 m s^{-1} . The wavefronts make an angle of 12° with the boundary.



- Calculate the angle the wavefronts in the water make with the boundary. [2]
 - Draw three wavefronts in the water. [2]
 - Use your answer to **ii** to suggest why a person swimming underwater near a noisy beach does not hear much noise. [2]
- 14 **a** Describe what is meant by **polarised light**. [1]
- b** Unpolarised light of intensity 320 W m^{-2} is incident on the first of three polarisers that are one behind the other. The first and third polarisers have vertical transmission axes. The middle polariser's transmission axis is rotated by an angle θ to the vertical. The transmitted intensity is 10 W m^{-2} . Determine θ . [3]
- c** Partially polarised light is a combination of completely unpolarised light and light that is polarised. Partially polarised light is transmitted through a polariser. As the polariser is rotated by 360° the ratio of the maximum to the minimum transmitted intensity is 7. Determine the fraction of the beam's intensity that is due to the polarised light. [3]
- d** A person is sitting behind a vertical glass wall. The person cannot be clearly seen because of the glare of reflections from the glass wall. Suggest how the use of a polariser makes it easier to see the person more clearly. [3]
- 15 In a two-slit experiment, red light is incident on two parallel slits. The light is observed on a screen far from the slits. The graph shows how the intensity of the light on the screen varies with distance y from M.





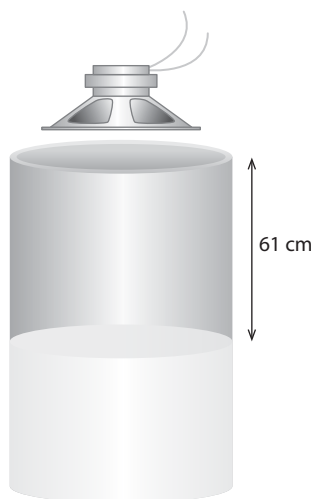
- a Explain why light is able to reach the middle of the screen. [2]
- b One of the slits is covered. State and explain the intensity of the light at M. [3]
- c State the feature of this graph that shows that the slit width is not negligibly small. [1]

The distance to the screen is 3.2 m and the separation of the slits is 0.39 mm.

- d Determine the wavelength of the light. [2]
- e The red light is replaced by blue light. Predict what (if anything) will happen to the separation of the bright fringes on the screen. [2]

- 16 a Outline how a standing wave may be formed. [2]

A source of sound is placed above a tube containing water. The longest length of the air column above the water for which a strong sound is heard from the tube is 61 cm. The next length of the air column for which another strong sound is heard is 49 cm.



- b
 - i Explain the origin of the loud sound from the tube. [2]
 - ii Suggest why a strong sound is heard for specific lengths of the air column. [2]
 - iii Predict the next length of the air column for which a loud sound will be heard. [1]
 - iv The frequency of the source is 1400 Hz. Estimate the speed of sound in the tube. [2]

- b** The magnitude of the decrease in kinetic energy of the proton is:

$$\Delta E_K = \frac{1}{2} \times 1.67 \times 10^{-27} [(4.4 \times 10^5)^2 - (2.2 \times 10^5)^2]$$

$$\Delta E_K = 1.2 \times 10^{-16} \text{ J}$$

Converting to electronvolts:

$$\begin{aligned} \Delta E_K &= \frac{1.2 \times 10^{-16} \text{ J}}{1.6 \times 10^{-19} \text{ J eV}^{-1}} \\ &= 750 \text{ eV} \end{aligned}$$

Hence $qV = 750 \text{ eV}$, implying $V = 750 \text{ volts}$.

Nature of science

The microscopic–macroscopic connection

If you are plumber, do you need to know the molecular structure of water? The flow of water in pipes is a macroscopic phenomenon whereas the detailed molecular structure of water is microscopic. We have a vast difference in scales of length in the two cases. In very many phenomena the presence of two different scales means that the detailed physics operating at one scale does not affect the physics at the other. This is also the case with current: it was possible to give detailed descriptions of the behaviour of current in circuits long before it was discovered that current is electrons moving in the same direction. (However, the most complicated problems in physics are those in which the physics at one length scale does affect the physics at the other scale.)

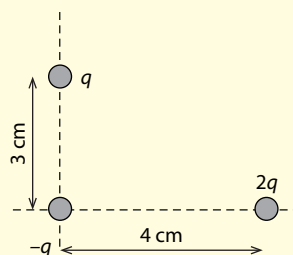
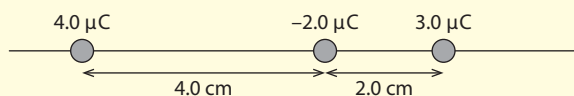


? Test yourself

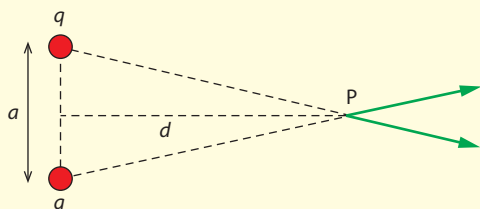
- a** Calculate the force between two charges q_1 of $2.0 \mu\text{C}$ and q_2 $4.0 \mu\text{C}$ separated by $r = 5.0 \text{ cm}$.

b Let the force calculated in **a** be F . In terms of F and without further calculations, state the force between these charges when:

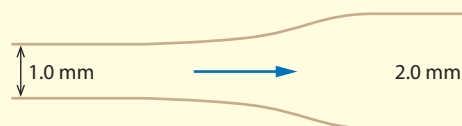
 - the separation r of the charges is doubled
 - q_1 and r are both doubled
 - q_1 , q_2 and r are all doubled.
- Three charges are placed on a straight line as shown in the diagram. Calculate the net force on the middle charge.
- In the previous question, determine the position of the middle charge so that it is in equilibrium.
- Calculate the force (magnitude and direction) on the charge q in the diagram where $q = 3.0 \mu\text{C}$.



- 5 Two plastic spheres each of mass 100.0 mg are suspended from very fine insulating strings of length 85.0 cm. When equal charges are placed on the spheres, the spheres repel and are in equilibrium when 10.0 cm apart.
- Determine the charge on each sphere.
 - Estimate how many electron charges this corresponds to.
- 6 Consider two people, each of mass 60 kg, a distance of 10 m apart.
- Assuming that all the mass in each person is made out of water, estimate how many electrons there are in each person.
 - Hence, estimate the electrostatic force of repulsion between the two people due to the electrons.
 - List any other simplifying assumptions you have made to make your estimate possible.
 - No such force is observed in practice. Suggest why this is so.
- 7 A charge of magnitude $+5.0 \mu\text{C}$ experiences an electric force of magnitude $3.0 \times 10^{-5} \text{ N}$ when placed at a point in space. Determine the electric field at that point.
- 8 The electric field is a vector and so two electric fields at the same point in space must be added according to the laws of vector addition. Consider two equal positive charges q , each $2.00 \mu\text{C}$, separated by $a = 10.0 \text{ cm}$ and a point P a distance of $d = 30.0 \text{ cm}$, as shown in the diagram. The diagram shows the directions of the electric fields produced at P by each charge. Determine the magnitude and direction of the net electric field at P.



- 9 Repeat the calculation of question 8 where the top charge is $+2.00 \mu\text{C}$ and the bottom charge is $-2.00 \mu\text{C}$.
- 10 The electron drift speed in a copper wire of diameter 1.8 mm is $3.6 \times 10^{-4} \text{ ms}^{-1}$. The number of free electrons per unit volume for copper is $8.5 \times 10^{28} \text{ m}^{-3}$. Estimate the current in the wire.
- 11 In the diagram, the current through the 1.0 mm diameter part of the wire is 1.2 A and the drift speed is $2.2 \times 10^{-4} \text{ ms}^{-1}$.



- Calculate **a** the current and **b** the drift speed in the part of the wire with 2.0 mm diameter.
- 12 Silver has 5.8×10^{28} free electrons per m^3 . If the current in a 2 mm radius silver wire is 5.0 A, find the velocity with which the electrons drift in the wire.
- 13 **a** If a current of 10.0 A flows through a heater, how much charge passes through the heater in 1 h?
b How many electrons does this charge correspond to?
- 14 A conducting sphere of radius 15.0 cm has a positive charge of $4.0 \mu\text{C}$ deposited on its surface. Calculate the magnitude of the electric field produced by the charge at distances from the centre of the sphere of:
- 0.0 cm
 - 5.0 cm
 - 15.0 cm
 - 20.0 cm.

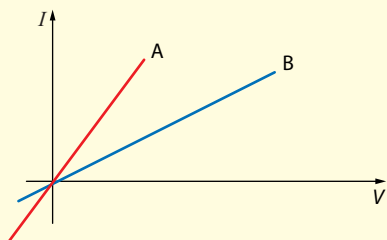
Nature of science

In 1825 in England Peter Barlow proposed a law explaining how wires conducted electricity. His careful experiments using a constant voltage showed good agreement, and his theory was accepted. At about the same time in Germany, Georg Ohm proposed a different law backed up by experimental evidence using a range of voltages. The experimental approach to science was not popular in Germany, and Ohm's findings were rejected. It was not until 1841 that the value of his work was recognised, first in England and later in Germany. In modern science, before research findings are published they are reviewed by other scientists working in the same area (peer review). This would have shown the errors in Barlow's work and given Ohm recognition sooner.



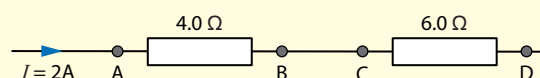
Test yourself

- 15 Outline the mechanism by which electric current heats up the material through which it flows.
- 16 Explain why doubling the length of a wire, at constant temperature, will double its resistance.
- 17 The graphs show the current as a function of voltage across the same piece of metal wire which is kept at two different temperatures.
 - a Discuss whether the wire obey Ohm's law.
 - b Suggest which of the two lines on the graph corresponds to the higher temperature.

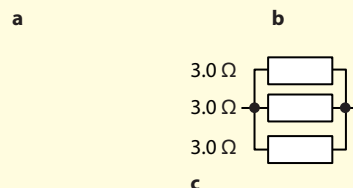
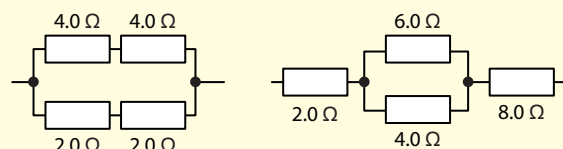


- 18 The current in a device obeying Ohm's law is 1.5 A when connected to a source of potential difference 6.0 V. What will the potential difference across the same device be when a current of 3.5 A flows in it?
- 19 A resistor obeying Ohm's law is measured to have a resistance of $12\ \Omega$ when a current of 3.0 A flows in it. Determine the resistance when the current is 4.0 A.
- 20 The heating element of an electric kettle has a current of 15 A when connected to a source of potential difference 220 V. Calculate the resistance of the heating element.

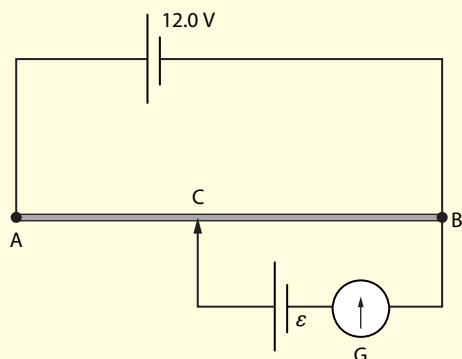
- 21 The diagram shows two resistors with a current of 2.0 A flowing in the wire.
 - a Calculate the potential difference across each resistor.
 - b State the potential between points B and C.



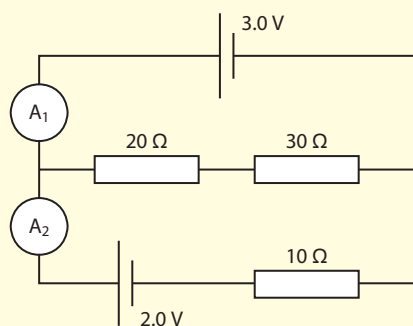
- 22 The filament of a lamp rated as 120 W at 220 V has resistivity $2.0 \times 10^{-6}\ \Omega\text{m}$.
 - a Calculate the resistance of the lamp when it is connected to a source of 220 V.
 - b The radius of the filament is 0.030 mm. Determine its length.
- 23 Determine the total resistance for each of the circuit parts in the diagram.



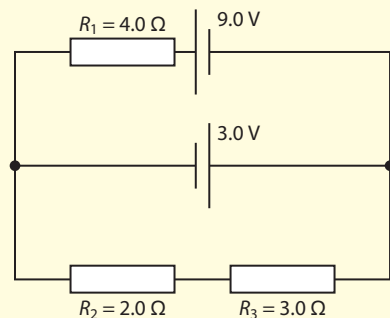
- 24 In the potentiometer in the diagram, wire AB is uniform and has a length of 1.00 m. When contact is made at C with $BC = 54.0$ cm, the galvanometer G shows zero current. Determine the emf of the second cell.



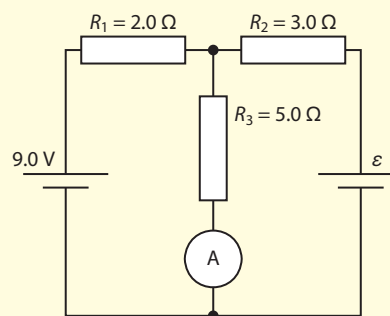
- 25 In the circuit shown the top cell has emf 3.0 V and the lower cell has emf 2.0 V. Both cells have negligible internal resistance. Calculate:
- the readings of the two ammeters
 - the potential difference across each resistor.



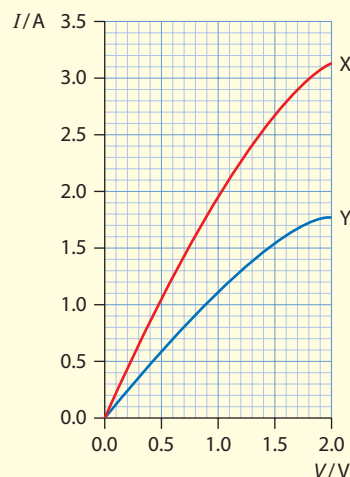
- 26 Calculate the current in each resistor in the circuit shown in the diagram.



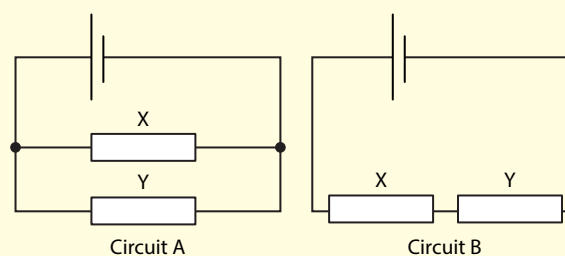
- 27 In the circuit in the diagram the ammeter reads 7.0 A. Determine the unknown emf ϵ .



- 28 Two resistors, X and Y, have I - V characteristics given by the graph.

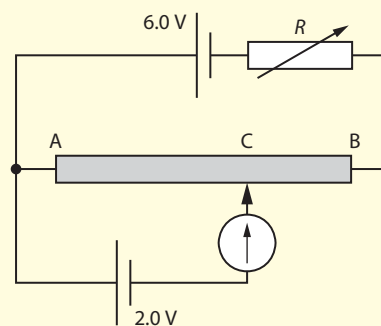


- a Circuit A shows the resistors X and Y connected in parallel to a cell of emf 1.5 V and negligible internal resistance. Calculate the total current leaving the cell.



- b In circuit B the resistors X and Y are connected in series to the same cell. Estimate the total current leaving the cell in this circuit.

- 29 The top cell in the circuit in the diagram has emf 6.0 V . The emf of the cell in the lower part of the circuit is 2.0 V . Both cells have negligible internal resistance. AB is a uniform wire of length 1.0 m and resistance $4.0\ \Omega$. When the variable resistor is set at $3.2\ \Omega$ the galvanometer shows zero current. Determine the length AC .



5.3 Electric cells

Batteries are now used to power watches, laptops, cars and entire submarines. Substantial advances in battery technology have resulted in batteries that store more energy, recharge faster and pose smaller environmental dangers.

Emf

We have already discussed that electric charges will not drift in the same direction inside a conductor unless a potential difference is established at the ends of the conductor. In a circuit we therefore need a source of potential difference. The most common is the connection of a **battery** in the circuit. (Others include a generator, a thermocouple or a solar cell.) What these sources do is to convert various forms of energy into **electrical energy**.

To understand the function of the battery, we can compare a battery to a pump that forces water through pipes up to a certain height and down again (Figure 5.42). The pump provides the gravitational potential energy mgh of the water that is raised. The water, descending, converts its gravitational potential energy into thermal energy (frictional losses) and mechanical work. Once the water reaches the pump, its gravitational potential energy has been exhausted and the pump must again perform work to raise the water so that the cycle repeats.

In an electric circuit a battery performs a role similar to the pump's. A battery connected to an outside circuit will force current in the circuit. Thus, the chemical energy of the battery is eventually converted into thermal energy (the current heats up the wires), into mechanical work (the circuit may contain a motor that may be used to raise a load) and into chemical energy again if it is used to charge another battery in the external circuit. Within the battery itself, negative ions are pushed from the negative to the positive terminal and positive ions in the opposite direction. This requires work that must be done on the ions (Figure 5.43). This work is provided by the **chemical energy** stored in the battery and is released by chemical reactions taking place inside the battery.

Learning objectives

- Distinguish between primary and secondary cells.
- Understand the presence of an internal resistance.
- Distinguish between emf and terminal potential difference.

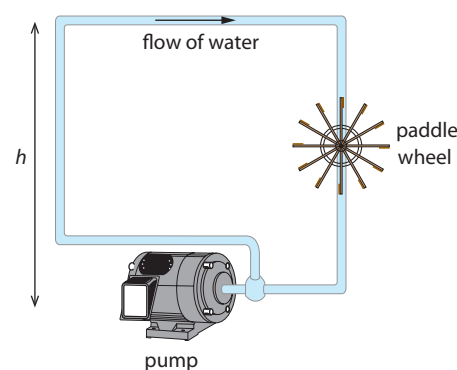


Figure 5.42 In the absence of the pump, the water flow would stop. The work done by the pump equals the work done to overcome frictional forces plus work done to operate devices, such as, for example, a paddle wheel.

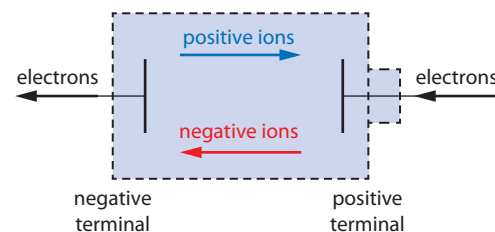


Figure 5.43 Inside the battery, negative ions move from the negative to the positive terminal of the battery. Positive ions move in the opposite direction. In the external circuit, electrons leave the negative battery terminal, travel through the circuit and return to the battery at the positive terminal.

Let us now calculate the power generated by the 12 V battery:

$$P = \mathcal{E}I = 12 \times 0.625 = 7.5 \text{ W}$$

The total resistance of the circuit is 16Ω and so the total power dissipated in the resistors is $16 \times 0.625^2 = 6.25 \text{ W}$. The remaining 1.25 W is stored in the 2.0 V battery that is being charged. This is the same as the power ‘dissipated’ by the battery: $2.0 \times (-0.625) = -1.25 \text{ W}$. We give the current a negative sign because it flows the ‘wrong’ way in the battery. The negative sign for the power means that this is power being stored, not being dissipated.

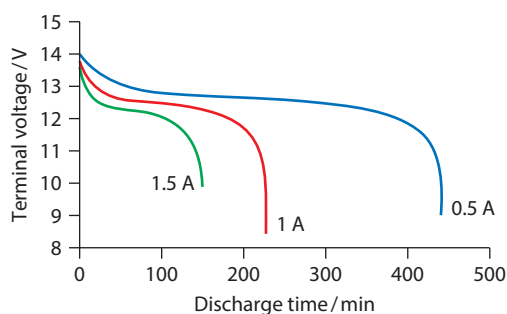


Figure 5.47 Discharge time for a cell for different currents.

Discharging a cell

A characteristic of a cell is the amount of charge it can deliver to an external circuit in its lifetime. This is known as the **capacity** of the cell. Suppose we connect a cell to an external resistor and monitor the potential difference across the cell, the terminal voltage. The general features are shown in **Figure 5.47**.

The bigger the current, the faster the cell discharges. After an initial sudden drop, the terminal voltage remains almost constant until the capacity of the cell is exhausted at the end of its lifetime, when there is again a sudden drop. The gentle drop in voltage for the majority of the cell’s lifetime is explained partly by an increasing internal resistance.

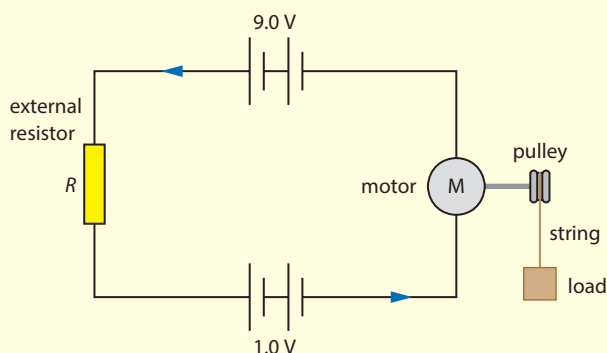
Nature of science

Consumers look for longer battery life in their electronic equipment, which drives research into electric cells. Mercury and cadmium are toxic components of some cells, and other cells contain flammable or otherwise dangerous materials. Scientists working to increase the storage capacity of cells need to balance the benefits (for example electric cars, which aim to be ‘greener’ than cars running on gasoline) with the long-term risks associated with the disposal of the chemical components when the batteries are discarded.



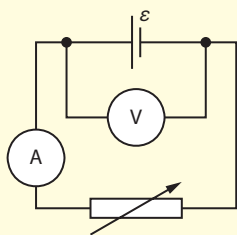
Test yourself

- 30** Describe the energy changes taking place in the circuit shown in the diagram.

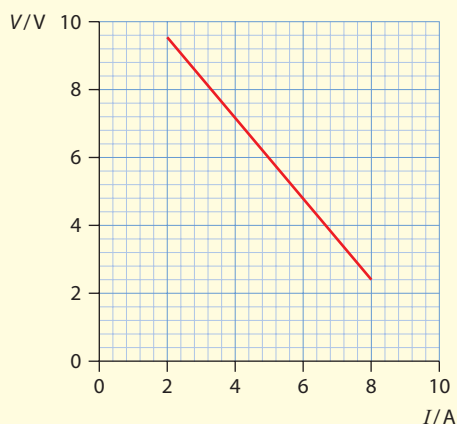


- 31** A battery has $\text{emf} = 10.0 \text{ V}$ and internal resistance 2.0Ω . The battery is connected in series to a resistance R . Make a table of the power dissipated in R for various values of R and then use your table to plot the power as a function of R . For what value of R is the power dissipated maximum?
- 32** A battery of $\text{emf } \mathcal{E}$ and internal resistance r sends a current I into a circuit.
- Sketch the potential difference across the battery as a function of the current.
 - What is the significance of **i** the slope and **ii** the vertical intercept of the graph?

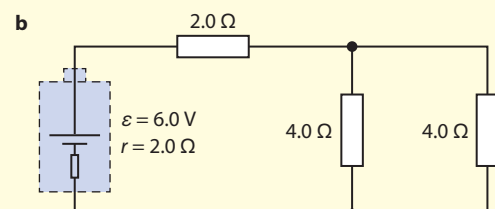
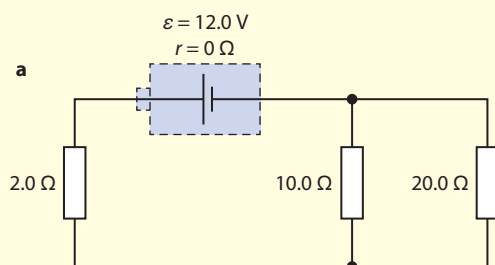
- 33 In an experiment, a voltmeter was connected across the terminals of a battery as shown in the diagram.



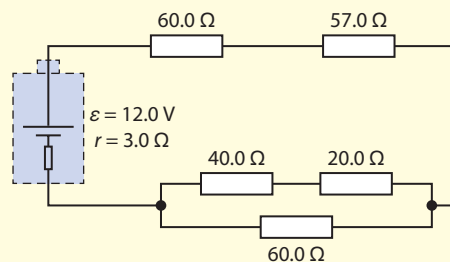
The current in the circuit is varied using the variable resistor. The graph shows the variation with current of the reading of the voltmeter.



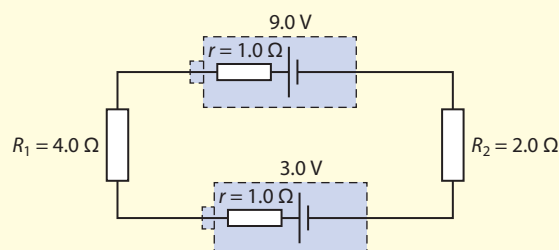
- a Calculate the internal resistance of the battery.
b Calculate the emf of the battery.
- 34 Calculate the current in, and potential difference across, each resistor in the circuits shown in the diagram.



- 35 When two resistors, each of resistance $4.0\ \Omega$, are connected in parallel with a battery, the current leaving the battery is 3.0 A . When the same two resistors are connected in series with the battery, the total current in the circuit is 1.4 A . Calculate:
- a the emf of the battery
b the internal resistance of the battery.



- 36 In the circuit shown in the diagram each of the cells has an internal resistance of $1.0\ \Omega$.



- a Determine the current in the circuit.
b Calculate the power dissipated in each cell.
c Comment on your answer to b.

The ampere is defined through the magnetic force between two parallel wires. If the force on a 1 m length of two wires that are 1 m apart and carrying equal currents is 2×10^{-7} N, then the current in each wire is defined to be 1 A.

The coulomb is defined in terms of the ampere as the amount of charge that flows past a certain point in a wire when a current of 1 A flows for 1 s.

The ampere is equal to a coulomb divided by a second; but it is defined as above.

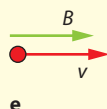
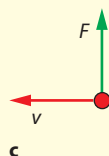
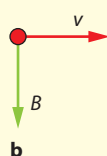
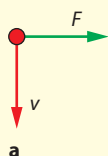
Nature of science

Introduced in the 19th century by Michael Faraday as 'lines of force', the concept of magnetic field lines allowed scientists to visualise the magnetic field around a magnet, and the magnetic field around a moving charge. A few years later, in one of the greatest unifications in physics, James Clerk Maxwell showed that all magnetic phenomena and electric phenomena are different sides of the same general phenomenon, electromagnetism, and that light is a combination of electric and magnetic fields. In the early 20th century, Albert Einstein showed that viewing electric and magnetic phenomena from different frames of reference leads naturally to the theory of relativity. At about the same time, trying to understand magnetism in different materials required the introduction of quantum theory.

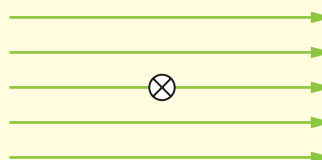


Test yourself

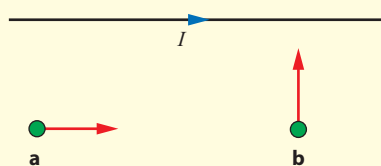
- 37 Draw the magnetic field lines for two parallel wires carrying equal currents into the page. Repeat for anti-parallel currents.
- 38 Determine the direction of the missing quantity from B , v and F in each of the cases shown in the diagram. The circle represents a positive charge.



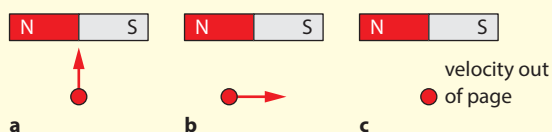
- 39 Draw the magnetic field lines that result when the magnetic field of a long straight wire carrying current into the page is superimposed on a uniform magnetic field pointing to the right that lies on the page.



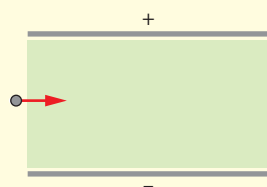
- 40 A long straight wire carries current as shown in the diagram. Two electrons move with velocities that are parallel and perpendicular to the current. Determine the direction of the magnetic force experienced by each electron.



- 41 A proton moves past a bar magnet as shown in the diagram. Find the direction of the force it experiences in each case.

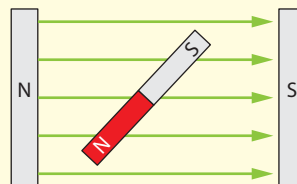


- 42 The diagram shows two parallel plates. The electric field is directed from top to bottom and has magnitude $2.4 \times 10^3 \text{ N C}^{-1}$. The shaded region is a region of magnetic field normal to the page.

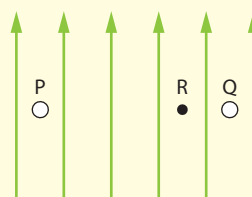


- Deduce the magnetic field magnitude and direction so that an electron experiences zero net force when shot through the plates with a speed of $2.0 \times 10^5 \text{ m s}^{-1}$.
- Suggest whether a proton shot with the same speed through the plates experiences zero net force.
- The electron's speed is doubled. Suggest whether the electron would still be undeflected for the same magnetic field found in **a**.

- 43 A bar magnet is placed in a uniform magnetic field as shown in the diagram.



- Suggest whether there is a net force on the bar magnet.
 - Determine how it will move.
- 44 A high-tension electricity wire running along a north-south line carries a current of 3000 A. The magnetic field of the Earth at the position of the wire has a magnitude of $5.00 \times 10^{-5} \text{ T}$ and makes an angle of 30° below the horizontal. Calculate the force experienced by a length of 30.0 m of the wire.
- 45 **a** An electron of speed v enters a region of magnetic field B directed normally to its velocity and is deflected into a circular path. Deduce an expression for the number of revolutions per second the electron will make.
- b** The electron is replaced by a proton. Suggest whether the answer to **a** changes.
- 46 A uniform magnetic field is established in the plane of the paper as shown in the diagram. Two wires carry **parallel** currents of equal magnitudes normally to the plane of the paper at P and Q. Point R is on the line joining P to Q and closer to Q. The magnetic field at position R is zero.
- Determine whether the currents are going into the paper or out of the paper.
 - The magnitude of the current is increased slightly. Determine whether the point where the magnetic field is zero moves to the right or to the left of R.



Exam-style questions

- 1 A small charge q is placed near a large spherical charge Q . The force experienced by both charges is F . The electric field created by Q at the position of q is:

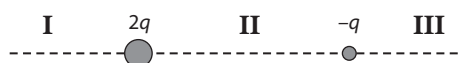
A $\frac{F}{Q}$

B $\frac{F}{q}$

C $\frac{F}{Qq}$

D $\frac{FQ}{q}$

- 2 Two charges are fixed as shown. The charges are $2q$ and $-q$. In which regions can the electric field strength due to the two particles be zero?



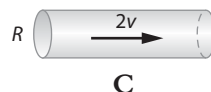
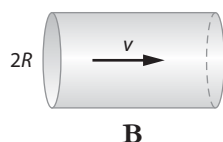
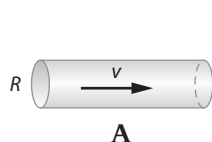
A I only

B II only

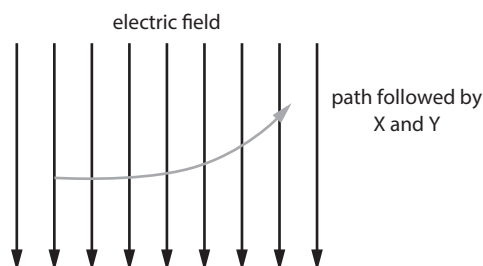
C III only

D I and III

- 3 The diagrams show equal lengths of wires made of the same material and various cross-sectional radii. The drift speed of electrons is indicated. In which wire is the current the greatest?



- 4 Two charged particles X and Y are projected horizontally with the same speed from the same point in a region of uniform electric field. Gravity is not negligible.



The two particles follow identical paths. What conclusion about X and Y can one draw from this?

- A** They have the same mass.
B They have the same charge.
C They have the same acceleration.
D They have the same momentum.

- 5 A charged particle moves in a circle of radius R in a region of uniform magnetic field. The magnetic field is at right angles to the velocity of the particle and exerts a force F on the particle. After half a revolution the change in the particle's kinetic energy is:

A 0

B πRF

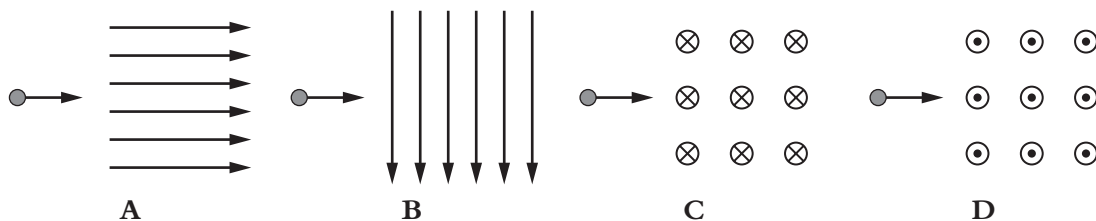
C $2\pi RF$

D RF

6 A negatively charged particle is at rest in a magnetic field B . The force on the particle is:

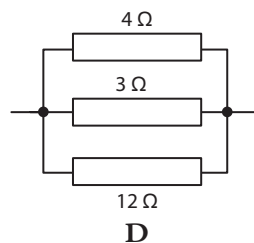
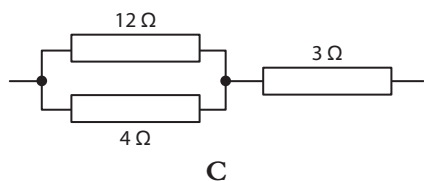
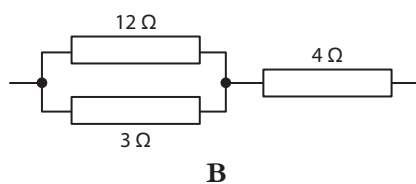
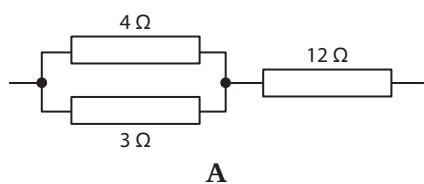
- A parallel to B
- B opposite to B
- C at right angles to B
- D zero.

7 An electron enters a region of magnetic field.

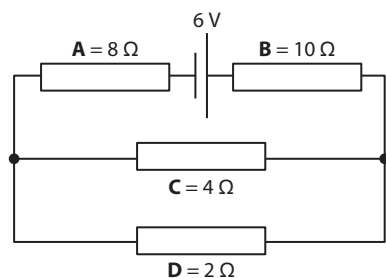


In which case is the initial force on the electron directed towards the bottom of the page?

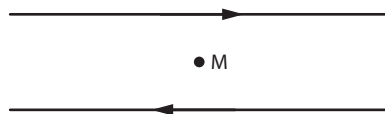
8 In which of the following arrangements is the total resistance 6Ω ?



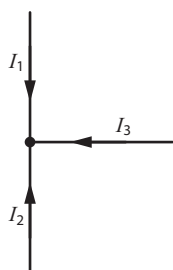
9 In which of the resistors in the circuit below is the power dissipated the least?



- 10 Two long parallel wires carry equal currents in opposite directions. What field do the two wires produce at point M, which is midway between the wires and on the plane of the paper?

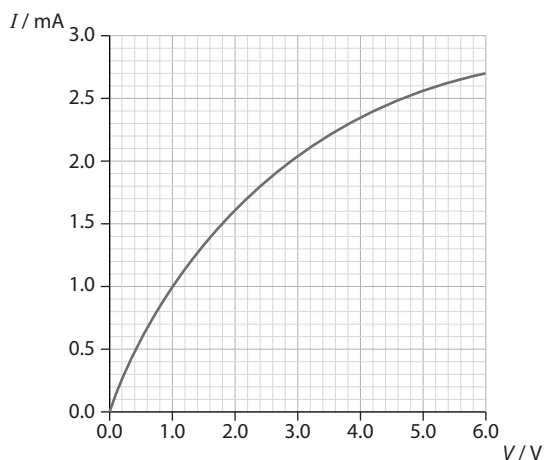


- A a magnetic field parallel to the wires
 B an electric field parallel to the wires
 C a magnetic field at right angles to the plane of the page
 D an electric field at right angles to the plane of the page
- 11 A student assigns currents at a junction in a circuit as shown in the diagram.



The student's calculations correctly give that $I_1 = 3\text{ A}$ and $I_2 = -2\text{ A}$. The value of I_3 is:

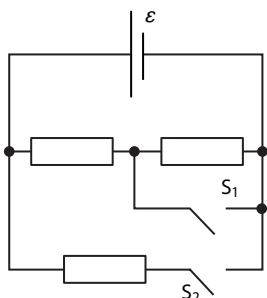
- A 1 A
 B -1 A
 C 5 A
 D -5 A
- 12 The graph shows the variation with voltage V across a filament lamp with the current I through the lamp.



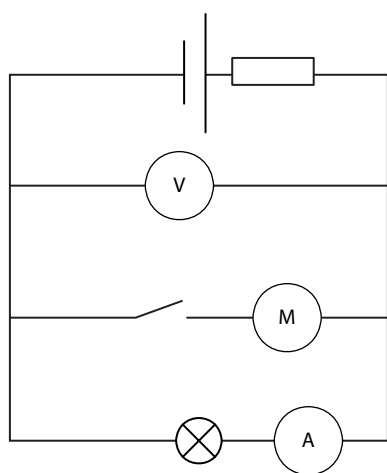
- a Suggest whether the resistor obeys Ohm's law. [1]
 b Calculate the resistance of the lamp when $V = 4.0\text{ V}$. [2]
 c The resistivity of the filament of the lamp at a voltage of 4.0 V is $3.0 \times 10^{-7}\ \Omega\text{ m}$. The radius of the filament is 0.25 mm . Calculate the length of the filament. [2]

- d Two lamps whose I - V characteristics are given by the graph above are connected in parallel to a battery of negligible internal resistance. The current leaving the battery is 2.0 mA. Estimate:
- i the emf of the battery [1]
 - ii the power dissipated in each lamp. [1]
- e Thermal energy is generated in a filament lamp when it is operating. Describe the mechanism by which this energy is generated. [3]

- 13 The three devices in the circuit shown are identical and may be assumed to have constant resistance. Each device is rated as 1500 W at 230 V. The emf of the source is 230 V and its internal resistance is negligible.



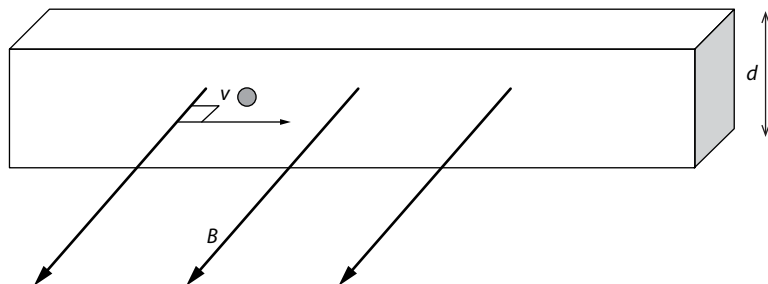
- a Calculate the resistance of **one** of the devices. [2]
- b Calculate the total power dissipated in the circuit when:
 - i S_1 is closed and S_2 is open [1]
 - ii S_1 is closed and S_2 is closed [1]
 - iii S_1 is open and S_2 is open [1]
 - iv S_1 is open and S_2 is closed. [1]
- c In the circuit below the cell has internal resistance $0.0500\ \Omega$. When the switch in series with a motor of resistance of $25.0\ \Omega$ is open, the voltmeter reads 11.5 V and the current in the ammeter is 9.80 A.



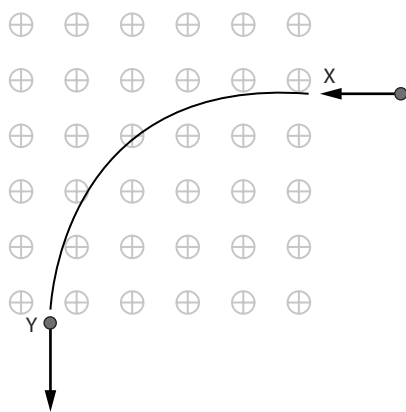
The switch is closed.

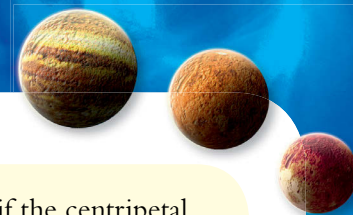
- i Determine the emf of the cell. [2]
- ii State and explain the effect, if any, of closing the switch on the brightness of the lamp. [2]
- iii Calculate the current through the motor. [2]

- 14 A current I is established in the conductor. The diagram shows one of the electrons making up the current moving with drift speed v . The conductor is exposed to a magnetic field B at right angles to the direction of motion of the electron.



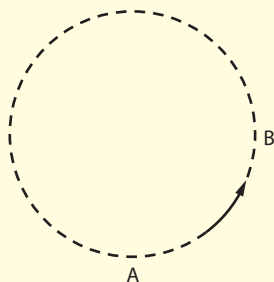
- a On a copy of the diagram, draw an arrow to indicate the direction:
 - i of the conventional current in the conductor [1]
 - ii the magnetic force on the electron. [1]
 - b Show that the current in the conductor is given by $I = qnAv$, where q is the charge of the electron, A the cross-sectional area of the conductor, v the drift speed of the electrons and n is the number of free electrons per unit volume. [3]
 - c Explain why a potential difference will be established between the top (T) and bottom (B) faces of the conductor. [3]
 - d
 - i The electric field between T and B is given by $E = \frac{V}{d}$ where V is the potential difference between T and B and d is their separation. Show that the voltage between T and B (the Hall voltage) is given by $V = vBd$. [2]
 - ii The current in the conductor is 0.50 A, the number density of electrons is $3.2 \times 10^{28} \text{ m}^{-3}$, the cross-sectional area of the wire is $4.2 \times 10^{-6} \text{ m}^2$ and the magnetic field is 0.20 T. Calculate the Hall voltage in this conductor. [3]
 - e Outline how the existence of the Hall voltage can be used to verify that the charge carriers in the conductor are negatively charged. [2]
- 15 A proton of mass m and electric charge q enters a region of magnetic field at point X and exits at point Y. The speed of the proton at X is v . The path followed by the proton is a quarter of a circle.



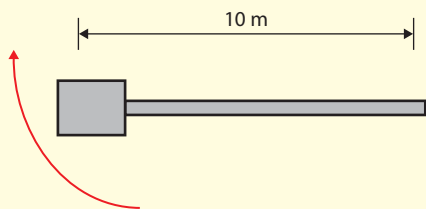


? Test yourself

- 1 **a** Calculate the angular speed and linear speed of a particle that completes a 3.50 m radius circle in 1.24 s.
b Determine the frequency of the motion.
- 2 Calculate the centripetal acceleration of a body that moves in a circle of radius 2.45 m making 3.5 revolutions per second.
- 3 The diagram shows a mass moving on a circular path of radius 2.0 m at constant speed 4.0 ms^{-1} .

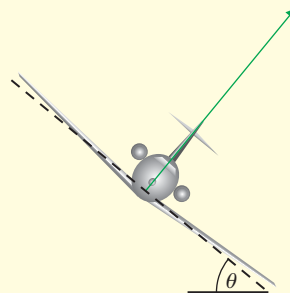


- a** Calculate the magnitude and direction of the average acceleration during a quarter of a revolution (from **A** to **B**).
 - b** Calculate the centripetal acceleration of the mass.
- 4 An astronaut rotates at the end of a test machine whose arm has a length of 10.0 m, as shown in the diagram. The acceleration she experiences must not exceed $5g$ (take $g = 10 \text{ ms}^{-2}$). Determine the maximum number of revolutions per minute of the arm.



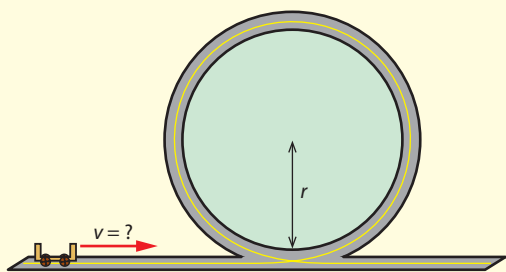
- 5 A body of mass 1.00 kg is tied to a string and rotates on a horizontal, frictionless table.
- a** The length of the string is 40.0 cm and the speed of revolution is 2.0 ms^{-1} . Calculate the tension in the string.
 - b** The string breaks when the tension exceeds 20.0 N. Determine the largest speed the mass can rotate at.
 - c** The breaking tension of the string is 20.0 N but you want the mass to rotate at 4.00 ms^{-1} . Determine the shortest length string that can be used.

- 6 Estimate the length of the day if the centripetal acceleration at the equator due to the spinning Earth was equal to the acceleration of free fall ($g = 9.8 \text{ ms}^{-2}$).
- 7 A neutron star has a radius of 50.0 km and completes one revolution every 25 ms.
 - a** Calculate the centripetal acceleration experienced at the equator of the star.
 - b** The acceleration of free fall at the surface of the star is $8.0 \times 10^{10} \text{ ms}^{-2}$. State and explain whether a probe that landed on the star could stay on the surface or whether it would be thrown off.
- 8 The Earth (mass = $6.0 \times 10^{24} \text{ kg}$) rotates around the Sun in an orbit that is approximately circular, with a radius of $1.5 \times 10^{11} \text{ m}$.
 - a** Estimate the orbital speed of the Earth around the Sun.
 - b** Determine the centripetal acceleration experienced by the Earth.
 - c** Deduce the magnitude of the gravitational force exerted on the Sun by the Earth.
- 9 A plane travelling at a speed 180 ms^{-1} along a horizontal circle makes an angle of $\theta = 35^\circ$ to the horizontal. The lift force L is acting in the direction shown. Calculate the radius of the circle.

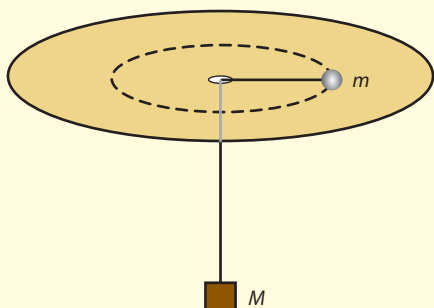


- 10 A cylinder of radius 5.0 m rotates about its vertical axis. A girl stands inside the cylinder with her back touching the side of the cylinder. The floor is suddenly lowered but the girl stays 'glued' to the wall. The coefficient of friction between the girl and the wall is 0.60.
- a** Draw a free body diagram of the forces on the girl.
 - b** Determine the minimum number of revolutions per minute for which the girl does not slip down the wall.

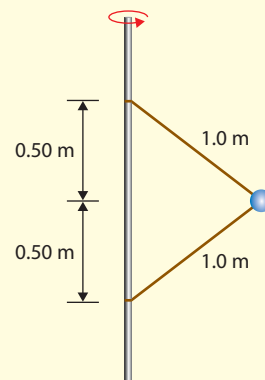
- 11 A loop-the-loop machine has radius r of 18 m.



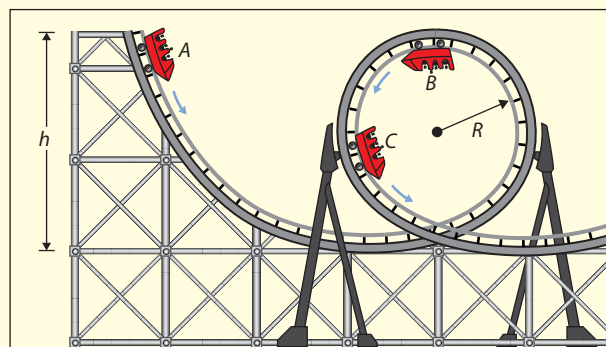
- Calculate the minimum speed with which a cart must enter the loop so that it does not fall off at the highest point.
 - Predict the speed at the top in this case.
- 12 The diagram shows a horizontal disc with a hole through its centre. A string passes through the hole and connects a mass m on top of the disc to a bigger mass M that hangs below the disc. Initially the smaller mass is rotating on the disc in a circle of radius r . Determine the speed of m be such that the big mass stands still.



- 13 The ball shown in the diagram is attached to a rotating pole with two strings. The ball has a mass of 0.250 kg and rotates in a horizontal circle at a speed of 8.0 ms^{-1} . Determine the tension in each string.



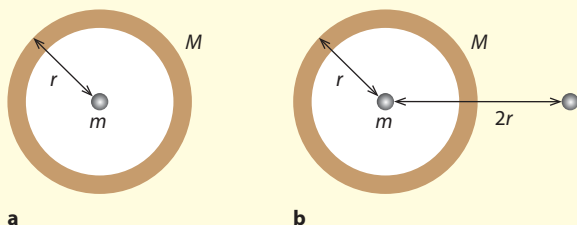
- 14 In an amusement park ride a cart of mass 300 kg and carrying four passengers each of mass 60 kg is dropped from a vertical height of 120 m along a frictionless path that leads into a loop-the-loop machine of radius 30 m. The cart then enters a straight stretch from A to C where friction brings it to rest after a distance of 40 m.



- Determine the velocity of the cart at A.
- Calculate the reaction force from the seat of the cart onto a passenger at B.
- Determine the acceleration experienced by the cart from A to C (assumed constant).

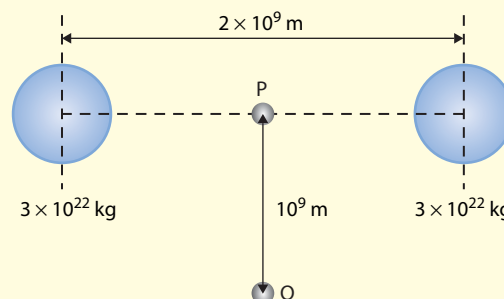
? Test yourself

- 15 Calculate the gravitational force between:
- the Earth and the Moon
 - the Sun and Jupiter
 - a proton and an electron separated by 10^{-10} m.
- 16 A mass m is placed at the centre of a thin, hollow, spherical shell of mass M and radius r , shown in diagram a.



- Determine the gravitational force the mass m experiences.
 - Determine the gravitational force m exerts on M .
- A second mass m is now placed a distance of $2r$ from the centre of the shell, as shown in diagram b.
- Determine the gravitational force the mass inside the shell experiences.
 - Suggest what gravitational force is experienced by the mass outside the shell.
- 17 Stars **A** and **B** have the same mass and the radius of star **A** is nine times larger than the radius of star **B**. Calculate the ratio of the gravitational field strength on star **A** to that on star **B**.
- 18 Planet **A** has a mass that is twice as large as the mass of planet **B** and a radius that is twice as large as the radius of planet **B**. Calculate the ratio of the gravitational field strength on planet **A** to that on planet **B**.
- 19 Stars **A** and **B** have the same density and star **A** is 27 times more massive than star **B**. Calculate the ratio of the gravitational field strength on star **A** to that on star **B**.
- 20 A star explodes and loses half its mass. Its radius becomes half as large. Determine the new gravitational field strength on the surface of the star in terms of the original one.
- 21 The mass of the Moon is about 81 times less than that of the Earth. Estimate the fraction of the distance from the Earth to the Moon where the gravitational field strength is zero. (Take into account the Earth and the Moon only.)

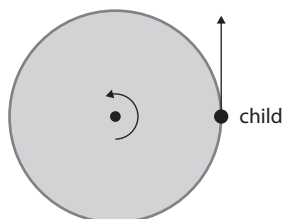
- 22 The diagram shows point P is halfway between the centres of two equal spherical masses that are separated by a distance of 2×10^9 m. Calculate the gravitational field strength at point P and state the direction of the gravitational field strength at point Q.



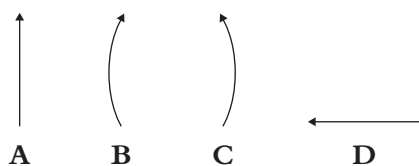
- 23 A satellite orbits the Earth above the equator with a period equal to 24 hours.
- Determine the height of the satellite above the Earth's surface.
 - Suggest an advantage of such a satellite.
- 24 The Hubble Space Telescope is in orbit around the Earth at a height of 560 km above the Earth's surface. Take the radius and mass of the Earth to be 6.4×10^6 m and 6.0×10^{24} kg, respectively.
- Calculate Hubble's speed.
 - In a servicing mission, a Space Shuttle spotted the Hubble telescope a distance of 10 km ahead. Estimate how long it took the Shuttle to catch up with Hubble, assuming that the Shuttle was moving in a circular orbit just 500 m below Hubble's orbit.
- 25 Assume that the force of gravity between two point masses is given by $F = \frac{Gm_1m_2}{r^n}$ where n is a constant.
- Derive the law relating period to orbit radius for this force.
 - Deduce the value of n if this law is to be identical with Kepler's third law.

Exam-style questions

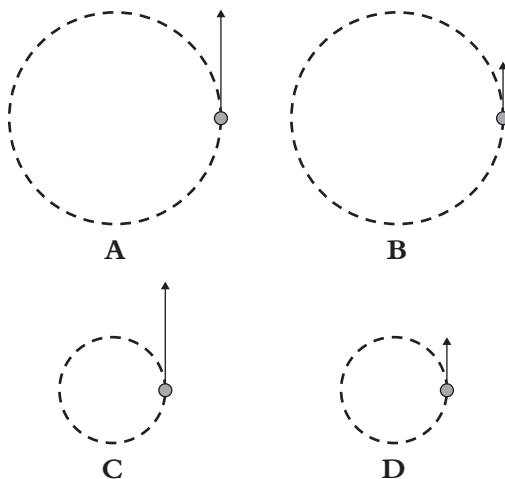
- 1 A child is sitting at the edge of a merry-go-round. The arrow shows the velocity of the child. At the instant shown, he releases a ball onto the ground.



Which is the path of the ball according to a stationary observer on the ground?

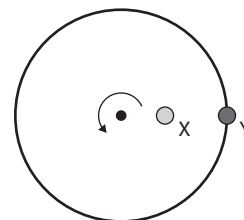


- 2 In which of the following examples of circular motion is the centripetal acceleration experienced by the particle the largest? In each case the arrows represent speed.



- 3 A horizontal disc rotates about a vertical axis through the centre of the disc. Two particles X and Y are placed on the disc.

The particles do not move relative to the disc. Which is correct about the angular speed ω and the linear speed v of X and Y?

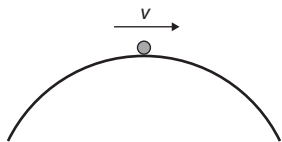


	ω	v
A	same	same
B	same	different
C	different	same
D	different	different

- 4 In the diagram for question 3 the ratio of distances of Y to X is 2. What is the ratio of the acceleration of Y to that of X?

A $\frac{1}{4}$ B $\frac{1}{2}$ C 2 D 4

- 5 A particle of mass m moves with speed v along a hill that may be assumed to be part of a circle of radius r .



What is the reaction force on the particle at the highest point on the hill?

A mg B $mg + \frac{mv^2}{r}$ C $mg - \frac{mv^2}{r}$ D $\frac{mv^2}{r} - mg$

- 6 A particle moves with speed v in a circular orbit of radius r around a planet. The particle is now moved to another circular orbit of radius $2r$. The new orbital speed is:

A $\frac{v}{2}$ B $\frac{v}{\sqrt{2}}$ C $v\sqrt{2}$ D $2v$

- 7 The mass of a landing module on the Moon is 2000 kg. The gravitational field strength on the Moon is one-sixth that on Earth. What is the weight of the landing module on Earth?

A 330 N B 2000 N C 12000 N D 20000 N

- 8 A planet has double the mass of Earth and half its radius. What is the gravitational field strength on the surface of this planet?

A 10 N kg^{-1} B 20 N kg^{-1} C 40 N kg^{-1} D 80 N kg^{-1}

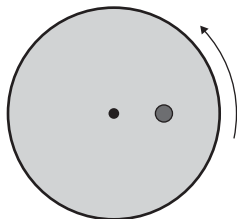
- 9 A satellite orbits the Earth in a circular orbit. The only force on the satellite is the gravitational force from the Earth. Which of the following is correct about the acceleration of the satellite?

A It is zero.
B It is constant in magnitude and direction.
C It is constant in magnitude but not in direction.
D It is not constant in magnitude or direction.

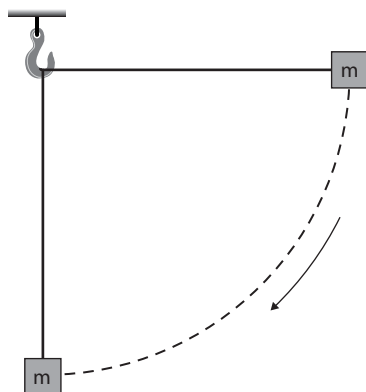
- 10 The two spherical bodies in the diagram have the same radius but the left mass has twice the mass of the other. At which point could the net gravitational field of the two masses have the greatest magnitude?



- 11 A horizontal disc of radius 45 cm rotates about a vertical axis through its centre. The disc makes one full revolution in 1.40 s. A particle of mass 0.054 kg is placed at a distance of 22 cm from the centre of the disc. The particle does not move relative to the disc.

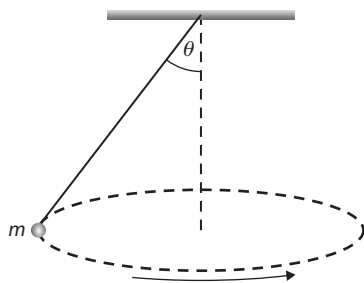


- On a copy of the diagram draw arrows to represent the velocity and acceleration of the particle. [2]
 - Calculate the angular speed and the linear speed of the particle. [2]
 - The coefficient of static friction between the disc and the particle is 0.82. Determine the largest distance from the centre of the disc where the particle can be placed and still not move relative to the disc. [3]
 - The particle is to remain at its original distance of 22 cm from the centre of the disc.
 - Determine the maximum angular speed of the disc so that the particle does not move relative to the disc. [2]
 - The disc now begins to rotate at an angular speed that is greater than the answer in **d i**. Describe qualitatively what happens to the particle. [2]
- 12 A block of mass of 5.0 kg is attached to a string of length 2.0 m which is initially horizontal. The mass is then released and swings as a pendulum. The diagram shows the mass falling to the position where the string is in the vertical position.



- Calculate the speed of the block when the string is in the vertical position. [2]
- Deduce the acceleration of the block. [1]
- On a copy of the diagram, draw arrows to represent the forces on the block. [2]
- For when the string is in the vertical position:
 - state and explain whether the block is in equilibrium [2]
 - calculate the tension in the string. [2]

- 13 A particle of mass m is attached to a string of length L whose other end is attached to the ceiling, as shown in the diagram. The particle moves in a horizontal circle making an angle of θ with the vertical. Air resistance may be neglected.



- a On a copy of the diagram draw arrows to represent the forces on the particle. [2]
 b State and explain whether the particle is in equilibrium. [2]
 c The linear speed of the particle is v and its angular speed is ω . Show that:

i $v = \sqrt{\frac{gL \sin^2 \theta}{\cos \theta}}$ [2]

ii $\omega = \sqrt{\frac{g}{L \cos \theta}}$ [2]

- d The length of the string is 45 cm and $\theta = 60^\circ$. Use the answer in c to evaluate:

i the linear speed [1]

ii the angular speed of the particle. [1]

Air resistance may no longer be neglected.

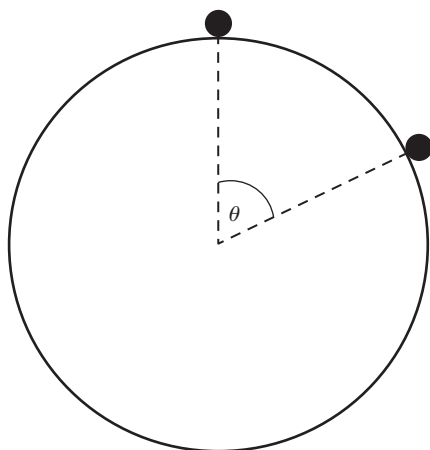
- e Suggest the effect of air resistance on:

i the linear speed of the particle [1]

ii the angle the string makes with the vertical [1]

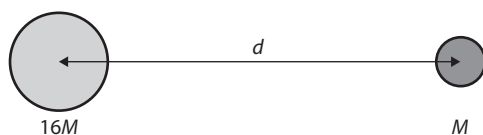
iii the angular speed of the particle. [1]

- 14 A marble rolls from the top of a big sphere, as shown in the diagram.



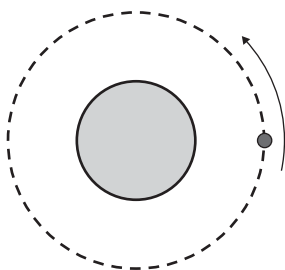
- a Show that when the marble has moved so that the line joining it to the centre of the sphere is θ , its speed is given by $v = \sqrt{2gR(1 - \cos \theta)}$. (Assume a very small speed at the top.) [3]
 b Deduce that at that instant, the normal reaction force on the marble from the sphere is given by $N = mg(3 \cos \theta - 2)$. [3]
 c Hence determine the angle θ at which the marble loses contact with the sphere. [1]

- 15 Consider two spherical bodies of mass $16M$ and M as in the diagram.



There is a point P somewhere on the line joining the masses where the gravitational field strength is zero.

- a Determine the distance of point P from the centre of the bigger mass in terms of d , the centre-to-centre distance separating the two bodies. [3]
 - b Draw a graph to show the variation of the gravitational field strength g due to the two masses with the distance x from the centre of the larger mass. [2]
 - c A small point mass m is placed at P.
 - i State the force on m . [1]
 - ii The small mass m is slightly displaced to the left of P. State and explain whether the net force on the point mass will be directed to the left or to the right. [2]
 - d Describe qualitatively the motion of the point mass after it has been displaced to the left of P. [2]
- 16 A satellite is in a circular orbit around a planet of mass M , as shown in the diagram.



- a
 - i On a copy of the diagram draw arrows to represent the velocity and acceleration of the satellite. [2]
 - ii Explain why the satellite has acceleration even though its speed is constant. [2]
- b Show that the angular speed ω is related to the orbit radius r by $r^3\omega^2 = GM$. [3]
- c Because of friction with the upper atmosphere, the satellite slowly moves into another circular orbit with a smaller radius than the answer in b. Suggest the effect of this on the satellite's:
 - i angular speed [1]
 - ii linear speed. [1]
- d Titan and Enceladus are two of Saturn's moons. Data about these moons are given in the table.

Moon	Orbit radius / m	Angular speed / rad s^{-1}
Titan	1.22×10^9	
Enceladus	2.38×10^8	5.31×10^{-5}

- i Determine the mass of Saturn. [2]
- ii Determine the period of revolution of Titan in days. [3]



The belief in unification

In the early 19th century there were three known forces: gravitational, electric and magnetic. Through the work of James Clerk Maxwell (1831–1879) physicists realised that electric and magnetic forces were two sides of the same force, the electromagnetic force. Thus began the notion (for some a belief, for others a prejudice) that all interactions, as more were being discovered, were part of the same ‘unified’ force. In the 20th century two new forces were discovered: the weak nuclear force and the strong nuclear force. In the late 1960s the electromagnetic and the weak nuclear force were unified in the standard model of particles. All efforts to unify this electroweak force with the strong nuclear force in a grand unified force have failed. All attempts to unify any of these forces with gravity have also failed. Yet, the dream of unification remains.

Nature of science

Accidental discovery

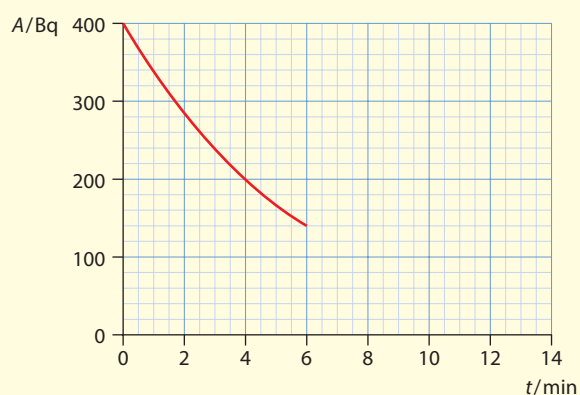
The discovery of radioactivity is an example of an accidental discovery. Henri Becquerel, working in Paris in 1896, believed that minerals made phosphorescent (emitting light) by visible light might give off X-rays. His idea was to wrap a photographic plate in black paper, and place on it a phosphorescent uranium mineral that had been exposed to bright sunlight. But the sun did not shine, and he stopped the experiment, placing the wrapped plate and the mineral in a drawer. A few days later he developed the photographic plate, expecting to see only a very weak image. To his surprise, there was a very strong image. Becquerel concluded that this image was formed by a new kind of radiation that had nothing to do with light. The radiation came from the uranium mineral. Becquerel conducted further experiments and showed that uranium minerals were the only phosphorescent minerals that had this effect.



Test yourself

- Discuss what is meant by the statement that the energy of atoms is discrete.
 - Outline the evidence for this discreteness.
- Explain why the dark lines of an absorption spectrum have the same wavelengths as the bright lines of an emission spectrum for the same element.
- Calculate the wavelength of the photon emitted in a transition from $n = 4$ to $n = 2$ in hydrogen. (Use [Figure 7.2.](#))
- Refer to [Figure 7.1.](#) Explain why the distance between the emission lines of hydrogen decreases as we move to the right.

- 5 A hydrogen atom is in its ground state.
- Explain the term **ground state**.
 - Photons of energy 10.4 eV are directed at hydrogen gas in its ground state. Suggest what, if anything, will happen to the hydrogen atoms.
 - In another experiment, a beam of electrons of energy 10.4 eV are directed at hydrogen gas atoms in their ground state. Suggest what, if anything, will happen to the hydrogen atoms and the electrons in the beam.
- 6 State the electric charge of the nucleus ${}^3_2\text{He}$.
- 7
 - State what is meant by the term **isotope**.
 - State **two** ways in which the nuclei of the isotopes ${}^{16}_8\text{O}$ and ${}^{18}_8\text{O}$ differ from each other (other than they have different neutrons).
- 8 Bismuth (${}^{210}_{83}\text{Bi}$) decays by beta minus decay, followed by gamma emission. State the equation for the reaction and the atomic and mass number of the nucleus produced.
- 9 Plutonium (${}^{239}_{94}\text{Pu}$) decays by alpha decay. State the equation for this reaction and name the nucleus plutonium decays into.
- 10 A radioactive source has a half-life of 3.0 min. At the start of an experiment 32.0 mg of the radioactive material is present. Determine how much will be left after 18.0 min.
- 11 The graph shows the variation with time of the activity of a radioactive sample.

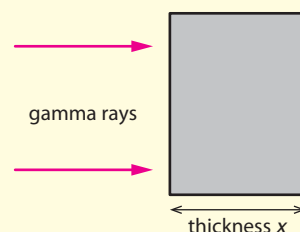


- State what is meant by **activity**.
- Use the graph to estimate the half-life of the sample.

- On a copy of the graph, extend the curve to show the variation of the activity for a time up to 12 minutes.
 - The sample contains a radioactive element X that decays in to a stable element Y. At $t=0$ no atoms of element Y are present in the sample. Determine the time after which the ratio of Y atoms to X atoms is 7.
- 12 In a study of the intensity of gamma rays from a radioactive source it is suspected that the counter rate C at a distance d from the source behaves as $C \propto \left(\frac{1}{d+d_0}\right)^2$ where d_0 is an unknown constant. A set of data for C and d is given. Outline how the data must be plotted in order to get a straight line.
- 13 The intensity of gamma rays of a specific energy (monochromatic rays) decreases exponentially with the thickness x of the absorbing material according to the equation:

$$I = I_0 e^{-\mu x}$$

where I_0 is the intensity at the face of the absorber and μ a constant depending on the material.



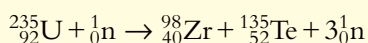
Discuss how the intensity I and thickness x should be plotted in order to allow an accurate determination of the constant μ .

- 14 State the name of the dominant force between two protons separated by a distance of:
- $1.0 \times 10^{-15} \text{ m}$
 - $1.0 \times 10^{-14} \text{ m}$.
- 15 Large stable nuclei have more neutrons than protons. Explain this observation by reference to the properties of the strong nuclear force.



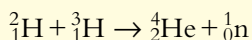
Test yourself

- 16 Find the binding energy and binding energy per nucleon of the nucleus ${}^{62}_{28}\text{Ni}$. The atomic mass of nickel is 61.928348 u.
- 17 How much energy is required to remove one proton from the nucleus of ${}^{16}_8\text{O}$? A rough answer to this question is obtained by giving the binding energy per nucleon. A better answer is obtained when we write a reaction that removes a proton from the nucleus. In this case ${}^{16}_8\text{O} \rightarrow {}^1_1\text{p} + {}^{15}_7\text{N}$. Calculate the energy required for this reaction to take place, known as the proton separation energy. Compare the two energy values. (The atomic mass of oxygen is 15.994 u; that of nitrogen is 15.000 u.)
- 18 The first excited state of the nucleus of uranium-235 is 0.051 MeV above the ground state.
- What is the wavelength of the photon emitted when the nucleus makes a transition to the ground state?
 - What part of the spectrum does this photon belong to?
- 19 Assume uranium-236 splits into two nuclei of palladium-117 (Pd). (The atomic mass of uranium is 236.0455561 u; that of palladium is 116.9178 u.)
- Write down the reaction.
 - What other particles must be produced?
 - What is the energy released?
- 20 A fission reaction involving uranium is:



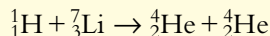
Calculate the energy released. (Atomic masses: U = 235.043922 u; Zr = 97.91276 u; Te = 134.9165 u.)

- 21 Calculate the energy released in the fusion reaction:



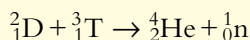
(Atomic masses: ${}^2_1\text{H} = 2.014102$ u; ${}^3_1\text{H} = 3.016049$ u; ${}^4_2\text{He} = 4.002603$ u.)

- 22 In the first nuclear reaction in a particle accelerator, hydrogen nuclei were accelerated and then allowed to hit nuclei of lithium according to the reaction:



Calculate the energy released. (The atomic mass of lithium is 7.016 u.)

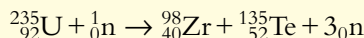
- 23 Show that an alternative formula for the mass defect is $\delta = ZM_{\text{H}} + (A - Z)m_{\text{n}} - M_{\text{atom}}$ where M_{H} is the mass of a hydrogen atom and m_{n} is the mass of a neutron.
- 24 Consider the nuclear fusion reaction involving the deuterium (${}^2_1\text{D}$) and tritium (${}^3_1\text{T}$) isotopes of hydrogen:



The energy released, Q_1 , may be calculated in the usual way, using the masses of the particles involved, from the expression:

$$Q_1 = (M_{\text{D}} + M_{\text{T}} - M_{\text{He}} - m_{\text{n}})c^2$$

Similarly, in the fission reaction of uranium:



the energy released, Q_2 , may be calculated from:

$$Q_2 = (M_{\text{U}} - M_{\text{Zr}} - M_{\text{Te}} - 2m_{\text{n}})c^2$$

- a Show that the expression for Q_1 can be rewritten as:

$$Q_1 = E_{\text{He}} - (E_{\text{D}} + E_{\text{T}})$$

where E_{He} , E_{D} and E_{T} are the binding energies of helium, deuterium and tritium, respectively.

- b Show that the expression for Q_2 can be rewritten as:

$$Q_2 = (E_{\text{Zr}} + E_{\text{Te}}) - E_{\text{U}}$$

where E_{Zr} , E_{Te} and E_{U} are the binding energies of zirconium, tellurium and uranium, respectively.

- c Results similar to the results obtained in a and b apply to all energy-releasing fusion and fission reactions. Use this fact and the binding energy curve in Figure 7.12 to explain carefully why energy is released in fusion and fission reactions.

The Higgs went undetected for about 40 years since its existence was proposed on theoretical grounds. In July 2012 physicists at CERN's Large Hadron Collider announced evidence that finally the Higgs had been discovered. Its mass is about $125 \text{ GeV } c^{-2}$.

Nature of science

At the time Gell-Mann proposed quarks, many hundreds of hadrons were known. Gell-Mann managed to explain the existence of each and every one of these by postulating the existence of just the three lightest quarks (the u, the d and the s). This was a purely mathematical explanation, as no quarks had been observed. Gell-Mann predicted the existence of a 'strangeness -3 ' baryon and used his quark model to predict its mass as well. In 1964 researchers at Brookhaven discovered the Ω^- with properties exactly as predicted. The bubble chamber photograph in Figure 7.32a shows the creation and subsequent decay of the Ω^- . Analysing these complex photographs and extracting relevant information is an enormously complicated task. Large-scale international collaboration later showed the existence of the other quarks and led to the Standard Model used today. Work on particles continues at CERN in Geneva (Figures 7.32b and 7.32c) and many other laboratories.

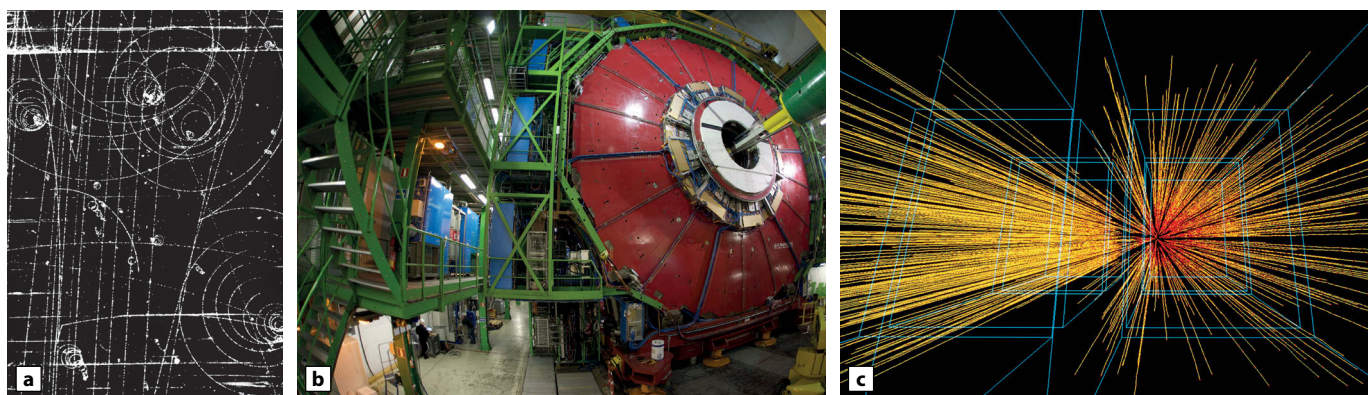


Figure 7.32 **a** A bubble chamber photograph of the creation and decay of the Ω^- . **b** The huge CMS detector at the Large Hadron Collider at CERN illustrates the complexity of the electronics required in particle physics. **c** Computer images of particle tracks after a collision illustrate the complexity of the analysis required.

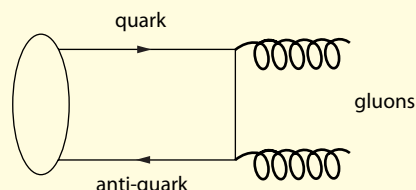


Test yourself

- 25 In the gold foil experiment explain why:
 - a** the foil was very thin
 - b** the experiment was done in an evacuated container.
- 26 Write down the quark structure of **a** the anti-neutron and **b** the anti-proton. Verify that the charges come out correctly.
- 27 Write down the quark structure of the anti-particle of the meson $K^+ = (u\bar{s})$.
- 28 State the baryon number of the hadron with quark content $\bar{c}\bar{c}\bar{c}$.
- 29 Determine whether the following reactions conserve or violate baryon number:
 - a** $p^+ \rightarrow e^+ + \gamma$
 - b** $p^+ + p^- \rightarrow \pi^+ + \pi^-$
 - c** $p^+ + p^- \rightarrow \pi^+ + \pi^- + n + \bar{n}$
 - d** $\Lambda^0 \rightarrow \pi^+ + \pi^-$ (The Λ particle has quark content uds .)

- 30 Suggest the reason that led to the introduction of the quantum number called strangeness.
- 31 The quark content of a certain meson is $(d\bar{s})$.
- Write down its charge and its strangeness.
 - Determine whether it can be its own anti-particle.
- 32 A charmed D meson is made out of $D = c\bar{d}$.
- Write down its charge.
 - Write down its strangeness.
- 33 Determine whether the following reactions conserve or violate strangeness: (use $\pi^0 = d\bar{d}$, $K^+ = u\bar{s}$, $\Lambda^0 = uds$, $K^0 = d\bar{s}$, $\Sigma^- = dds$)
- $\pi^- + p^+ \rightarrow K^0 + \Lambda^0$
 - $p^0 + n \rightarrow K^+ + \Sigma^-$
 - $K^0 \rightarrow \pi^- + \pi^+$
 - $\pi^- + p^+ \rightarrow \pi^- + \Sigma^+$
- 34 In the reactions listed below, various neutrinos appear (denoted ν). In each case, identify the correct neutrino (ν_e , ν_μ , ν_τ or the anti-particles of these).
- $\pi^+ \rightarrow p^0 + e^+ + \nu$
 - $\pi^+ \rightarrow p^0 + \mu^+ + \nu$
 - $\tau^+ \rightarrow \pi^- + \pi^+ + \nu$
 - $p^+ + \nu \rightarrow n + e^+$
 - $\tau^- \rightarrow e^- + \nu + \nu$
- 35 The reactions listed below are all impossible because they violate one or more conservation laws. In each case, identify the law that is violated.
- $K^+ \rightarrow \mu^- + \bar{\nu}_\mu + e^+ + e^+$
 - $\mu^- \rightarrow e^+ + \gamma$
 - $\tau^+ \rightarrow \gamma + \bar{\nu}_\tau$
 - $p + n \rightarrow p + p^0$
 - $e^+ \rightarrow \mu^+ + \bar{\nu}_\mu + \bar{\nu}_e$
 - $p \rightarrow \pi^+ + \pi^-$
- 36 Explain whether the electric force acts:
- on quarks
 - on neutrinos.
- 37 The neutron is electrically neutral. Could it possibly have electromagnetic interactions?
- 38 The neutral meson $\eta_c = (c\bar{c})$ is its own anti-particle, but the neutral $K^0 = (d\bar{s})$ is not. Explain why.

- 39 a Outline what is meant by the term **confinement** in the context of quarks.
- b The Feynman diagram shows the decay of a quark–anti-quark pair in a meson into two gluons. With reference to your answer in a, suggest what might happen to the gluons produced in this decay.



- 40 a The rest mass of the proton is $938 \text{ MeV } c^{-2}$ and that of the neutron is $940 \text{ MeV } c^{-2}$. Using the known quark contents of the proton and the neutron, calculate the masses of the u and d quarks.
- b Using the values you calculated in a, predict the mass of the meson π^+ (which is made out of a u quark and an d anti-quark).
- c The actual value of the rest mass of the π^+ is about $140 \text{ MeV } c^{-2}$. Suggest how this enormous disagreement is resolved.
- 41 Describe the significance of the Higgs particle in the standard model of quarks and leptons.
- 42 Use the electromagnetic vertex to draw a Feynman diagram for the scattering of a photon off a positron.
- 43 Beta-minus decay involves the decay of a neutron into a proton according to the reaction $n \rightarrow p^+ + e^- + \bar{\nu}_e$.
- Describe this decay in terms of quarks.
 - Draw a Feynman diagram for the process.
- 44 Using the basic weak interaction vertex involving a W boson and two quarks or leptons given in Figure 7.28, draw Feynman diagrams to represent the following processes:
- $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$
 - $e^- + \bar{\nu}_e \rightarrow \mu^- + \bar{\nu}_\mu$
 - $\pi^+ \rightarrow \mu^+ + \nu_\mu$ (quark structure of positive pion is $u\bar{d}$)
 - $K^- \rightarrow \mu^- + \bar{\nu}_\mu$ (quark structure of negative kaon is $s\bar{u}$).



45 Using the basic weak interaction vertex involving a W boson and two quarks and leptons given in Figure 7.30, state **three** possible ways in which the W boson can decay.

46 Using the basic weak interaction vertex involving a Z boson and two quarks and leptons given in Figure 7.30, draw Feynman diagrams to represent the following processes:

a $e^- + e^+ \rightarrow \bar{\nu}_\mu + \nu_\mu$

b $e^- + \nu_\mu \rightarrow e^- + \nu_\mu$

c $e^- + e^+ \rightarrow e^- + e^+$

Exam-style questions

1 How would the decay of a nucleus of $^{60}_{27}\text{Co}$ into a nucleus of $^{60}_{28}\text{Ni}$ be described?

- A alpha decay B beta minus decay C beta plus decay D gamma decay

2 What are the number of neutrons and the number of electrons in the neutral atom of $^{195}_{78}\text{Pt}$?

	Number of neutrons	Number of electrons
A	117	195
B	117	78
C	195	78
D	195	117

3 The activity of a sample containing a radioactive element is 6400 Bq. After 36 minutes the activity is 800 Bq. What is the half-life of the sample?

- A 4.0 minutes B 8.0 minutes C 12 minutes D 18 minutes

4 A sample contains a small quantity of a radioactive element with a very long half-life. The activity is constant and equal to A . The temperature of the sample is increased. What are the effects if any, on the half-life and activity of the sample?

	Effect on half-life	Effect on activity
A	none	none
B	none	increase
C	increase	none
D	increase	increase

5 What is the common characteristic of most nuclei with mass number greater than about 20?

- A binding energy per nucleon
B binding energy
C decay pattern
D half-life

- 6 The binding energy per nucleon for ${}^{11}_5\text{B}$ is about 7 MeV. What is the minimum energy needed to separate the nucleons of ${}^{11}_5\text{B}$?

A 7 MeV **B** 35 MeV **C** 42 MeV **D** 77 MeV

- 7 The reaction $p + n \rightarrow p + \pi^0$ is impossible. Which conservation law would be violated if the reaction occurred?

A charge **B** lepton number **C** baryon number **D** strangeness

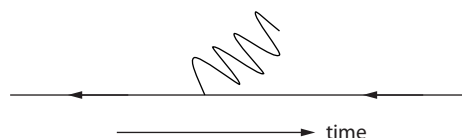
- 8 Which is the neutral exchange particle of the weak interaction?

A photon **B** gluon **C** W **D** Z

- 9 What are the charge Q and strangeness S of the baryon $\Lambda = (uds)$?

	Q	S
A	0	+1
B	+1	+1
C	0	-1
D	+1	-1

- 10 What process does this Feynman diagram represent?

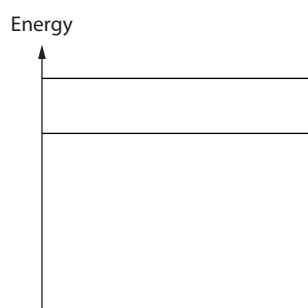


- A** electron emitting photon
B electron absorbing photon
C positron emitting photon
D positron absorbing photon

- 11 **a** Explain how the emission lines in the spectrum of a gas are evidence for discrete energy levels within atoms.

[3]

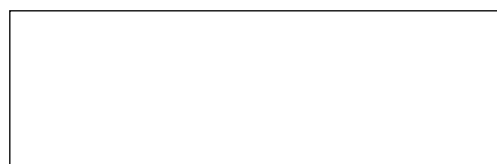
The diagram shows three energy levels of a vapour.



Transitions between these three levels give rise to photons of three different wavelengths. Two of these wavelengths are 486 nm and 656 nm.

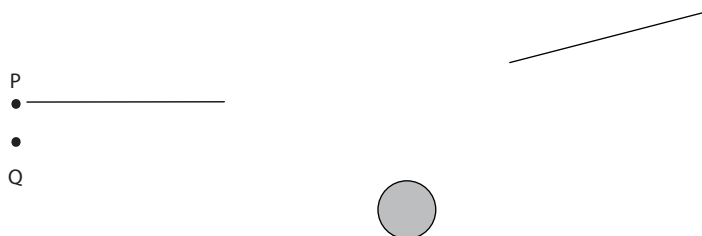


- b** On a copy of the diagram draw arrows to identify the transitions that give rise to the wavelengths of 656 nm and 486 nm. [2]
- c** Calculate the wavelength of the photon that corresponds to the third transition. [3]
- d** White light containing wavelengths that vary from 400 nm to 700 nm is transmitted through the vapour. On a copy of the diagram below, draw lines to show the absorption lines in the transmitted light. [2]



increasing wavelength →

- 12 a** Explain why in their experiment Geiger and Marsden used:
- i** an evacuated enclosure [1]
 - ii** a gold foil that was very thin [1]
 - iii** a beam of alpha particles that was very narrow. [1]
- b** State the name of the force responsible for the deflection of the alpha particles. [1]
- c**
- i** Describe the deflections of the alpha particles by the gold foil. [2]
 - ii** Outline how the results of this experiment led to the Rutherford model of the atom. [3]
- d** The diagram shows a partially completed path of an alpha particle that left point P as it scatters past a nucleus of gold.

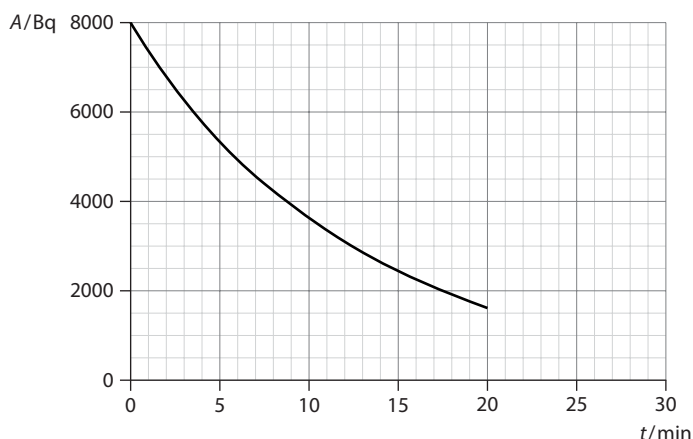


On a copy of the diagram:

- i** complete the path [1]
 - ii** draw lines to clearly show the angle of deflection of this alpha particle [2]
 - iii** draw an arrow to indicate the direction of the force on the alpha particle at the point of closest approach. [1]
- e**
- i** A second alpha particle is shot at the nucleus from position Q with identical kinetic energy, in a direction parallel to that of the alpha particle at P. On your diagram, draw the path of this particle. [2]
 - ii** Discuss how, if at all, the answer to **e i** would change if the nucleus of gold were replaced by a nucleus of another, heavier, isotope of gold. [2]

13 a Radioactive decay is random and spontaneous. State what you understand by this statement. [4]

b The graph shows how activity of a sample containing a radioactive isotope of thorium $^{225}_{90}\text{Th}$ varies with time.



i State what is meant by an **isotope**. [1]

ii Determine the half-life of thorium. [2]

iii State **one** assumption made in obtaining the answer to ii. [1]

iv Draw on a copy of the graph to show the variation of the activity with time to 30 minutes. [2]

c i Thorium undergoes alpha decay. Complete the reaction equation:



ii Calculate the energy released, in MeV. (Atomic masses: thorium 226.024903 u, radium 221.013917 u, helium 4.0026603 u.) [2]

d The nuclei of thorium are at rest when they decay. Determine the fraction of the energy released that is carried by the alpha particle. [3]

14 A possible fission reaction is given by the equation:



a i Calculate the number x of neutrons produced. [1]

ii Use the binding energy per nucleon curve in Figure 7.12 to estimate the energy released in this reaction. [3]

b Suggest why most nuclei with $A > 20$ have roughly the same binding energy per nucleon. [3]

c Use the diagram in Figure 7.12 to explain why energy is released in nuclear fusion. [2]



15 a Explain, in terms of quarks, the difference between a baryon and a meson.

[2]

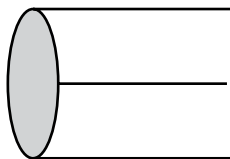
b In a copy of the table below, put a check mark (✓) to identify the interaction(s) that apply to hadrons and to leptons.

[2]

	strong	weak
hadrons		
leptons		

c Copy and complete the Feynman diagram to represent the beta minus decay of a neutron, making sure that you label all particles involved.

[5]



d For this part of the question it is given that $K^- = s\bar{u}$, $\pi^+ = u\bar{d}$ and that Σ^- has strangeness -1 .

i Using the fact that the reaction $K^- + p \rightarrow \pi^+ + \Sigma^-$ occurs, determine whether Σ^- would be classified as a baryon or as a meson.

[2]

ii Using the fact that the reaction $K^- \rightarrow \mu^- + \bar{\nu}$ occurs, determine whether the reaction takes place through the strong, the weak or the electromagnetic interaction.

[2]

iii State and explain whether the anti-neutrino in d ii is an electron, muon or tau anti-neutrino.

[2]

16 A student suggest that the muon decays according to the reaction equation $\mu^- \rightarrow e^- + \gamma$.

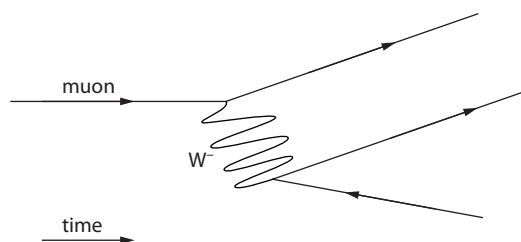
a i State **one** similarity and **one** difference between the electron and the muon.

[2]

ii Explain why the reaction equation proposed by the student is incorrect.

[2]

b In fact, the muon decays according to $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$. A Feynman diagram for this decay is shown.



i Identify the **three** unlabelled particles in this diagram.

[3]

ii Using the diagram above to construct a new Feynman diagram representing the scattering of an electron anti-neutrino off a muon.

[2]

iii Write down the reaction equation representing the decay μ^+ , which is the anti-particle of the μ^- .

[2]

c The interaction responsible for the decay of the muon has very short range. State the property of the exchange particle that is responsible for the short range.

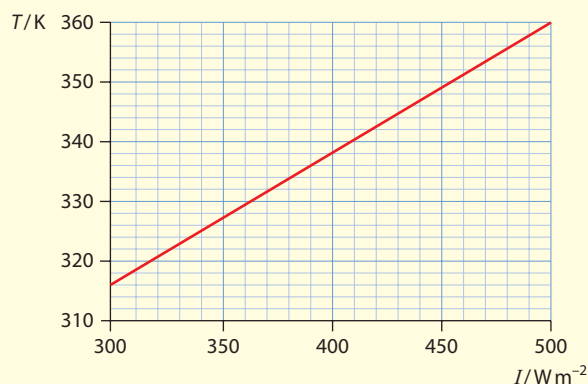
[1]



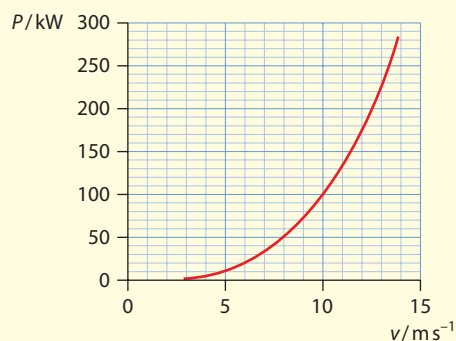
? Test yourself

- 1
 - a Distinguish between specific energy and energy density of a fuel.
 - b Estimate the energy density of water that falls from a waterfall of height 75 m and is used to drive a turbine.
- 2 A power plant produces 500 MW of power.
 - a Determine the energy produced in one second. Express your answer in joules.
 - b Determine the energy (in joules) produced in one year.
- 3 A power plant operates in four stages. The efficiency in each stage is 80%, 40%, 12% and 65%.
 - a Find the overall efficiency of the plant.
 - b Sketch a Sankey diagram for the energy flow in this plant.
- 4 A coal power plant with 30% efficiency burns 10 million kilograms of coal a day. (Take the specific energy of coal to be 30 MJ kg^{-1} .)
 - a Calculate the power output of the plant.
 - b Estimate the rate at which thermal energy is being discarded by this plant.
 - c The discarded thermal energy is carried away by water whose temperature is not allowed to increase by more than 5°C . Calculate the rate at which water must flow away from the plant.
- 5 One litre of gasoline releases 34 MJ of energy when burned. The efficiency of a car operating on this gasoline is 40%. The speed of the car is 9.0 ms^{-1} when the power developed by the engine is 20 kW. Calculate how many kilometres the car can go with one litre of gasoline when driven at this speed.
- 6 A coal-burning power plant produces 1.0 GW of electricity. The overall efficiency of the power plant is 40%. Taking the specific energy of coal to be 30 MJ kg^{-1} , calculate the amount of coal that must be burned in one day.
- 7 In the context of nuclear fission reactors, state what is meant by:
 - a uranium enrichment
 - b moderator
 - c critical mass.
- 8
 - a Calculate the energy released in the fission reaction:
$${}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{92}^{236}\text{U} \rightarrow {}_{54}^{140}\text{Xe} + {}_{38}^{94}\text{Sr} + 2{}_0^1\text{n}$$
(Mass data: uranium-235, ${}_{92}^{235}\text{U} = 235.043\,923 \text{ u}$; xenon-140, ${}_{54}^{140}\text{Xe} = 139.921\,636 \text{ u}$; strontium-94, ${}_{38}^{94}\text{Sr} = 93.915\,360 \text{ u}$; neutron, ${}_0^1\text{n} = 1.008\,665 \text{ u}$.)
 - b The power output is 200 MW. Estimate the number of fission reactions per second.
- 9 The energy released in a typical fission reaction involving uranium-235 is 200 MeV.
 - a Calculate the specific energy of uranium-235.
 - b Estimate how much coal (specific energy 30 MJ kg^{-1}) must be burned in order to give the same energy as that released in nuclear fission with 1 kg of uranium-235.
- 10
 - a A 500 MW nuclear power plant converts the energy released in nuclear reactions into electrical energy with an efficiency of 40%. Calculate how many fissions of uranium-235 are required per second. Take the energy released per reaction to be 200 MeV.
 - b What mass of uranium-235 is required to fission per second?
- 11
 - a Draw a diagram of a fission reactor, explaining the role of **i** fuel rods, **ii** control rods and **iii** moderator.
 - b In what form is the energy released in a fission reactor?
- 12 Distinguish between a solar panel and a photovoltaic cell.
- 13 Sunlight of intensity 700 W m^{-2} is captured with 70% efficiency by a solar panel, which then sends the captured energy into a house with 50% efficiency.
 - a The house loses thermal energy through bad insulation at a rate of 3.0 kW. Find the area of the solar panel needed in order to keep the temperature of the house constant.
 - b Draw a Sankey diagram for the energy flow from the panel to the house.

- 14 A solar heater is to heat 300 kg of water initially at 15 °C to a temperature of 50 °C in a time of 12 hours. The amount of solar radiation falling on the collecting surface of the solar panel is 240 W m^{-2} and is collected at an efficiency of 65%. Calculate the area of the collecting panel that is required.
- 15 A solar heater is to warm 150 kg of water by 30 K. The intensity of solar radiation is 600 W m^{-2} and the area of the panels is 4.0 m^2 . The specific heat capacity of water is $4.2 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$. Estimate the time this will take, assuming a solar panel efficiency of 60%.
- 16 The graph shows the variation with incident solar intensity I of the temperature of a solar panel used to heat water. Thermal energy is extracted from the water at a rate of 320 W. The area of the panel is 2.0 m^2 . Calculate, for a solar intensity of 400 W m^{-2} :



- a the temperature of the water
b the power incident on the panel
c the efficiency of the panel.
- 17 The graph shows the power curve of a wind turbine as a function of the wind speed. For a wind speed of 10 m s^{-1} , calculate the energy produced in the course of one year, assuming that the wind blows at this speed for 1000 hours in the year.



- 18 a State the expected increase in the power extracted from a wind turbine when:
- the length of the blades is doubled
 - the wind speed is doubled
 - both the length of the blades and the wind speed are doubled.
- b Outline reasons why the actual increase in the extracted power will be less than your answers.
- 19 Wind of speed v is incident on the blades of a wind turbine. The blades present the wind with an area A . The maximum theoretical power that can be extracted is given by:

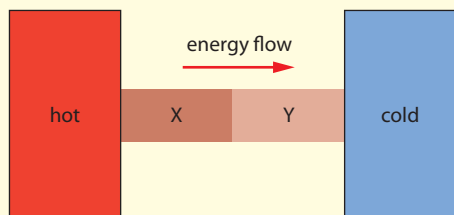
$$P = \frac{1}{2} \rho A v^3$$

State the assumptions made in deriving this equation.

- 20 Air of density 1.2 kg m^{-3} and speed 8.0 m s^{-1} is incident on the blades of a wind turbine. The radius of the blades is 1.5 m. Immediately after passing through the blades, the wind speed is reduced to 3.0 m s^{-1} and the density of air is 1.8 kg m^{-3} . Estimate the power extracted from the wind.
- 21 Calculate the blade radius of a wind turbine that must extract 25 kW of power out of wind of speed 9.0 s^{-1} . The density of air is 1.2 kg m^{-3} . State any assumptions made in this calculation.
- 22 Find the power developed when water in a waterfall with a flow rate of 500 kg s^{-1} falls from a height of 40 m.
- 23 Water falls from a vertical height h at a flow rate (volume per second) Q . Deduce that the maximum theoretical power that can be extracted is given by $P = \rho Q g h$.
- 24 A student explaining pumped storage systems says that the water that is stored at a high elevation is allowed to move lower, thus producing electricity. Some of this electricity is used to raise the water back to its original height, and the process is then repeated. What is wrong with this statement?
- 25 Make an annotated energy flow diagram showing the energy changes that are taking place in each of the following:
- a conventional electricity-producing power station using coal
 - a hydroelectric power plant
 - an electricity-producing wind turbine
 - an electricity-producing nuclear power station.

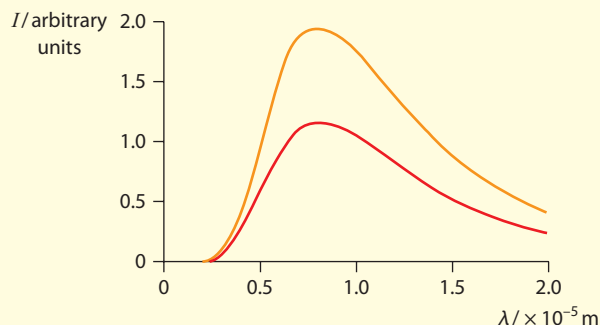
? Test yourself

- 26 A cylindrical solid tube is made out of two smaller tubes, X and Y, of different material. X and Y have the same length and cross-sectional area. The tube is used to conduct energy from a hot to a cold reservoir.



State and explain whether the following are equal or not:

- the rates of flow of energy through X and Y
 - the temperature differences across X and Y.
- 27 Suggest whether there is any point in using a ceiling fan in winter.
- 28 Calculate the ratio of the power radiated per unit area from two black bodies at temperature 900 K and 300 K.
- 29
- State what you understand by the term **black body**.
 - Give an example of a body that is a good approximation to a black body.
 - By what factor does the rate of radiation from a body increase when the temperature is increased from 50 °C to 100 °C?
- 30 The graph shows the variation with wavelength of the intensity of radiation emitted by two bodies of identical shape.



- Explain why the temperature of the two bodies is the same.
- The upper line corresponds to a black body. Calculate the emissivity of the other body.

- 31 The total power radiated by a body of area 5.00 km² and emissivity 0.800 is 1.35×10^9 W. Assume that the body radiates into a vacuum at temperature 0 K. Calculate the temperature of the body.

- 32 Assume that the distance d between the Sun and the Earth decreases. Then the Earth's average temperature T will go up. The fraction of the power radiated by the Sun that is received on Earth is proportional to $\frac{1}{d^2}$; the power radiated by the Earth is proportional to T^4 .

a Deduce the dependence of the temperature T of the Earth on the distance d .

b Hence estimate the expected rise in temperature if the distance decreases by 1.0%. Take the average temperature of the Earth to be 288 K.

- 33 a Define the term **intensity** in the context of radiation.

b Estimate the intensity of radiation emitted by a naked human body of surface area 1.60 m², temperature 37 °C and emissivity 0.90, a distance of 5.0 m from the body.

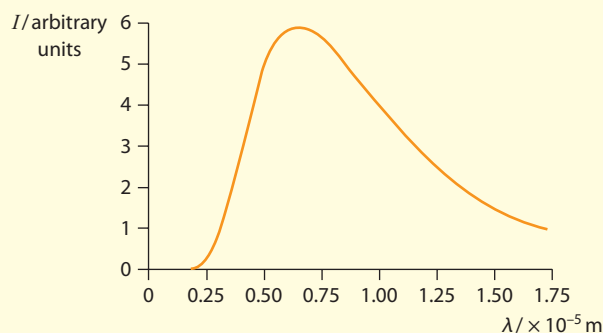
- 34 A body radiates energy at a rate (power) P .

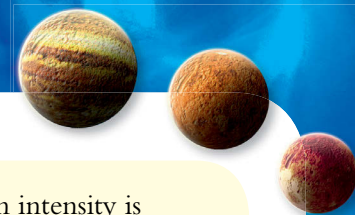
a Deduce that the intensity of this radiation at distance d from the body is given by:

$$I = \frac{P}{4\pi d^2}$$

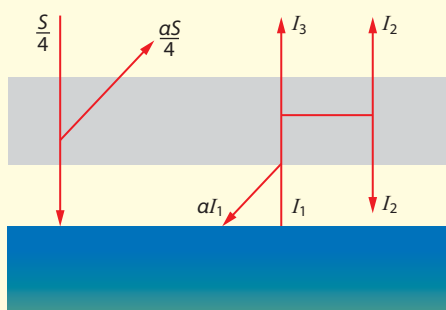
b State **one** assumption made in deriving this result.

- 35 The graph shows the variation with wavelength of the intensity of the radiation emitted by a black body.





- a Determine the temperature of the black body.
b Copy the diagram and, on the same axes, draw a graph to show the variation with wavelength of the intensity of radiation emitted by a black body of temperature 600 K.
- 36 a Define the term **albedo**.
b State **three** factors that the albedo of a surface depends on.
- 37 a State what is meant by the **greenhouse effect**.
b State the main greenhouse gases in the Earth's atmosphere, and for each give **three** natural and **three** man-made sources.
- 38 A researcher uses the following data for a simple climatic model of an Earth without an atmosphere (see Worked example 8.10): incident solar radiation = 350 W m^{-2} , absorbed solar radiation = 250 W m^{-2} .
a Make an energy flow diagram for these data.
b Determine the average albedo for the Earth that is to be used in the modelling.
c Determine the intensity of the outgoing long-wave radiation.
d Estimate the temperature of the Earth according to this model, assuming a constant Earth temperature.
- 39 The diagram shows a more involved model of the greenhouse effect.



Exam tip

You will not get anything as complicated as this in the exam, but this is excellent practice in understanding energy balance equations.

The average incoming radiation intensity is $\frac{S}{4} = 350 \text{ W m}^{-2}$. The albedo of the atmosphere is 0.300. Assume that only a fraction t of the energy radiated by the Earth actually escapes the Earth and that the surface behaves as a black body. The model assumes that part of the radiation from the Earth is reflected back down from the atmosphere.

- a The intensity radiated by the Earth is I_1 , the intensity radiated by the atmosphere is I_2 and the fraction of the intensity escaping the Earth is I_3 . By examining the energy balance of the atmosphere and the surface separately, show that:

$$I_1 = \frac{2}{1-a+t} \times \frac{(1-a)S}{4},$$

$$I_2 = \frac{1-a-t}{1-a+t} \times \frac{(1-a)S}{4} \quad \text{and}$$

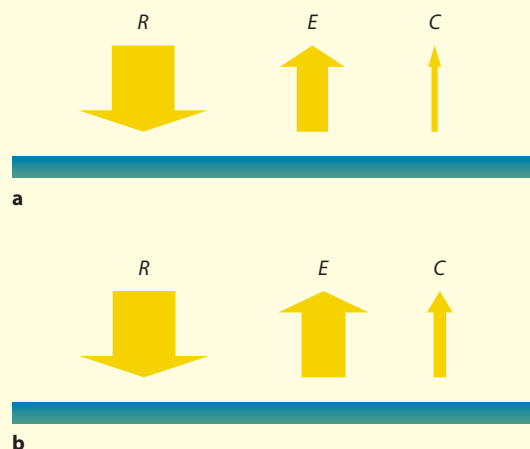
$$I_3 = \frac{2t}{1-a+t} \times \frac{(1-a)S}{4}$$

- b Show that as much energy enters the Earth-atmosphere system as leaves it.
c Show that a surface temperature of $T \approx 288 \text{ K}$ implies that $t = 0.556$.
d i Explain why the emissivity of the atmosphere is $1 - t - a$.
ii Calculate the temperature of the atmosphere.
- 40 Outline the main ways in which the surface of the Earth loses thermal energy to the atmosphere and to space.
- 41 a Compare the albedo of a subtropical, warm, dry land with that of a tropical ocean.
b Suggest mechanisms through which the subtropical land and the tropical ocean lose thermal energy to the atmosphere.
c If the sea level were to increase, sea water would cover dry land. Suggest **one** change in the regional climate that might come about as a result.

42 Evaporation is a method of thermal energy loss. Explain whether you would expect this method to be more significant for a tropical ocean or an arctic ocean.

43 The diagram shows two energy flow diagrams for thermal energy transfer to and from specific areas of the surface of the Earth. R represents the net energy incident on the surface in the form of radiation, E is the thermal energy lost from the Earth due to evaporation, and C is the thermal energy conducted to the atmosphere because of the temperature difference between the surface and the atmosphere.

Suggest whether the Earth area in each diagram is most likely to be dry and cool or moist and warm.



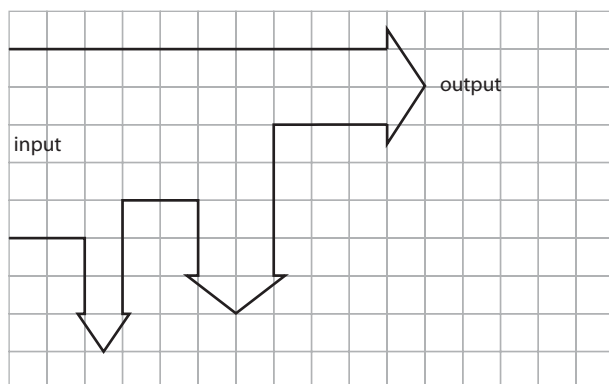
44 It is estimated that a change of albedo by 0.01 will result in a 1°C temperature change. A large area of the Earth consists of 60% water and 40% land. Calculate the expected change in temperature if melting ice causes a change in the proportion of the area covered by water from 60% to 70%. Take the albedo of dry land to be 0.30 and that of water to be 0.10.

Exam-style questions

- 1 A power plant produces 500 MW of electrical power with an overall efficiency of 20%. What is the input power to the plant?
A 100 MW B 400 MW C 625 MW D 2500 MW
- 2 The specific energy of a fuel is the:
A energy that can be extracted from a unit volume of the fuel
B energy that can be extracted from a unit mass of the fuel
C energy contained in a unit volume of the fuel
D energy contained in a unit mass of the fuel.



3 What is the efficiency of a system whose Sankey diagram is shown below?

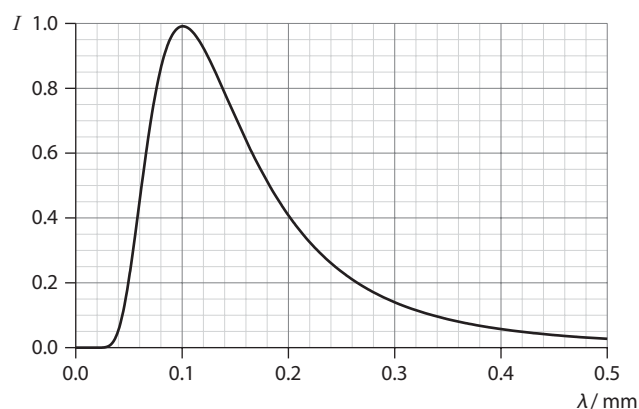


- A** 10% **B** 20% **C** 30% **D** 40%
- 4 Which of the following lists contains one renewable and one non-renewable source of power?
- A** uranium, coal
 - B** natural gas, biomass
 - C** wind power, wave power
 - D** hydropower, solar power
- 5 A plastic ruler and a metallic ruler are in the same room. The metallic ruler ‘feels’ colder when touched. What is the reason for this?
- A** Plastic has a lower specific heat capacity than metal.
 - B** Plastic has a higher specific heat capacity than metal.
 - C** Plastic is a better conductor of heat than metal
 - D** Plastic is a worse conductor of heat than metal.
- 6 A fireplace warms a room by:
- A** conduction
 - B** convection
 - C** radiation
 - D** conduction, convection and radiation
- 7 A star explodes in the vacuum of space. The thermal energy transferred by the star takes place through:
- A** radiation **B** conduction **C** convection **D** evaporation

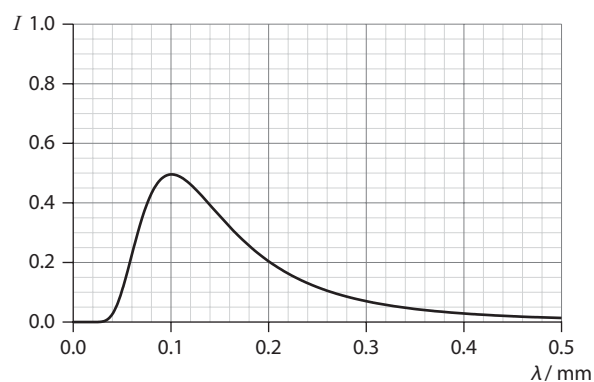
- 8 Four different rooms are losing energy to the outside through a wall. The temperature difference between the inside and the outside of the rooms is the same. Which combination of wall area and wall thickness results in the smallest rate of heat loss?

	Area	Thickness
A	S	d
B	$2S$	$\frac{d}{2}$
C	S	$2d$
D	$2S$	$2d$

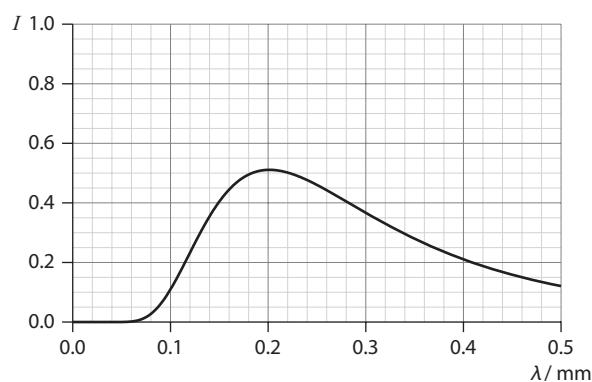
- 9 The graph shows the variation with wavelength of the intensity from a unit area of a black body. The scale on the vertical axis on all graphs in this question is the same.



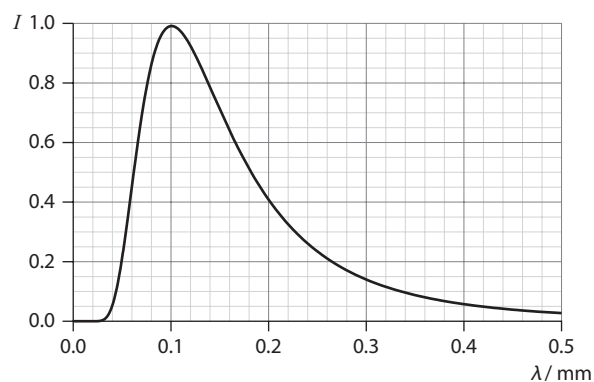
The area and the temperature of the black body are both halved. Which graph now shows the correct variation with wavelength of the intensity from a unit area of the body?



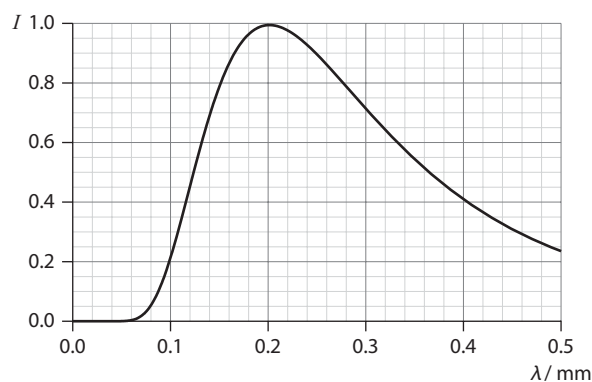
A



B

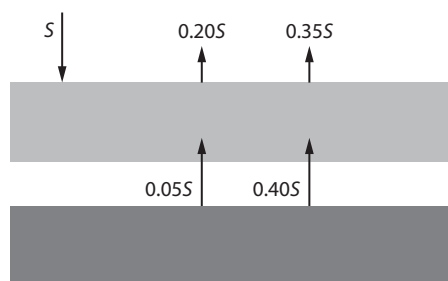


C



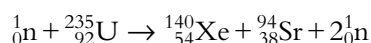
D

- 10 The intensity of solar radiation incident on a planet is S . The diagram represents the energy balance of the planet. The atmosphere reflects an intensity $0.20S$ and radiates $0.35S$. The surface reflects $0.05S$ and radiates $0.40S$.



What is the albedo of the planet?

- A** 0.05 **B** 0.20 **C** 0.25 **D** 0.60
- 11 A nuclear power plant produces 800 MW of electricity with an overall efficiency of 0.32. The fission reaction taking place in the core of the reactor is:



- a i** Using the masses provided below show that the energy released in one fission reaction is about 180 MeV. [2]

$${}_0^1\text{n} = 1.009 \text{ u}, {}_{92}^{235}\text{U} = 235.044 \text{ u}, {}_{54}^{140}\text{Xe} = 139.922 \text{ u}, {}_{38}^{94}\text{Sr} = 93.915 \text{ u}.$$

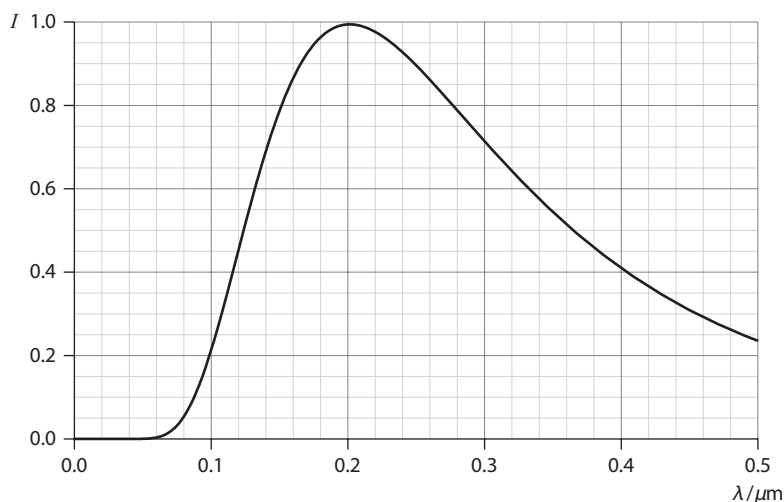
- ii** Estimate the specific energy of uranium-235. [2]
iii Show that the mass of uranium-235 undergoing fission in one year is about 1500 kg. [3]
- b** In a nuclear fission reactor, describe the role of: [2]
i the moderator [2]
ii the control rods [2]
iii the heat exchanger. [2]
- c** Suggest what might happen to a nuclear fission reactor that does not have a moderator. [2]
d State **one** advantage and **one** disadvantage of nuclear power. [2]

- 12 In a pumped storage system, the high reservoir of water has area $4.8 \times 10^4 \text{ m}^2$ and an average depth of 38 m. When water from this reservoir falls to the lower reservoir the centre of mass of the water is lowered by a vertical distance of 225 m. The water flows through a turbine connected to a generator at a rate of $350 \text{ m}^3 \text{ s}^{-1}$.

- a** Calculate the mass of the water in the upper reservoir. [1]
b Determine the loss of gravitational potential energy when the upper reservoir has been completely emptied. [2]
c Estimate the power supplied by the falling water. [2]
The efficiency of the plant in converting this energy into electricity is 0.60. The price of electricity sold by this power station at peak times is \$0.12 per kWh. The plant can buy off-peak electrical power at \$0.07 per kWh. The efficiency at which water can be pumped back up to the high reservoir is 0.64.
d Estimate the profit made by the power plant for a single emptying and refilling of the high reservoir. [3]

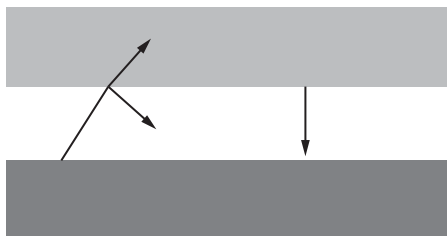
- 13 a Outline, in the context of a wind turbine, the meaning of **primary** and **secondary** energy. [2]
- b The power that can be theoretically extracted by a wind turbine of blade radius R in wind of speed v is $P_{\max} = \frac{1}{2}\pi\rho R^2 v^3$
- i State **one** assumption that has been made in deriving this expression. [1]
- ii Explain **one** other reason why the actual power derived from the wind turbine will be less than $P_{\max}$. [2]
- c A wind turbine has an overall efficiency of 0.30. The following data are available:
- Density of air entering turbine = 1.2 kg m^{-3}
- Density of air leaving turbine = 1.9 kg m^{-3}
- Speed of air entering turbine = 8.2 m s^{-1}
- Speed of air leaving turbine = 5.3 m s^{-1}
- Blade radius = 12 m
- Estimate the power extracted by this wind turbine. [3]

- 14 a On a hot summer day there is usually a breeze from the sea to the shore. Explain this observation. [3]
- b Explain why walking on a day when the temperature is 22°C would be described as very comfortable but swimming in water of the same temperature would be described as cool. [2]
- A black body has temperature T . The graph shows the variation with wavelength of the intensity of radiation emitted by a unit area of the body. The units on the vertical axis are arbitrary.



- c i Describe what is meant by a **black body**. [2]
- ii Estimate T . [2]
- d On a copy of the axes above sketch a graph to show the variation with wavelength of the intensity of radiation emitted by a unit area of:
- i a grey body of emissivity 0.5 and temperature T (label this graph G) [2]
- ii a black body of temperature $\frac{2T}{3}$ (label this graph B). [2]

- 15 The diagram shows a black body of temperature T_1 emitting radiation towards a grey body of lower temperature T_2 and emissivity e . No radiation is transmitted through the grey body.



- a** Using all or some of the symbols T_1 , T_2 , e and σ , state expressions for the intensity:
- i** radiated by the black body [1]
 - ii** radiated by the grey body [1]
 - iii** absorbed by the grey body [1]
 - iv** reflected by the grey body. [1]
- b** The black and the grey bodies in **a** gain as much energy as they lose. Deduce that their temperatures must be the same. [2]
- 16 The power radiated by the Sun is P and the Earth–Sun distance is d . The albedo of the Earth is α .
- a**
- i** Deduce that the solar constant (i.e. the intensity of the solar radiation) at the position of the Earth is $S = \frac{P}{4\pi d^2}$ [2]
 - ii** State what is meant by **albedo**. [1]
- b**
- i** Explain why the average intensity absorbed by the Earth surface is $\frac{S(1 - \alpha)}{4}$ [3]
 - ii** $P = 3.9 \times 10^{26} \text{ W}$, $d = 1.5 \times 10^{11} \text{ m}$ and $\alpha = 0.30$. Assuming the Earth surface behaves as a black body, estimate the average equilibrium temperature of the Earth. [2]
- c** The average Earth temperature is much higher than the answer to **b ii**. Suggest why this is so. [3]

- b The angular frequency is found from $\omega = \frac{2\pi}{T}$:

$$\omega = \frac{2\pi}{0.35} = 17.95 \text{ rad s}^{-1}$$

The maximum kinetic energy is given by $E_{\text{Kmax}} = \frac{1}{2}m\omega^2 x_0^2$ and so:

$$0.80 = \frac{1}{2} \times 0.25 \times 17.95^2 \times x_0^2$$

Rearranging:

$$x_0^2 = \sqrt{\frac{2 \times 0.80}{0.25 \times 17.95^2}}$$

$$x_0 \approx 0.14 \text{ m}$$

- c We know the displacement so use a formula that gives kinetic energy in terms of displacement, i.e. $E_{\text{K}} = \frac{1}{2}m\omega^2(x_0^2 - x^2)$. This gives:

$$E_{\text{K}} = \frac{1}{2} \times 0.25 \times 17.95^2 (0.142^2 - 0.0802^2)$$

$$E_{\text{K}} = 0.53 \text{ J.}$$

Exam tip

More significant figures are being used here. This is all right – this is an intermediate calculation.

Nature of science

The complex can be understood in terms of the simple

The equation for SHM can be solved in terms of simple sine and cosine functions. These simple solutions help physicists to visualise how an oscillator behaves. Although real oscillations are very complex, a powerful mathematical machinery called Fourier analysis allows the decomposition of complex oscillations, sounds, noise and waves in general, in terms of sines and cosines. Energy exchange in oscillating electrical circuits is modelled using this type of analysis. Therefore the simple descriptions used in this topic can also be used in more complex problems as well.



Test yourself

- 1 Explain why the oscillations of a pendulum are, in general, not simple harmonic. State the condition that must be satisfied for the oscillations to become approximately simple harmonic.
- 2 It can be shown that the two solutions for simple harmonic oscillations, $x = x_0 \cos(\omega t)$ or $x = x_0 \sin(\omega t)$ are special cases of the more general $x = x_0 \cos(\omega t + \phi)$ where the angle ϕ is known as the phase of the motion.
 - a State the phase of the motion when $x = x_0 \sin(\omega t)$.
 - b Show explicitly that, if $x = x_0 \cos(\omega t + \phi)$, the period of the motion is given by $T = \frac{2\pi}{\omega}$ independently of x_0 and ϕ .
- 3 The displacement of a particle executing SHM is given by $y = 5.0 \cos(2t)$, where y is in millimetres and t is in seconds. Calculate:
 - a the initial displacement of the particle
 - b the displacement at $t = 1.2$ s
 - c the time at which the displacement first becomes -2.0 mm
 - d the displacement when the velocity of the particle is 6.0 mm s^{-1} .
- 4 a Write down an equation for the displacement of a particle undergoing SHM with an amplitude equal to 8.0 cm and a frequency of 14 Hz, assuming that at $t = 0$ the displacement is 8.0 cm and the particle is at rest.
 - b Find the displacement, velocity and acceleration of this particle at a time of 0.025 s.

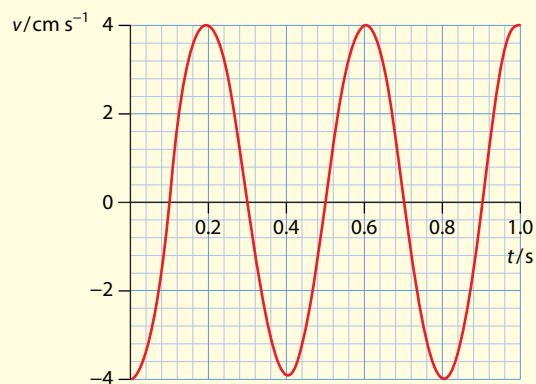
- 5 A point on a guitar string oscillates in SHM with an amplitude of 5.0 mm and a frequency of 460 Hz. Determine the maximum velocity and acceleration of this point.
- 6 A guitar string, whose two ends are fixed oscillates as shown in the diagram.



The vertical displacement of a point on the string a distance x from the left end is given by $y = 6.0 \cos(1040\pi t) \sin(\pi x)$, where y is in millimetres, x is in metres and t is in seconds.

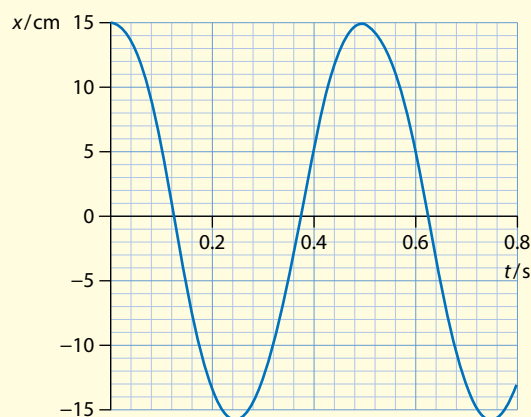
Use this expression to:

- deduce that all points on the string execute SHM with a common frequency and common phase, and determine the common frequency
 - deduce that different points on the string have different amplitudes
 - determine the maximum amplitude of oscillation
 - calculate the length L of the string
 - calculate the amplitude of oscillation of the point on the string where $x = \frac{3}{4}L$.
- 7 The graph shows the variation with time t of the velocity v of a particle executing SHM.



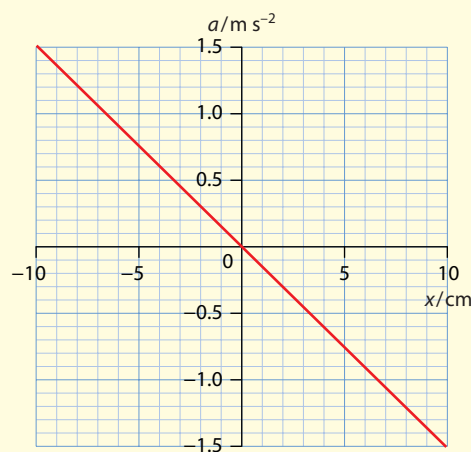
- Using the graph, estimate the area between the curve and the time axis from 0.10 s to 0.30 s.
- State what this area represents.
- Hence write down an equation giving the displacement of the particle as a function of time.

- 8 The graph shows the variation with time t of the displacement x of a particle executing SHM.



Draw a graph to show the variation with displacement x of the acceleration a of the particle (put numbers on the axes).

- 9 The graph shows the variation with displacement x of the acceleration a of a body of mass 0.150 kg.



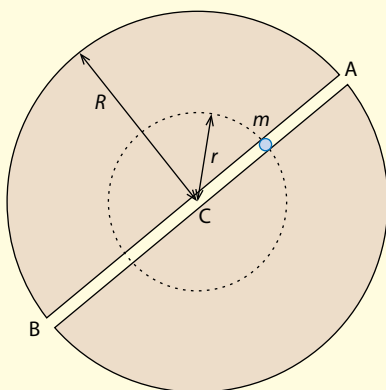
- Use the graph to explain why the motion of the body is SHM. Determine the following:
- the period of the motion
- the maximum velocity of the body during an oscillation
- the maximum net force exerted on the body
- the total energy of the body.

- 10 A body of mass 0.120 kg is placed on a horizontal plate. The plate oscillates vertically in SHM making five oscillations per second.
- Determine the largest possible amplitude of oscillations such that the body never loses contact with the plate.
 - Calculate the normal reaction force on the body at the lowest point of the oscillations when the amplitude has the value found in **a**.

- 11 This is a very unrealistic but interesting 'thought experiment' involving SHM.

Imagine boring a straight tunnel from one place (A) on the surface of the Earth to another place (B) diametrically opposite, and then releasing a ball of mass m at point A. The ball then moves under the influence of gravity.

To answer the following questions, you need to know that when the ball is at the position shown in the diagram, the gravitational force it experiences is the force of gravitation from the mass inside the dotted circle *only*. In addition, this mass inside the dotted circle may be considered to be concentrated at the centre of the Earth. Assume that the density of the Earth is uniform.

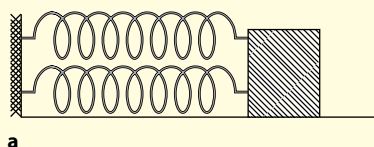


- Denoting the mass of the Earth by M and its radius by R , derive an expression for the mass inside the dotted circle (of radius x).
- Derive an expression for the gravitational force on the ball when at the position shown in the diagram, a distance x from the centre of the Earth.
- Hence deduce that the motion of the ball is simple harmonic.

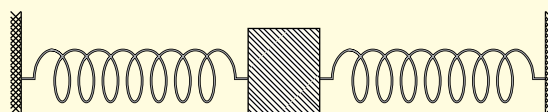
- Determine the period of the motion.
- Evaluate this period using:
 $M = 6.0 \times 10^{24}\text{ kg}$ $R = 6.4 \times 10^6\text{ m}$
and $G = 6.67 \times 10^{-11}\text{ N kg}^{-2}\text{ m}^2$.
- Compare the period of this motion with the period of rotation of a satellite around the Earth in a circular orbit of radius R .

- 12 A body of mass 2.0 kg is connected to two springs, each of spring constant $k = 120\text{ N m}^{-1}$.

- The springs are connected as in part **a** of the diagram. Calculate the period of the oscillations of this mass when it is displaced from its equilibrium position and then released.
- The springs are now connected as in part **b** of the diagram. State and explain whether the period changes.



a



b

- 13 A woman bungee-jumper of mass 60 kg is attached to an elastic rope of natural length 15 m . The rope behaves like a spring of spring constant $k = 220\text{ N m}^{-1}$. The other end of the spring is attached to a high bridge. The woman jumps from the bridge.

- Determine how far below the bridge she falls, before she instantaneously comes to rest.
- Calculate her acceleration at the position you found in **a**.
- Explain why she will perform SHM, and find the period of oscillations.
- The woman will eventually come to rest at a specific distance below the bridge. Calculate this distance.
- The mechanical energy of the woman after she comes to rest is less than the woman's total mechanical energy just before she jumped. Explain what happened to the 'lost' mechanical energy.

Nature of science

The interference patterns seen in diffraction through a single slit provide evidence of the wave nature of light. The pattern we see is very different from the simple geometrical shadow expected if light consisted of particles. In this section, you have seen how summing the different waves leads to the pattern observed, and how the width of the slit affects the intensity pattern. In a similar way, the waves diffracted around objects can be summed. **Figure 9.16** shows the result of diffraction of light around a small circular object. There is a bright spot at the centre of the disc, where a particle model of light would predict darkness! It was this spot, predicted by Fresnel's wave theory and observed by François Arago, that led to the acceptance of the wave theory in the 19th century (the debate between Fresnel and Poisson was described in the NOS for **Subtopic 4.4**). This is an example of how theory can be used to predict what should be observed. This can then be tested by experiment, to give a result supporting the theory. The bright spot in the centre is now called Fresnel's spot or Arago's spot.

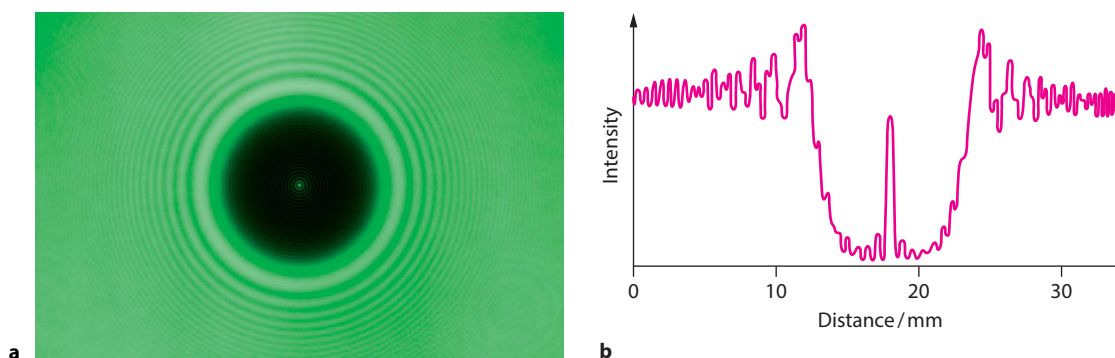
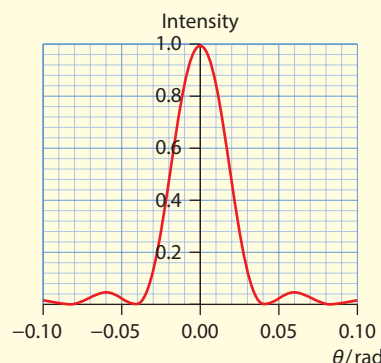


Figure 9.16 **a** The Fresnel spot. **b** The intensity of light as a function of horizontal distance showing the peak in the middle of the disc.



Test yourself

- 14 A single slit of width $1.50\mu\text{m}$ is illuminated with light of wavelength 500.0 nm . Determine the angular width of the central maximum. (Use the approximate formula.)
- 15 In a single-slit diffraction experiment the slit width is 0.12 mm and the wavelength of the light used is $6.00 \times 10^{-7}\text{ m}$. Calculate the width of the central maximum on a screen 2.00 m from the slit.
- 16 The intensity pattern for single-slit diffraction is shown in the diagram. (The vertical units are arbitrary.) The wavelength of the light used is λ .
 - a** Find the width of the slit b in terms of λ .
 - b** On a copy of the axes draw a graph to show how intensity varies with diffraction angle for a slit with:
 - i** width $\frac{b}{2}$ and wavelength λ
 - ii** width $\frac{b}{2}$ and wavelength $\frac{\lambda}{2}$.
 (You do not need numbers on the intensity axis.)



Nature of science

Curiosity and serendipity

The shimmering colours of a peacock's tail feathers, the colours inside the paua shell, and the glistening surface of a golden stag beetle's coat are all examples of iridescence in nature. Similar colours are seen in soap bubbles and oil films. Curious as to what caused these beautiful colours and effects in nature, scientists speculated on different mechanisms. One suggestion was that the surface pigments of iridescent feathers appeared to reflect different colours when viewed from different angles. Another suggestion was that the colours were formed as a result of the structure of the surface, not anything to do with the pigments. With the acceptance of the wave theory of light, scientists in the late 19th century developed a theory of interference in thin films that could explain these colours. Developments in microscopy meant that finer and finer structure could be seen, and using an electron microscope in the 20th century it was possible to see complex thin-film structures in iridescent bird feathers.

In 1817 Joseph Fraunhofer accidentally produced a thin film coating when he left nitric acid on a polished glass surface. Experimenting with different glasses, he was able to produce a surface with the same vivid colours seen in a soap bubbles. When he observed the same colours appearing as a coating of alcohol evaporated from a polished metal surface, he concluded that such colours would appear in any transparent thin film. This was the start of the technology of thin films, developed in the 20th century to make optical coatings for glass.

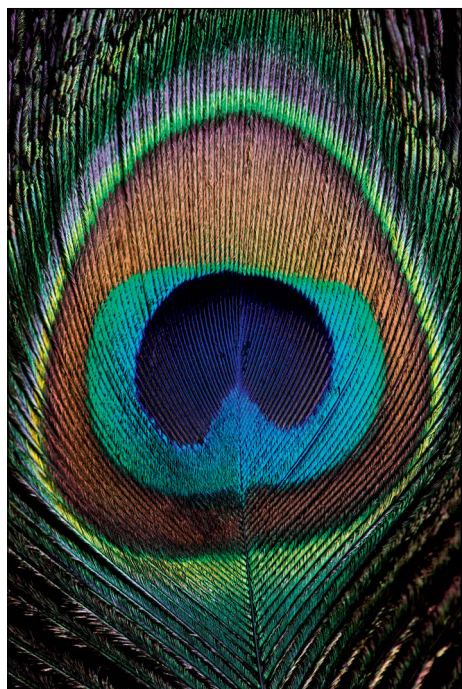


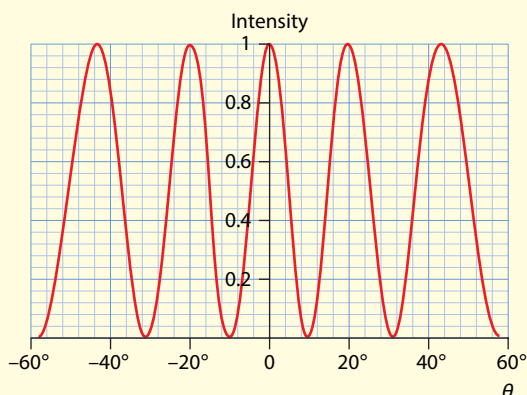
Figure 9.29 The iridescent 'eye' of a peacock's tail feather.



Test yourself

- 17 In a Young's two-slit experiment, a coherent source of light of wavelength 680 nm is used to illuminate two very narrow slits a distance of 0.12 mm apart. A screen is placed at a distance of 1.50 m from the slits. Calculate the separation of two successive bright spots.
- 18 Explain why two identical flashlights pointing light to the same spot on a screen will never produce an interference pattern.
- 19 In a Young's two-slit experiment it is found that an n th-order maximum for a wavelength of 680.0 nm coincides with the $(n+1)$ th maximum of light of wavelength 510.0 nm . Determine n .
- 20 Light is incident normally on two narrow parallel slits a distance of 1.00 mm apart. A screen is placed a distance of 1.2 m from the slits. The distance on the screen between the central maximum and the centre of the $n=4$ bright spot is measured to be 3.1 mm .
 - a Determine the wavelength of light.
 - b This experiment is repeated in water (of refractive index 1.33). Suggest how the distance of 3.1 mm would change, if at all.

- 21 The graph shows the intensity pattern from a two-slit interference experiment.



- a** Determine the separation of the slits in terms of the wavelength of light used.
- b** Suggest how the pattern in **a** changes if the slit separation is halved.
- 22 A grating with 400 lines per mm is illuminated with light of wavelength 600.0 nm.
- a** Determine the angles at which maxima are observed.
- b** Determine the largest order that can be seen with this grating and this wavelength.
- 23 A piece of glass of index of refraction 1.50 is coated with a thin layer of magnesium fluoride of index of refraction 1.38. It is illuminated with light of wavelength 680 nm. Determine the minimum thickness of the coating that will result in no reflection.
- 24 A thin soap bubble of index of refraction 1.33 is viewed with light of wavelength 550.0 nm and appears very bright. Predict a possible value of the thickness of the soap bubble.
- 25 Two very narrow, parallel slits separated by a distance of 1.4×10^{-5} m are illuminated by coherent, monochromatic light of wavelength 7.0×10^{-7} m.
- a** Describe what is meant by coherent and monochromatic light.
- b** Draw a graph to show the intensity of light observed on a screen far from the slits.
- c** By drawing another graph on the same axes, illustrate the effect on the intensity distribution of increasing the width of the slits to 2.8×10^{-5} m.
- 26 **a** Draw a graph to show the variation with angle of the intensity of light observed on a screen some distance from two very narrow, parallel slits when coherent monochromatic light falls on the slits.
- b** Describe how the graph you drew in **a** changes when:
- i** the number of slits increases but their separation stays the same
 - ii** the number of slits stays at two but their separation decreases.

Worked example

9.17 A beam of light containing different wavelengths is incident on a diffraction grating. The grating has 600 lines per mm and is 2.0 cm wide. The average wavelength in the beam is 620 nm. Calculate the least difference in wavelength that can be resolved by this grating in the second order.

From the formula for resolving power we have:

$$\Delta\lambda = \frac{\lambda_{\text{avg}}}{mN}$$

Since the grating is 20 mm wide we find:

$$N = 600 \times 20 = 12\,000$$

We are using the second order so $m = 2$ and therefore:

$$\Delta\lambda = \frac{\lambda_{\text{avg}}}{mN} = \frac{620}{2 \times 12\,000} = 0.0258 \approx 0.026 \text{ nm}$$

Nature of science

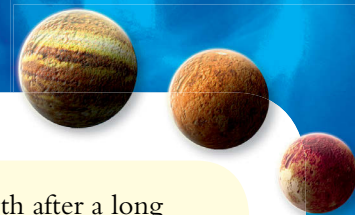
Pushing the limits of resolution

The Rayleigh criterion determines the detail that can be resolved for a given wavelength. Advances in technology have pushed the limits of what can be resolved using both large and small wavelengths. Larger diameter receiving dishes give greater resolution to radio telescopes, which use very long wavelengths. Even greater resolution is obtained using vast arrays of radio telescopes connected together, made possible through advances in signal processing. At the other extreme, small objects need small wavelengths. All other things being equal, blue light is better than red light for distinguishing detail. A huge technological advance was made when it was realised that the wave used to 'see' a small object does not even have to be an electromagnetic wave. In [Topic 12](#) we will find out that all particles, and in particular electrons, show wave-like behaviour, and this is the basis of the images obtained by the electron microscope. The reason particle physicists need particle accelerators is because the high energies obtained give particles very short wavelengths, which can then be used to probe the structure of other tiny particles.



Test yourself

- 27** Determine whether a telescope with an objective lens of diameter 20 cm can resolve two objects a distance of 10 km away separated by 1 cm. (Assume we are using a wavelength of 600 nm.)
- 28 a** The headlights of a car are separated by a distance of 1.4 m. Estimate the distance these would be resolved as two separate sources by a lens of diameter 5 cm with a wavelength of 500 nm.
- b** Discuss the effect, if any, of decreasing the wavelength on the distance in **a**.



- 29 Assume that the pupil of the human eye has a diameter of 4.0 mm and receives light of wavelength 5.0×10^{-7} m.
- Calculate the smallest angular separation that can be resolved by the eye at this wavelength.
 - Estimate the least distance between features on the Moon (a distance of 3.8×10^8 m away) that can be resolved by the human eye.
- 30 The Jodrell Bank radio telescope in Cheshire, UK, has a diameter of 76 m. Assume that it receives electromagnetic waves of wavelength 21 cm.
- Calculate the smallest angular separation that can be resolved by this telescope.
 - Determine whether this telescope can resolve the two stars of a binary star system that are separated by a distance of 3.6×10^{11} m and are 8.8×10^{16} m from Earth (assume a wavelength of 21 cm).
- 31 The Arecibo radio telescope has a diameter of 300 m. Assume that it receives electromagnetic waves of wavelength 8.0 cm. Determine if this radio telescope will see the Andromeda galaxy (a distance of 2.5×10^6 light years away) as a point source of light or an extended object. Take the diameter of Andromeda to be 2.2×10^5 light years.
- 32 A spacecraft is returning to Earth after a long mission far from Earth. Estimate the distance from Earth, at which an astronaut in the spacecraft will first see the Earth and the Moon as distinct objects with a naked eye. Take the separation of the Earth and the Moon to be 3.8×10^8 m, and assume a pupil diameter of 4.5 mm and light of wavelength 5.5×10^{-7} m.
- 33 The Hubble Space Telescope has a mirror of diameter 2.4 m.
- Estimate the resolution of the telescope, assuming that it operates at a wavelength of 5.5×10^{-7} m.
 - Suggest why the Hubble Space Telescope has an advantage over Earth-based telescopes of similar mirror diameter.
- 34 The spectrum of sodium includes two lines at wavelengths 588.995 nm and 589.592 nm. A sodium lamp is viewed by a diffraction grating that just manages to resolve these two lines in the third order at 12° . Determine:
- the slit spacing d of the grating
 - the total number of rulings on the grating.
- 35
- A diffraction grating is 5.0 mm wide and has 600 lines per mm. A beam of light containing a range of wavelengths is incident on the grating. The average wavelength is 550 nm. Determine the least wavelength range that can be resolved in second order.
 - You can increase the resolving power by increasing the order m or the number of lines N . Suggest whether these two ways are equivalent.

9.5 The Doppler effect

This section looks at the Doppler effect, the change in the observed frequency of a wave when there is relative motion between the source and the observer. The Doppler effect is a fundamental wave phenomenon with many applications. The phenomenon applies to all waves, but only sound and light waves are considered here.

Wavefront diagrams

If you stand by the edge of a road and a car moving at high speed approaches you will hear a high-pitched sound. The instant the car moves past you the frequency of the sound will drop abruptly and will stay low

Learning objectives

- Understand the Doppler effect through wavefront diagrams.
- Solve problems involving frequency and wavelength shifts and speeds of source or observer.



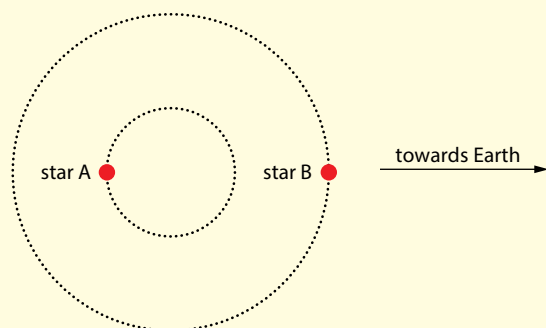
Test yourself

Take the speed of sound in still air to be 340 m s^{-1} .

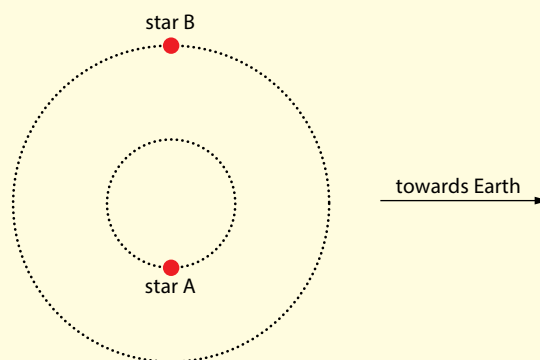
- 36** Explain, with the help of diagrams, the Doppler effect. Show clearly the cases of a source that **a** moves towards and **b** goes away from a stationary observer, as well the case of a moving observer.
- 37** A source approaches a stationary observer at 40 m s^{-1} emitting sound of frequency 500 Hz .
 - a** Determine the frequency the observer measures.
 - b** Calculate the wavelength of the sound as measured by **i** the source and **ii** the observer.
- 38** A source is moving away from a stationary observer at 32 m s^{-1} emitting sound of frequency 480 Hz .
 - a** Determine the frequency the observer measures.
 - b** Calculate the wavelength of the sound as measured by **i** the source and **ii** the observer.
- 39** A sound wave of frequency 512 Hz is emitted by a stationary source toward an observer who is moving away at 12 m s^{-1} .
 - a** Determine the frequency the observer measures.
 - b** Calculate the wavelength of the sound as measured by **i** the source and **ii** the observer.
- 40** A sound wave of frequency 628 Hz is emitted by a stationary source toward an observer who is approaching at 25 m s^{-1} .
 - a** Determine the frequency the observer measures.
 - b** Calculate the wavelength of the sound as measured by **i** the source and **ii** the observer.
- 41** A source of sound is directed at an approaching car. The sound is reflected by the car and is received back at the source. Carefully explain what changes in frequency the observer at the source will detect.
- 42** A sound wave of frequency 500 Hz is emitted by a stationary source toward a receding observer. The signal is reflected by the observer and received by the source, where the frequency is measured and found to be 480 Hz . Calculate the speed of the observer.
- 43** A sound wave of frequency 500 Hz is emitted by a moving source toward a stationary observer. The signal is reflected by the observer and received by the source, where the frequency is measured and found to be 512 Hz . Calculate the speed of the source.
- 44** Consider the general case when both the source and the observer move towards each other. Let u_s be the velocity of the source and u_o that of the observer. In the frame of reference in which the observer is at rest, the waves appear to move with velocity $v + u_o$ and the source appears to move with velocity $u_s + u_o$. Show that the frequency received by the observer is:
$$f_o = f_s \frac{c + v_o}{c - v_s}$$
- 45** Ultrasound of frequency 5.000 MHz reflected from red blood cells moving in an artery is found to show a frequency shift of 2.4 kHz . The speed of ultrasound in blood is $v = 1500 \text{ m s}^{-1}$.
 - a** Explain why the appropriate formula for the frequency shift is $\frac{\Delta f}{f} = 2\frac{u}{v}$, where u is the speed of the blood cells.
 - b** Estimate the speed of the blood cells.
 - c** In practice, a range of frequency shifts is observed. Explain this observation.
- 46** Calculate the speed of a galaxy emitting light of wavelength $5.48 \times 10^{-7} \text{ m}$ which when received on Earth is measured to have a wavelength of $5.65 \times 10^{-7} \text{ m}$.
- 47** Light from a nearby galaxy is emitted at a wavelength of 657 nm and is observed on Earth at a wavelength of 654 nm .
 - a** Deduce the speed of this galaxy.
 - b** State what, precisely, can be deduced about the direction of the velocity of this galaxy.
- 48** The Sun rotates about its axis with a period that may be assumed to be constant at 27 days. The radius of the Sun is $7.00 \times 10^8 \text{ m}$. Discuss the shifts in frequency of light emitted from the Sun's equator and received on Earth. Assume that the Sun emits monochromatic light of wavelength $5.00 \times 10^{-7} \text{ m}$.

49 In a binary star system, two stars orbit a common point and move so that they are always in diametrically opposite positions. Light from both stars reaches an observer on Earth. Assume that both stars emit light of wavelength $6.58 \times 10^{-7} \text{ m}$.

a When the stars are in the position shown in the diagram below, the observer on Earth measures a wavelength of light of $6.58 \times 10^{-7} \text{ m}$ from both stars. Explain why there is no Doppler shift in this case.

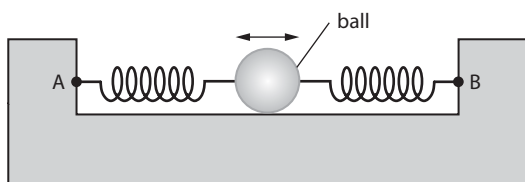


b When the stars are in the position shown in the diagram below, the observer on Earth measures two wavelengths in the received light, $6.50 \times 10^{-7} \text{ m}$ and $6.76 \times 10^{-7} \text{ m}$. Determine the speed of each of the stars.



Exam-style questions

- 1 A ball of mass m is attached to two identical springs of spring constant k as shown. Initially the springs have their natural length. The ball is displaced a small distance to the right and is then released.



What is the period of oscillation of the ball?

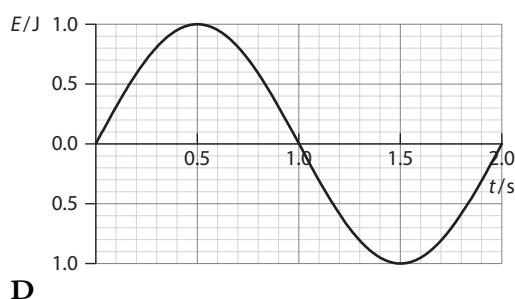
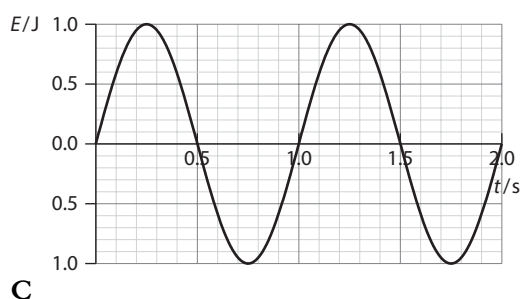
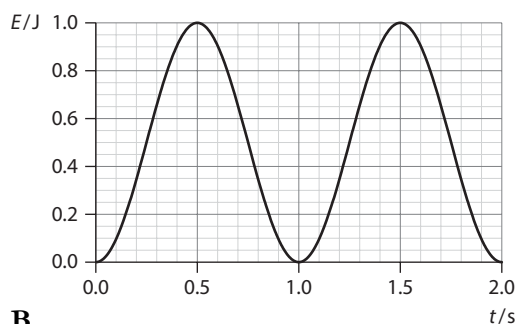
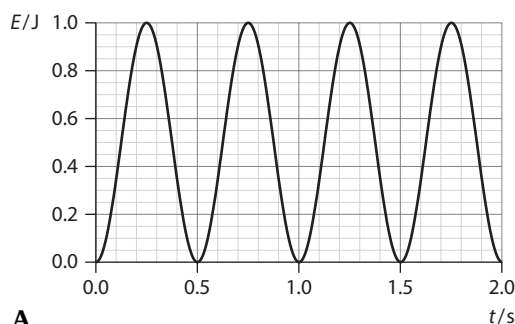
- A $2\pi\sqrt{\frac{m}{k}}$ B $2^{\frac{1}{2}}\pi\sqrt{\frac{m}{k}}$ C $2^{\frac{3}{2}}\pi\sqrt{\frac{m}{k}}$ D $2^{-\frac{1}{2}}\pi\sqrt{\frac{m}{k}}$

- 2 A particle performs simple harmonic oscillations with amplitude 0.1 mm and frequency 100 Hz . What is the maximum acceleration of this particle (in m s^{-2})?

- A 0.2π B 0.4π C $2\pi^2$ D $4\pi^2$



- 3 Which of the following graphs shows the variation with time of the potential energy of a particle undergoing simple harmonic oscillations with a period of 1.0 s?



- 4 Light of wavelength λ is incident normally on a slit of width b . A screen is placed a distance D from the slit. What is the width of the central maximum measured along the screen?

A $\frac{\lambda}{b}$

B $\frac{2\lambda}{b}$

C $\frac{\lambda D}{b}$

D $\frac{2\lambda D}{b}$

- 5 Light of wavelength λ is incident on two parallel slits. An interference pattern is formed on a screen behind the slits. The separation of the slits and the width of the slits are both decreased. Which of the following is correct about the separation and the width of the bright fringes?

	separation	width
A	increases	increases
B	increases	decreases
C	decreases	increases
D	decreases	decreases

- 6 Two radio wave emitting stars are separated by a distance d and are both a distance r from Earth. Radio waves of wavelength λ are received by a radio telescope of diameter b on Earth. The two stars will be well resolved by the telescope if which condition is satisfied?

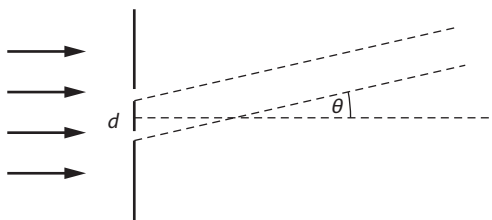
A $\frac{d}{r} < 1.22 \times \frac{\lambda}{b}$

B $\frac{d}{r} > 1.22 \times \frac{\lambda}{b}$

C $\frac{r}{d} < 1.22 \times \frac{\lambda}{b}$

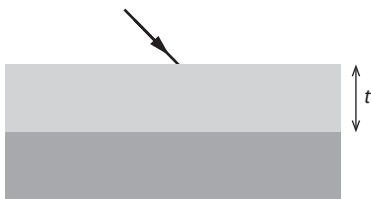
D $\frac{r}{d} > 1.22 \times \frac{\lambda}{b}$

- 7 Coherent light of wavelength λ is incident on two parallel slits that are separated by a distance d . Angle θ is the smallest angle for which the two rays shown interfere destructively on a screen far from the slits.



Which of the following could be correct?

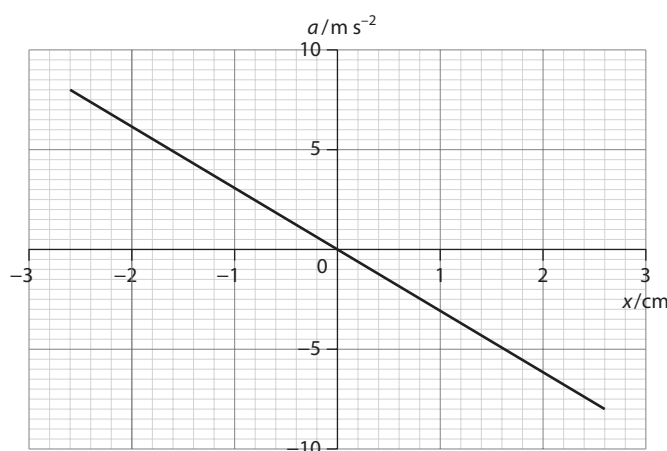
- A** $d \sin \theta = \frac{\lambda}{2}$ **B** $d \sin \theta = \lambda$ **C** $d \sin \theta = \frac{3\lambda}{2}$ **D** $\sin \theta = 2\lambda$
- 8 A train sounding its horn goes past a train station without stopping. The train moves at constant speed. Which is correct about the frequency of the horn measured by the observer?
- A** keeps increasing then keeps decreasing
B keeps decreasing then keeps increasing
C is constant and high then constant and low
D is constant and low then constant and high
- 9 Light is incident essentially normally on a thin film of thickness t and refractive index 1.5. The film is on transparent glass of refractive index 2.5.



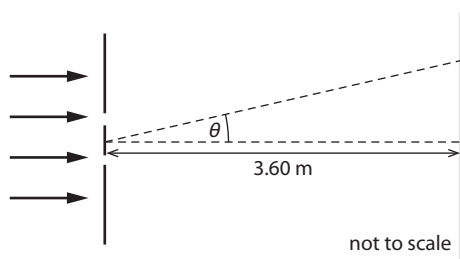
Which of the following conditions on the wavelength in oil leads to destructive interference of the reflected light?

- A** $\lambda = 6t$
B $\lambda = 3t$
C $\lambda = t$
D $\lambda = 1.5t$
- 10 Light is incident on N very thin parallel slits and an interference pattern is formed on a screen a distance away. The number of slits is increased while the separation of two consecutive slits stays the same. Which is correct as N increases?
- A** the number of secondary maxima decreases
B the intensity of the secondary maxima increases
C the primary maxima become narrower
D the distance between the central maximum and the first primary maximum to the side increases

- 11 The graph shows how the acceleration of a particle varies with displacement from a fixed equilibrium position.



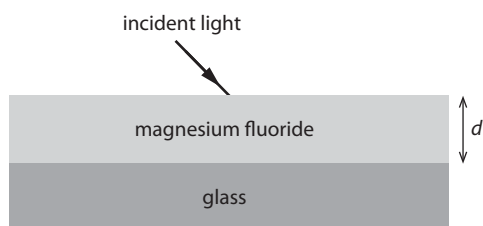
- a Use the graph to explain why the particle is performing simple harmonic oscillations. [3]
 - b Determine:
 - i the amplitude of oscillations [1]
 - ii the frequency of the motion. [2]
 - c
 - i The mass of the particle is 0.25 kg. Calculate the maximum potential energy of the particle. [2]
 - ii Determine the speed of the particle when its kinetic energy equals its potential energy. [3]
 - d Sketch a graph to show how the kinetic energy varies with displacement. [2]
- 12 In a Young two-slit experiment, a source of light of unknown wavelength is used to illuminate two very narrow slits that are a distance 0.120 mm apart.



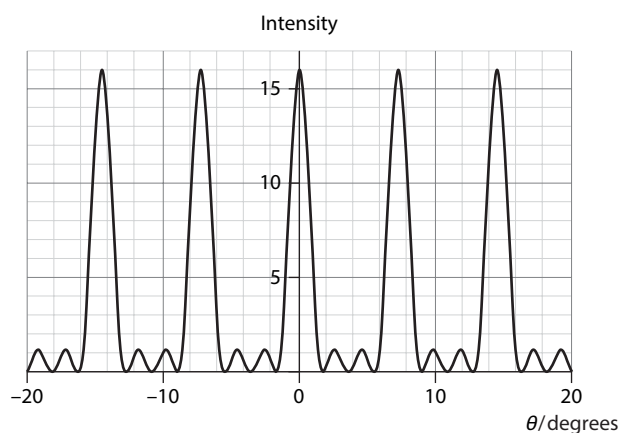
Bright fringes are observed on a screen a distance of 3.60 m from the slits. The separation between the bright fringes is 1.86 cm.

- a
 - i Explain how the bright fringes are formed. [2]
 - ii Determine the wavelength of light. [3]
- b Draw a graph to show how the intensity of the light observed on the screen varies with the angle θ . The intensity at the screen due to one slit alone is 20 W m^{-2} . You may neglect the slit width. [3]
- c
 - i The two slits are replaced by a diffraction grating. The light in **a** makes a second order maximum at an angle of 58° . Calculate the number of rulings per mm for this grating. [2]
 - ii Another wavelength of visible light creates a maximum at the same angle as in **c i** but at a different order. Determine this wavelength and the order of its maximum. [4]

- 13 Monochromatic light of wavelength $5.0 \times 10^{-7} \text{ m}$ in air is incident on a rectangular piece of glass of refractive index 1.60 that is coated by a thin layer of magnesium fluoride of refractive index 1.38.

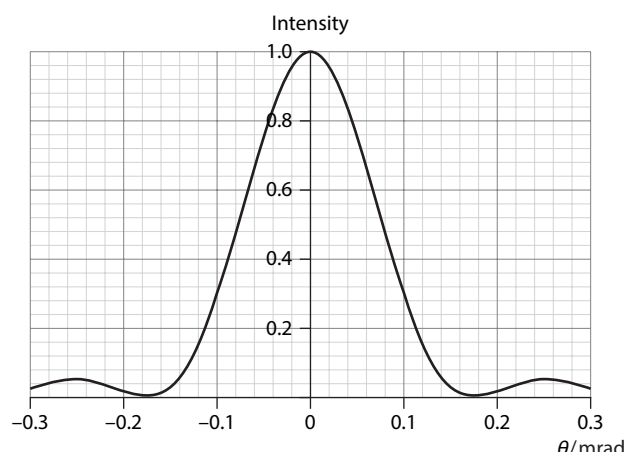


- Copy and complete this diagram by drawing the paths of the two rays, originating with the incident ray, that will interfere in the eye of an observer looking down on the glass from above. [2]
 - Indicate on the diagram points at which reflected rays undergo phase changes. [2]
 - Calculate the least thickness d of this coating that will result in no light being reflected. Assume that the ray is incident normally. [3]
- 14 The graph shows the interference pattern for a number of very thin parallel slits.



- Justify that the number of slits is 4. [2]
- List a total of **four** ways in which this pattern changes as:
 - the number of slits increases but their separation stays the same
 - the number of slits stays the same but their separation decreases. [4]
- It is required to resolve two lines in the spectrum of hydrogen: a line with wavelength 656.45 nm in ^1H and a line of wavelength 656.27 nm in the isotope ^2H . Calculate the least number of slits per mm required to resolve these two lines in the second order at an angle of 15° . [3]

- 15 The graph shows the single-slit diffraction pattern for monochromatic light from one point source.



The wavelength of light is 5.0×10^{-7} m.

- a Calculate the slit diameter assuming that it is circular. [2]
- b A second source is placed at distance of 3.0 cm from the other source.
- i On a copy of the axes above draw the diffraction pattern from the second source in the case in which the two sources are just barely resolved. [2]
- ii Calculate the distance of the two sources from the slit. [2]
- 16 a State what is meant by the **Doppler effect**. [1]
- b Illustrate the Doppler effect for the case of a moving source using wavefront diagrams. [2]
- c Outline **one** practical application of the Doppler effect. [2]
- d A disc of radius 0.20 m rotates about its axis making eight revolutions per second. Sound of frequency 2400 Hz is emitted in all directions from a source on the circumference of the disc. The sound is received by an observer far away from the disc. Determine the range of frequencies and the range of wavelengths that the observer measures. [4]

Similarities between electricity and gravitation

As is clear from a comparison between Newton's law of gravitation and Coulomb's law, there are many similarities between electricity and gravitation (both force laws are inverse square laws). The biggest difference is, of course, the existence of positive as well as negative electric charge, which implies that the electric force can be attractive or repulsive. The one sign of the mass leads to attractive forces only. The constant G is very small compared to k . This implies that the force of gravitation is significant only when one or both of the bodies have enormous mass.

Nature of science

The Sun influences the motion of the Earth a distance of 1.5×10^{11} m away. How is the 'influence' of the Sun 'transmitted' to the position of the Earth? If the Sun were to suddenly disappear, how long would it take the Earth to leave its orbit and plunge into darkness? To answer such questions required a huge shift in scientific thinking, known as a paradigm shift. Fields were introduced, which replaced the idea of direct, observable action with a mechanism for 'action at a distance'. Fields were further refined with the concept of waves. If a charge were to suddenly appear, an electric field would be established in space at the speed of light. The 'information' about the existence of the charge is carried by electromagnetic waves travelling through vacuum at the speed of light. It is similarly believed that gravitational waves carry the 'information' about the existence of mass. But unlike electromagnetic waves, gravity waves have not yet been observed, even though no one doubts their existence.

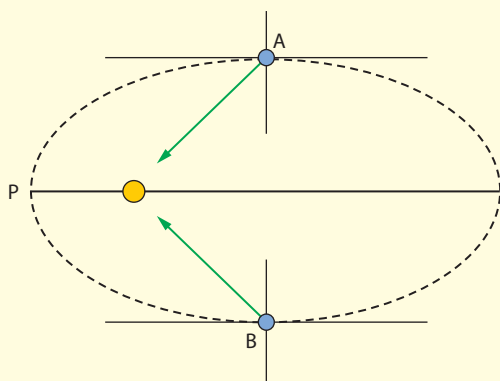
? Test yourself

- 1 Consider two particles of mass m and $16m$ separated by a distance d .
 - a Deduce that at point P, a distance $\frac{d}{5}$ from the particle with mass m , the gravitational field strength is zero.
 - b Determine the value of the gravitational potential at P.
- 2
 - a What is the gravitational potential at a distance from the Earth's centre equal to 5 Earth radii?
 - b What is the gravitational potential energy of a 500 kg satellite placed at a distance from the Earth's centre equal to 5 Earth radii?
- 3
 - a What is the gravitational potential energy stored in the gravitational field between the Earth and the Moon?
 - b What is the Earth's gravitational potential at the position of the Moon?
 - c Find the speed with which the Moon orbits the Earth. (The Earth–Moon distance is 3.8×10^8 m. Take the mass of the Earth to be 5.97×10^{24} kg.)
- 4 A spacecraft of mass 30 000 kg leaves the Earth on its way to the Moon and lands on the Moon. Plot the spacecraft's potential energy as a function of its distance from the Earth's centre. (The Earth–Moon distance is 3.8×10^8 m. Take the mass of the Earth to be 5.97×10^{24} kg and the mass of the Moon to be 7.35×10^{22} kg.)

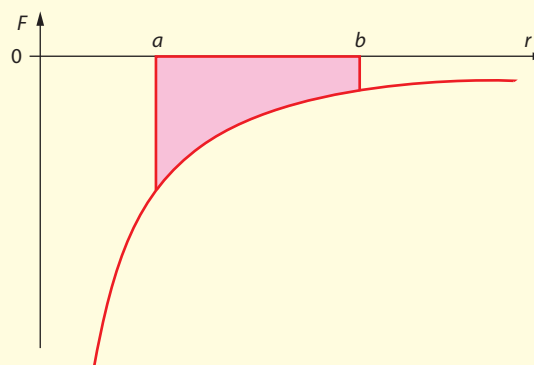
- 5 The diagram shows the variation with distance from the centre of the planet of the gravitational potential due to the planet and its moon. The planet's centre is at $r=0$ and the centre of the moon is at $r=1$. The units of separation are arbitrary. At the point where $r=0.75$ the gravitational field is zero.



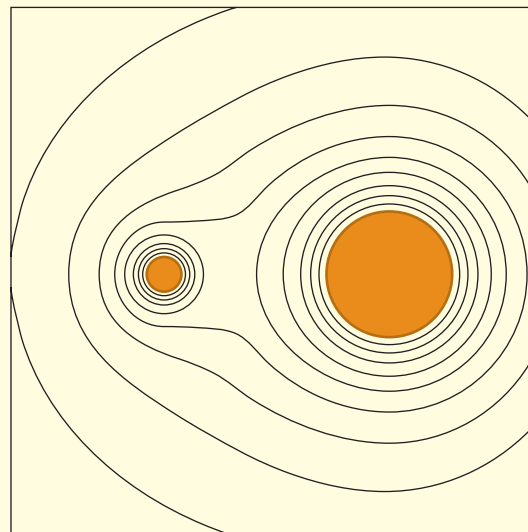
- a Determine the ratio of the mass of the planet to that of the moon.
- b With what speed must a probe be launched from the surface of the planet in order to arrive on the surface of the moon?
- 6 The diagram shows a planet orbiting the Sun counter-clockwise, at two positions – A and B. Also shown is the gravitational force acting on the planet at each position. By decomposing the force into components normal and tangential to the path (dotted lines), explain why it is only the tangential component that does work. Hence explain why the planet will accelerate from A to P but will slow down from P to B.



- 7 The diagram shows the variation of the gravitational force with distance between two masses. What does the shaded area represent?

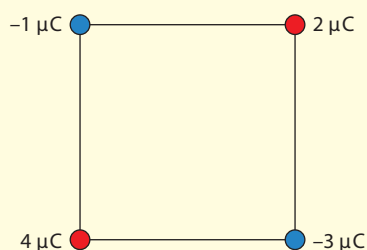


- 8 The diagram shows equipotential surfaces due to two spherical masses.



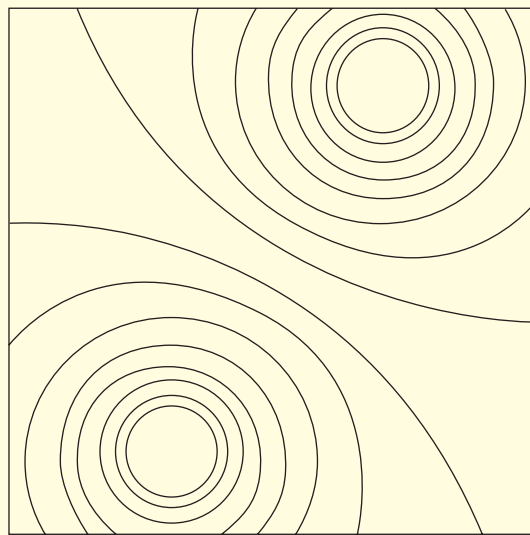
- a Using the diagram, explain how it can be deduced that the masses are unequal.
- b Copy the diagram and draw in the gravitational field lines due to the two masses.
- c Explain why the equipotential surfaces are spherical very far from the two masses.
- 9 a Determine the electric potential at the mid-point of the line joining two equal positive charges q in terms of q , the Coulomb constant and the charge separation d .
- b Repeat a for two equal but opposite charges.
- 10 Two charges, $q_1 = 2.0 \mu\text{C}$ and $q_2 = -4.0 \mu\text{C}$, are 0.30 m apart. Find the electric potential at a point P, which is 0.40 m from q_1 and 0.60 m from q_2 .

- 11 Four equal charges of $5.0\ \mu\text{C}$ are placed at the vertices of a square of side $10\ \text{cm}$.
- Calculate the value of the electric potential at the centre of the square.
 - Determine the electric field at the centre of the square.
 - How do you reconcile your answers to **a** and **b** with the fact that the electric field is the derivative of the potential?
- 12 A charge q of $10.0\ \text{C}$ is placed somewhere in space. What is the work required to bring a charge of $1.0\ \text{mC}$ from a point X, $10.0\ \text{m}$ from q , to a point Y, $2.0\ \text{m}$ from q ? Does the answer depend on which path the charge follows?
- 13 An electron is brought from infinity to a distance of $10.0\ \text{cm}$ from a charge of $-10.0\ \text{C}$. How much work was done on the electron?
- 14 An electron moves from point A where the potential is $100.0\ \text{V}$ to point B where the potential is $200.0\ \text{V}$. The electron started from rest. Calculate the speed of the electron as it passes the point B.
- 15 Four charges are placed at the vertices of a square of side $5.00\ \text{cm}$, as shown in the diagram.



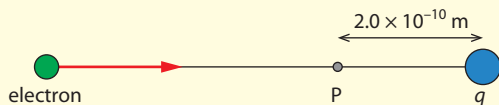
- On a copy of the diagram, show the forces acting on the $2.0\ \mu\text{C}$ charge. Find the magnitude and direction of the net force on the $2.0\ \mu\text{C}$ charge.
- Calculate the value of the electric potential at the centre of the square.
- Determine the work that must be done in order to move a charge of $1\ \text{nC}$ initially at infinity to the centre of the square.

- 16 Two conducting spheres are separated by a distance that is large compared with their radii. The first sphere has a radius of $10.0\ \text{cm}$ and has a charge of $2.00\ \mu\text{C}$ on its surface. The second sphere has a radius of $15.0\ \text{cm}$ and is neutral. The spheres are then connected by a long conducting wire.
- Find the charge on each sphere.
 - Calculate the charge density on each sphere (charge density is the total charge on the sphere divided by the surface area of the sphere).
 - Calculate the electric field on the surface of each sphere.
 - Comment on your result in the light of your answer to part **b**. Why is it stated that the wire is long?
- 17 The diagram shows the equipotential lines for two equal and opposite charges. Draw the electric field lines for these two charges.

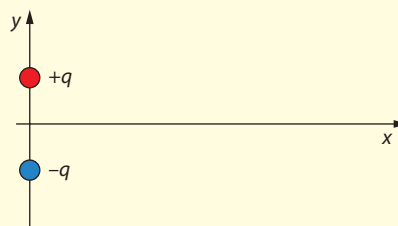


- 18 Two long parallel plates are separated by a distance of $15.0\ \text{cm}$. The bottom plate is kept at a potential of $-250\ \text{V}$ and the top at $+250\ \text{V}$. A charge of $-2.00\ \mu\text{C}$ is placed at a point $3.00\ \text{cm}$ from the bottom plate.
- Find the electric potential energy of the charge.
- The charge is then moved vertically up to a point $3.00\ \text{cm}$ from the top plate.
- What is the electrical potential energy of the charge now?
 - How much work was done on the charge?

- 19 An electron is shot with a speed equal to $1.59 \times 10^6 \text{ m s}^{-1}$ from a point where the electric potential is zero toward an immovable negative charge q (see the diagram).



- Determine the potential at P so that the electron stops momentarily at P and then turns back.
 - Calculate the magnitude of q .
- 20 Two equal and opposite charges are placed at points with coordinates $x=0, y=a$ and $x=0, y=-a$, as shown in the diagram.



- Find the electric field at the point with coordinates $x=d, y=0$.
 - Repeat for two equal negative charges $-q$ on the y -axis.
 - Sketch graphs to show the variation of these fields with the distance d .
- 21 Three protons are initially very far apart. Calculate the work that must be done in order to bring these protons to the vertices of an equilateral triangle of side $5.0 \times 10^{-16} \text{ m}$.

10.2 Fields at work

This section deals with the application of Newton's law of gravitation to the motion of the planets around the Sun and satellites around the Earth.

Orbital motion

A satellite of mass m orbits a planet of mass M with speed v . The radius of the orbit is r (Figure 10.26). The total energy E_T of this system is the sum of the kinetic energy E_K and the gravitational potential energy E_P .

$$E_K = \frac{1}{2}mv^2 \quad \text{and} \quad E_P = -\frac{GMm}{r}$$

So:

$$E_T = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

Note that we do not include any kinetic energy for the planet, as we assume it does not move.

Since the satellite is orbiting in a circle, it must have an acceleration towards the centre of the circle of magnitude $\frac{v^2}{r}$. From Newton's second law of motion, to provide this acceleration there must be a force F on the satellite directed towards the centre of the circle:

$$F = \frac{mv^2}{r}$$

Learning objectives

- Derive an expression for the orbital speed.
- Solve problems on orbital motion, including total energy.
- Use the concept of escape speed.
- Explain weightlessness.
- Understand the inverse square law behaviour of the electric and gravitational force.

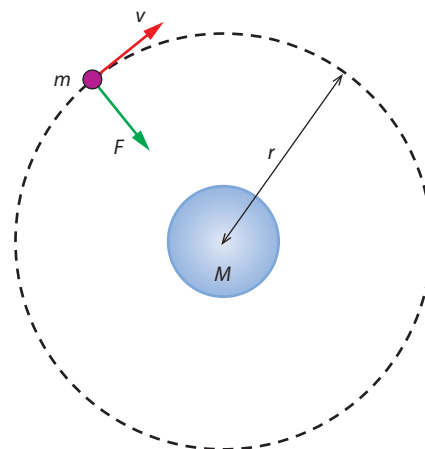


Figure 10.26 A system of a satellite orbiting a planet.

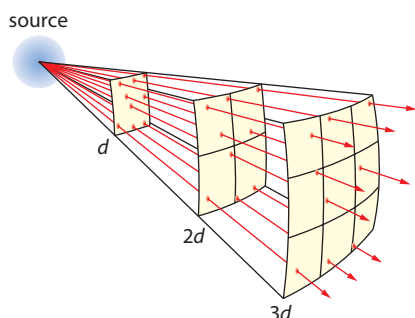


Figure 10.32 As the distance from the source doubles the area presented to the source quadruples. The effect of the source per unit area decreases by a factor of 4.

Why is this so? A purely geometric explanation is that if we have a mass or charge, then its ‘influence’ a distance d away may be thought to be spread uniformly on the surface of a sphere of radius d , as shown in [Figure 10.32](#). Then the ‘influence’ per unit area is:

for a mass, m : $\frac{m}{4\pi d^2}$

for a charge, q : $\frac{q}{4\pi d^2}$

So it looks as if the inverse square law is a consequence of just geometry. But there is something much deeper. The precise inverse square law for electricity and gravitation is related to the fact that the photon and the graviton are particles with strictly zero mass – see [Topic 7](#).

Nature of science

The Newtonian theory of gravity and mechanics applied to the motion of planets gives spectacular agreement with the observed motion of the planets. However, during the 19th century very small discrepancies between theory and observations began to arise. One of them had to do with the motion of the planet Uranus. What could be the cause of these discrepancies? One possibility was that Newtonian gravity was not correct – an unlikely possibility, given the previous successes of the theory. The French astronomer Urbain Le Verrier (1811–1877) argued that a new, unknown planet was affecting the orbit of Uranus and was responsible for the erratic behaviour of its orbit. Working backwards, Le Verrier calculated the possible orbit of the unknown planet that would give rise to the observed effects on Uranus. He communicated his results to the Berlin observatory. His letter arrived at the observatory on 23 September 1846, and on that same evening the planet Neptune was discovered!

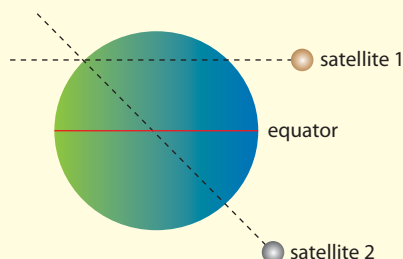
? Test yourself

- 22 **a** Calculate the speed of a satellite that orbits the Earth at a height of 500 km.
- b** Determine the period of revolution of this satellite.
- 23 Show by applying Newton’s law of gravitation and the second law of motion that a satellite (or planet) in a circular orbit of radius R around the Earth (or the Sun) has a period (i.e. time to complete one revolution) given by:
- 24 A satellite that always looks down at the same spot on the Earth’s surface is called a geosynchronous satellite.
- a** Find the distance of this satellite from the surface of the Earth.
- b** Could the satellite be looking down at any point on the surface of the Earth?

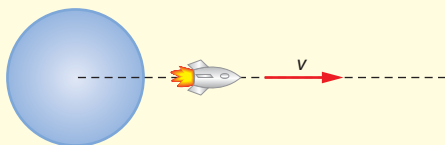
$$T^2 = \frac{4\pi^2 R^3}{GM}$$

where M is the mass of the attracting body (Earth or Sun). This is Kepler’s third law.

- 25 The diagram shows cross-sections of two satellite orbits around the Earth. (To be in orbit means that only gravity is acting on the satellite.) Discuss whether either of these orbits is possible.

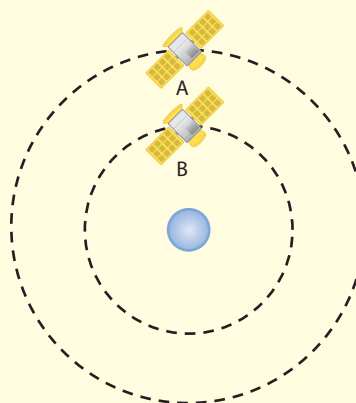


- 26 In the text it was calculated that the acceleration due to gravity at a height of 300 km above the Earth's surface is far from negligible, yet astronauts orbiting in a space shuttle at such a height feel weightless. Explain why.
- 27 A rocket is launched from the surface of a planet. At the position shown in the diagram, the rocket is a distance of $2R$ from the planet (where R is the radius of the planet) and its speed is $v = \sqrt{\frac{GM}{2R}}$. At that point the fuel runs out.



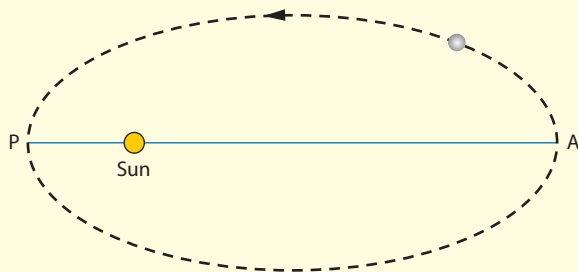
- Explain why the probe will eventually crash onto the surface of the planet.
- Calculate, in terms of R , the maximum distance from the centre of the planet the rocket travels to.
- Determine the speed with which the rocket crashes onto the planet surface.
- Draw a graph to show how the speed of the rocket varies with distance r from the centre of the planet as the rocket begins to fall back towards the planet.

- 28 Prove that the total energy of the Earth (mass m) as it orbits the Sun (mass M) can be expressed as either $E = -\frac{1}{2}mv^2$ or $E = -\frac{GMm}{2r}$ where r is the radius of the Earth's circular orbit. Calculate this energy numerically.
- 29 The diagram shows two identical satellites in circular orbits. Which satellite has the larger:
- kinetic energy
 - potential energy
 - total energy?



- 30 The total energy of a satellite during launch from the Earth's surface is $E = -\frac{GMm}{5R}$, where R is the radius of the Earth.
- Explain why this satellite will not escape the Earth.
 - The satellite eventually settles into a circular orbit. Calculate the radius of the orbit in terms of R .
- 31 A satellite is in a circular orbit around the Earth. The satellite turns on its engines and the satellite now finds itself in a new circular orbit of larger radius. State and explain whether the work done by the engines is positive, zero or negative.

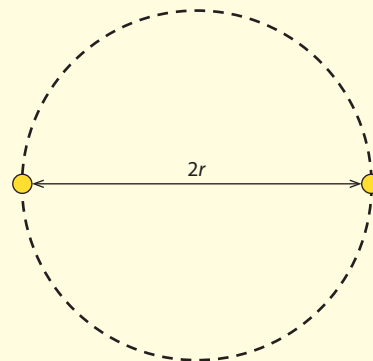
- 32 The diagram shows a planet orbiting the Sun. Explain why at points A and P of the orbit the potential energy of the planet assumes its minimum and maximum values, and determine which is which. Hence determine at what point in the orbit the planet has the greatest speed.



- 33 Show that the escape speed from the surface of a planet of radius R can be written as $v_{\text{esc}} = \sqrt{2gR}$ where g is the gravitational field strength on the planet's surface.

- 34 a Deduce that a satellite orbiting a planet of mass M in a circular orbit of radius r has a period of revolution given by $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$
- b A grazing orbit is one in which the orbit radius is approximately equal to the radius R of the planet. Deduce that the period of revolution in a grazing orbit is given by $T = \sqrt{\frac{3\pi}{G\rho}}$ where ρ is the density of the planet.
- c The period of a grazing orbit around the Earth is 85 minutes and around the planet Jupiter it is 169 minutes. Deduce the ratio $\frac{\rho_{\text{Earth}}}{\rho_{\text{Jupiter}}}$.
- 35 a The acceleration of free fall at the surface of a planet is g and the radius of the planet is R . Deduce that the period of a satellite in a very low orbit is given by $T = 2\pi\sqrt{\frac{R}{g}}$
- b Given that $g = 4.5 \text{ m s}^{-2}$ and $R = 3.4 \times 10^6 \text{ m}$, deduce that the orbital period of the low orbit is about 91 minutes.
- c A spacecraft in orbit around this planet has a period of 140 minutes. Deduce the height of the spacecraft from the surface of the planet.

- 36 Two stars of equal mass M orbit a common centre as shown in the diagram. The radius of the orbit of each star is R . Assume that each of the stars has a mass equal to 1.5 solar masses (solar mass $= 2.0 \times 10^{30} \text{ kg}$) and that the initial separation of the stars is $2.0 \times 10^9 \text{ m}$.



- a State the magnitude of the force on each star in terms of M , R and G .
- b Deduce that the period of revolution of each star is given by the expression:

$$T^2 = \frac{16\pi^2 r^3}{GM}$$

- c Evaluate the period numerically.
- d Show that the total energy of the two stars is given by:

$$E = -\frac{GM^2}{4R}$$

- e The two-star system loses energy as a result of emitting gravitational radiation. Deduce that the stars will move closer to each other.

- f i Explain why the fractional loss of energy per unit time may be calculated from the expression:

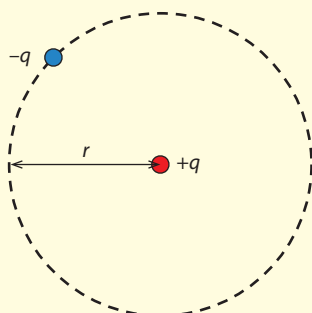
$$\frac{\Delta E/E}{\Delta t} = \frac{3}{2} \frac{\Delta T/T}{\Delta t}$$

where $\frac{\Delta T/T}{\Delta t}$ is the fractional decrease in period per unit time. (Hint: Use ideas of error propagation.)

- ii The orbital period decreases at a rate of $\frac{\Delta T/T}{\Delta t} = 72 \mu\text{s yr}^{-1}$. Estimate the fractional energy loss per year.

- g The two stars will collapse into each other when $\Delta E \approx E$. Estimate the lifetime, in years, of this binary star system.

- 37** A charge $-q$ whose mass is m moves in a circle of radius r around another positive stationary charge q located at the centre of the circle, as shown in the diagram.



- a** Draw the force on the moving charge.
b Show that the velocity of the charge is given by:

$$v^2 = \frac{1}{4\pi\epsilon_0} \frac{q^2}{mr}$$

- c** Show that the total energy of the charge is given by:

$$E = -\frac{1}{8\pi\epsilon_0} \frac{q^2}{r}$$

- d** Hence determine how much energy must be supplied to the charge if it is to orbit around the stationary charge at a radius equal to $2r$.

- 38** An electron of charge $-e$ and mass m orbits the proton in a hydrogen atom as in the previous problem.

- a** Show that the period of revolution of the electron is given by $T^2 = \frac{4\pi^2 m}{ke^2} r^3$ where k is the Coulomb constant and r the radius of the orbit.
b Calculate this period for an orbit radius of 0.5×10^{-10} m.
c Using the results of the previous problem calculate the energy that must be supplied to the electron so it orbits the proton in an orbit of radius 2.0×10^{-10} m.

Exam-style questions

- 1 Two identical solid steel spheres touch. The gravitational force between them is F . The spheres are now replaced by two touching solid steel spheres of double the radius. What is the force between the spheres now?

A $\frac{F}{4}$

B $\frac{F}{16}$

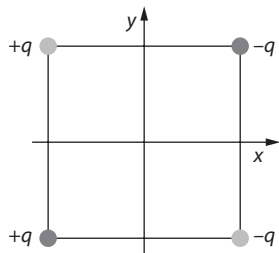
C $4F$

D $16F$

- 2 A planet has double the mass of the Earth and double the radius. The gravitational potential at the surface of the Earth is V and the magnitude of the gravitational field strength is g . The gravitational potential and gravitational field strength on the surface of the planet are:

	Potential	Field
A	V	$\frac{g}{4}$
B	$2V$	$\frac{g}{2}$
C	V	$\frac{g}{2}$
D	$2V$	$\frac{g}{4}$

- 3 Four charges that are equal in magnitude are put at the vertices of a square, as shown in the diagram.



Where is the electric potential zero?

- A** At the origin only.
B Along the x -axis.
C Along the y -axis.
D Along both axes.

- 4 Consider two spherical masses, each of mass M , whose centres are a distance d apart. Which of the following is true at the point midway on the line joining the two centres?

	Potential	Gravitational field strength
A	zero	zero
B	zero	non-zero
C	non-zero	zero
D	non-zero	non-zero

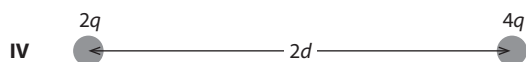
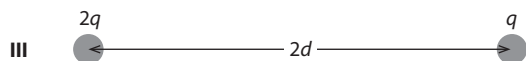
- 5 A probe of mass m is in a circular orbit of radius r around a planet of mass M . The probe is moved to a higher circular orbit of orbit radius $2r$. What is the work done on the probe?

A $-\frac{GMm}{2r}$ B $\frac{GMm}{2r}$ C $\frac{GMm}{4r}$ D $-\frac{GMm}{4r}$

- 6 A positive charge q is placed half way between two long parallel plates that are separated by a distance $2d$. The charge on one of the plates is Q and the charge on the other plate is $-Q$. The potential difference between the plates is V . What is the magnitude of the force on the charge q ?

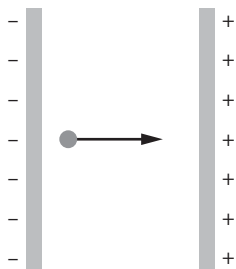
A $k\frac{Qq}{d^2}$ B $k\frac{2Qq}{d^2}$ C $\frac{qV}{2d}$ D $\frac{qV}{d}$

- 7 Shown are four arrangements of two unequal positive point charges separated by various distances. Which **two** arrangements result in the same electric potential energy?

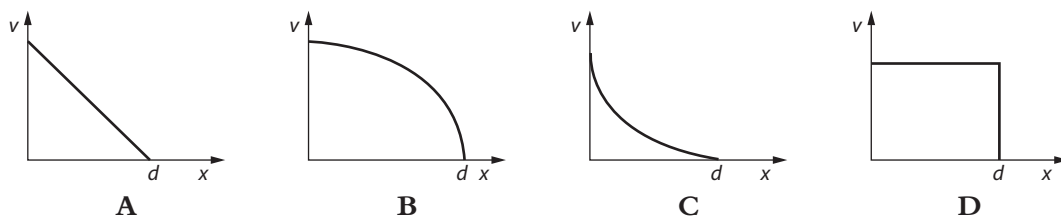


- A I and II B II and IV C III and IV D I and III

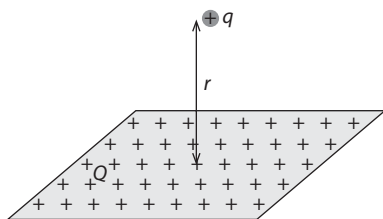
- 8 The figure shows two oppositely charged parallel plates a distance d apart.



A proton is launched from the negative plate with initial speed u . The proton just reaches the positive plate. Which graph represents the variation of the speed v of the proton with distance x from the negative plate?



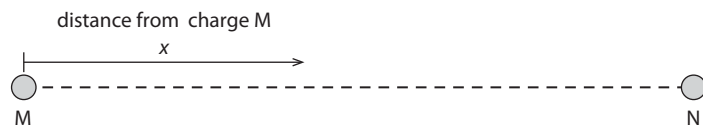
- 9 An amount of positive charge Q is placed uniformly on a large plane surface. A small positive point charge q placed a perpendicular distance r from the surface experiences a force F .



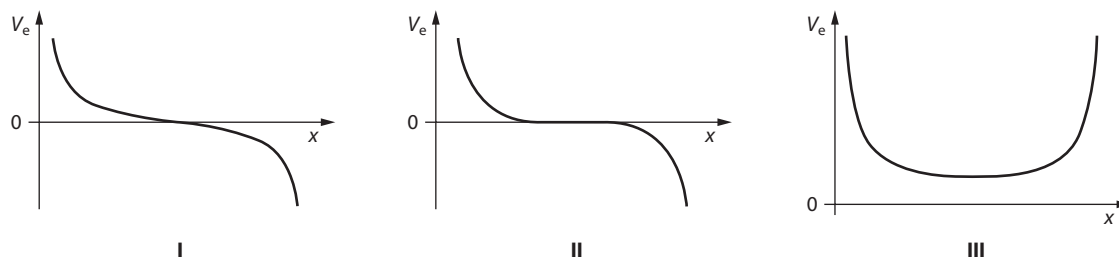
What is the magnitude of the electric field at the position of the small charge q ?

- A $\frac{kQ}{r^2}$ B $\frac{F}{Q}$ C $\frac{F}{q}$ D $\frac{kQ}{r}$

- 10 Two charges M and N are separated by a certain distance, as shown in the diagram.

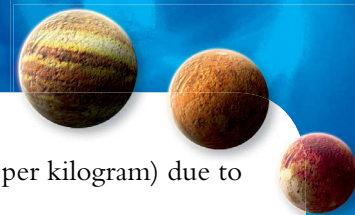


Graphs I, II and III show the variation of electric potential V_e with distance x from the centre of charge M.

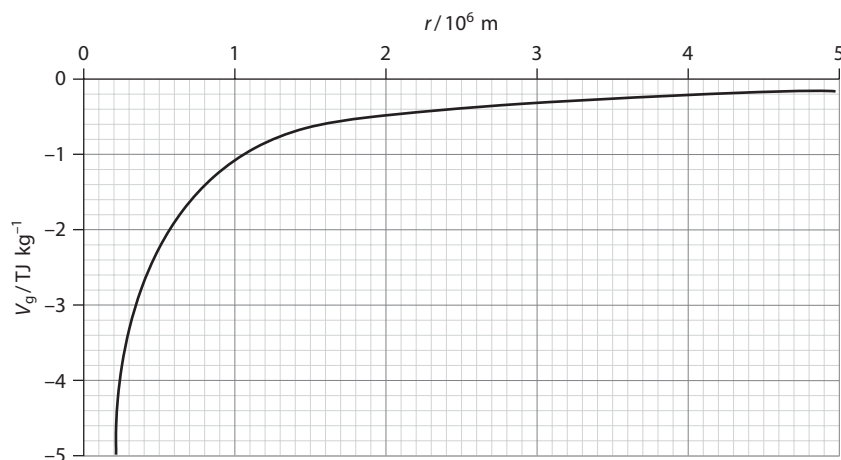


Which of the possibilities below applies to the case of two equal and opposite charges?

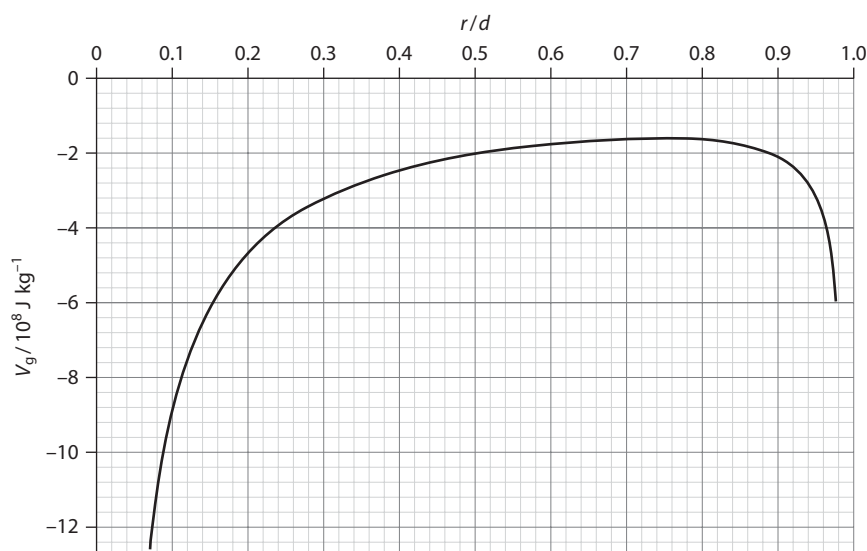
- A I only B II only C III only D I, II or III



- 11 The graph shows the variation with distance r of the gravitational potential V_g (in terajoules per kilogram) due to a planet of radius 2.0×10^5 m.



- Calculate the mass of the planet. [2]
 - Show that the escape speed from the surface of the planet may be written as $v_{\text{esc}} = \sqrt{-2V}$, where V is the gravitational potential on the planet's surface. [3]
 - Use the graph to determine the escape speed from this planet. [2]
 - Calculate how much energy is required to move a rocket of mass 1500 kg from the surface of the planet to a distance of 1.0×10^6 m from the centre. [2]
 - Determine the additional energy required to put the rocket in orbit at the distance in part d. [2]
 - A probe is released from rest at a distance from the planet's centre of 0.50×10^6 m and allowed to crash onto the planet's surface. Determine the speed with which the probe hits the surface. [2]
- 12 The graph shows the variation with distance r from the centre of a planet of the combined gravitational potential V_g due to the planet (of mass M) and its moon (of mass m) along the line joining the planet and the moon. The horizontal axis is labelled $\frac{r}{d}$, where d is the centre-to-centre separation of the planet and the moon.



- a The distance d is equal to 4.8×10^8 m. Use the graph to calculate the magnitude of the gravitational field strength at the point where $\frac{r}{d} = 0.20$. [3]
- b Explain the physical significance of the point where $\frac{r}{d} = 0.75$. [2]
- c Using the graph, calculate the ratio M/m . [3]

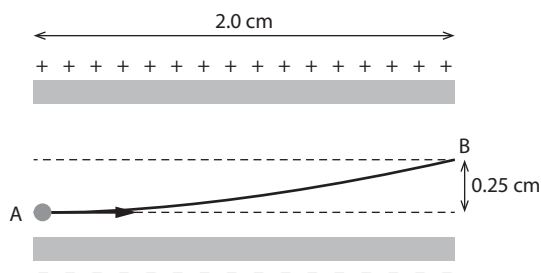
- 13 A sphere of radius 0.25 m has positive charge $+8.8 \mu\text{C}$ uniformly distributed on its surface. A small pellet of mass 0.075 kg and charge $+2.4 \mu\text{C}$ is directed radially at the sphere. When the pellet is at a distance of 0.75 m from the centre of the sphere its speed is 3.2 m s^{-1} .

- a Determine the distance from the centre of the sphere at which the pellet will stop. [3]
- b Describe qualitatively the subsequent motion of the pellet. [2]
- c Determine the speed of the pellet after it moves very far from the sphere. [2]

- 14 An electron is accelerated from rest by a potential difference of 29.1 V.

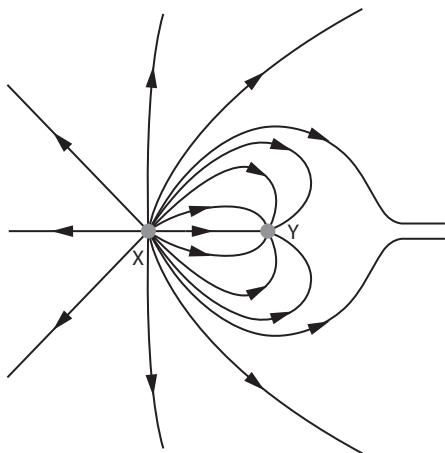
- a Show that the electron acquires a speed of $3.2 \times 10^6 \text{ m s}^{-1}$. [2]

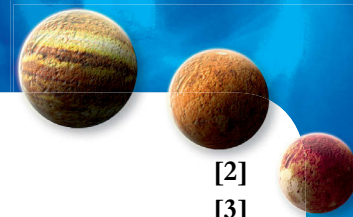
The accelerated electron enters the region between two parallel oppositely charged plates at point A. The electron exits the plates at point B after having moved a vertical distance of 0.25 cm. The length of the plates is 2.0 cm.



- b Calculate the time the electron spends within the plates. [2]
- c Determine the magnitude of the electric field within the plates. [2]
- d Calculate the angle that the velocity of the electron makes with the horizontal at point B. [2]
- e Calculate the work done on the electron from A to B. [2]
- f Using your answer to e, state the potential difference between points A and B. [1]

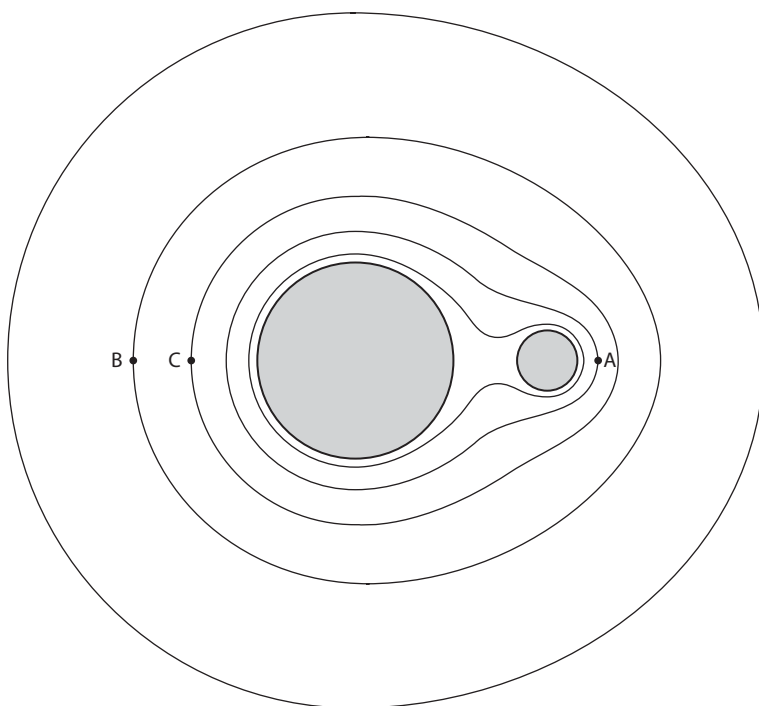
- 15 The diagram shows electric field lines for two point charges X and Y.





- a State what is meant by field lines. [2]
- b State **three** properties of electric field lines. [3]
- c Determine the signs of X and Y. [1]
- d
 - i Copy the diagram. On your copy, indicate where the electric field is zero. [1]
 - ii By making appropriate measurements on the diagram estimate the ratio charge X: charge Y. [2]

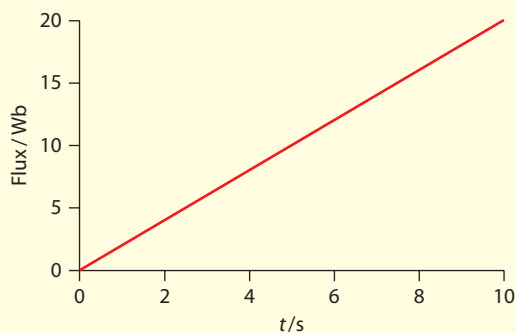
- 16 a State what is meant by an equipotential surface. [1]
- b The diagram shows five equipotential lines around two spherical masses.



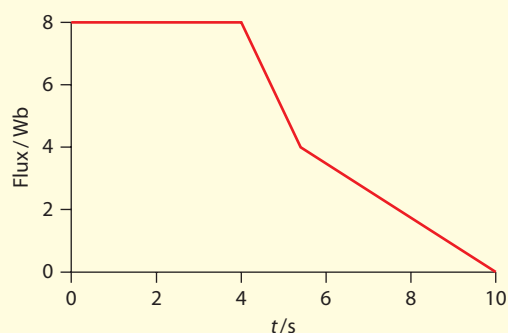
- i On a copy of the diagram, draw lines to represent field lines for this arrangement of masses. You must draw **six** field lines. [2]
 - ii Two consecutive lines are separated by a potential difference of 10^6 J kg^{-1} and the innermost line has potential $-15.0 \times 10^6 \text{ J kg}^{-1}$. Calculate the work done to move a mass of 1500 kg from point A to point B. [2]
 - iii The distance between points B and C is $4.0 \times 10^6 \text{ m}$. Estimate the average gravitational field strength between B and C. [2]
 - iv The equipotential lines tend to become circular as the distance from the sources increases. Explain this observation. [2]
- c State and explain whether the pattern for equipotential lines shown above could also apply to electric charges. [3]

? Test yourself

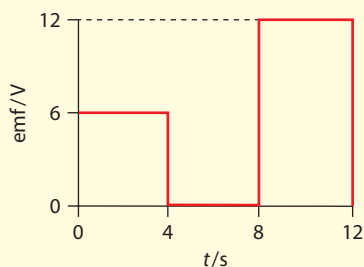
- 1 The flux through a loop as a function of time is given by the graph in the diagram. Sketch a graph of the emf induced in the loop as a function of time.



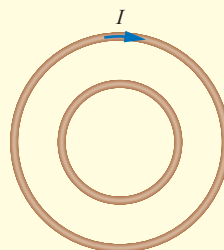
- 2 The flux through a loop as a function of time is given by the graph. Sketch a graph of the emf induced in the loop as a function of time.



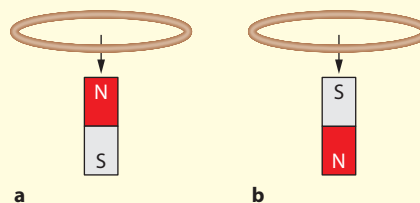
- 3 The graph shows the emf induced in a loop as a result of a changing flux in the loop.
- Sketch a possible flux versus time graph that would give rise to such an emf.
 - Explain why there isn't a unique answer.



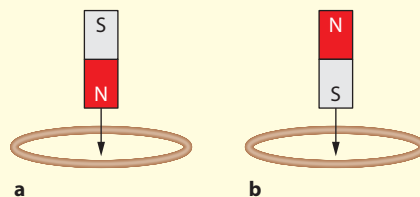
- 4 The diagram shows a top view of two solenoids with their axes parallel, one with a smaller diameter so that it fits inside the other. The bigger solenoid has a current flowing in the clockwise direction (looked at from above) and the current is increasing in magnitude; find the direction of the induced current in the smaller solenoid.



- 5 A metallic ring is dropped from a height above a bar magnet as shown in the diagram. Determine the direction of the induced current in the ring as the ring falls over the magnet in each case, giving full explanations for your choices.

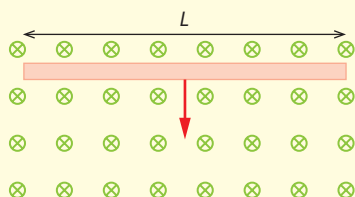


- 6 A magnet is dropped from above into a metallic ring as shown in the diagram. Determine the direction of the current induced in the ring in each case.

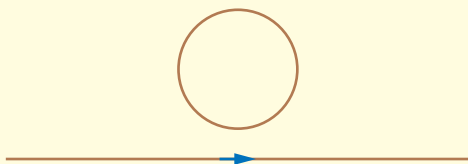


- 7 For the diagram in question 5a, determine the direction of the magnetic force on the ring as it **a** enters and **b** leaves the magnetic field.

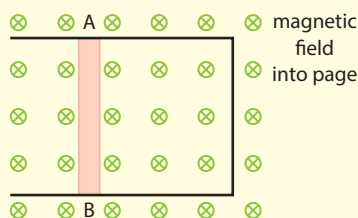
- 8 A metallic rod of length L is dragged with constant velocity v in a region of magnetic field directed into the page (shaded region), as shown in the diagram. By considering the force on electrons inside the rod, show that the ends of the rod will become oppositely charged. Determine the end that is positively charged.



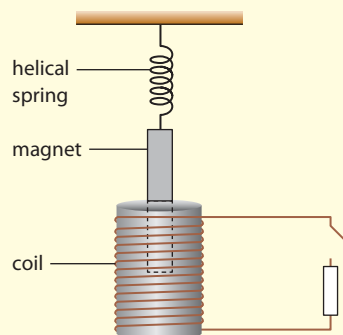
- 9 Find the direction of the current in the loop shown in the diagram as the current in the straight wire:
- increases
 - decreases.



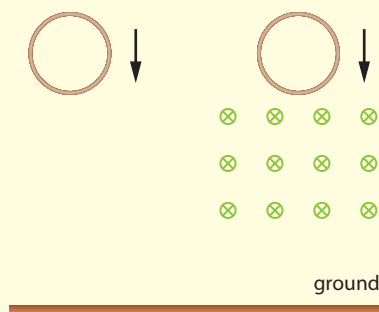
- 10 A large coil has a smaller coil inserted inside it so that their axes are parallel. The smaller coil has 200 turns and a diameter of 2.0 cm. A changing current in the large coil causes the magnetic field to be increasing at a rate of 0.45 T s^{-1} . Calculate the emf induced in the smaller coil.
- 11 Look at the diagram. The rod AB is free to move. The magnetic field is increasing. Determine what will happen to the rod AB.



- 12 A magnet is attached to a spring. The magnet oscillates in and out of a coil, as shown in the diagram.
- Draw a sketch graph to show the variation with time of the displacement of the magnet when **i** the switch is open and **ii** the switch is closed.
 - Explain your sketches in part **a**.

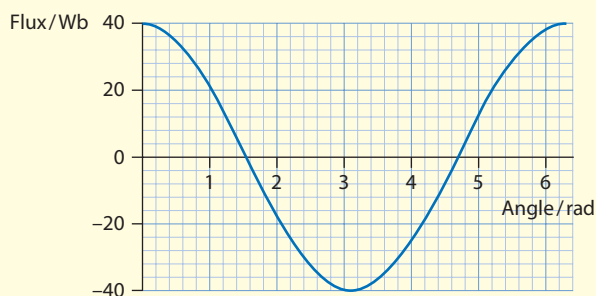


- 13 Two identical rings made out of conducting material are released from rest, from the same height above the ground. One ring will fall through a region of a horizontal magnetic field. State and explain which ring will reach the ground first, given that they are released at the same time.

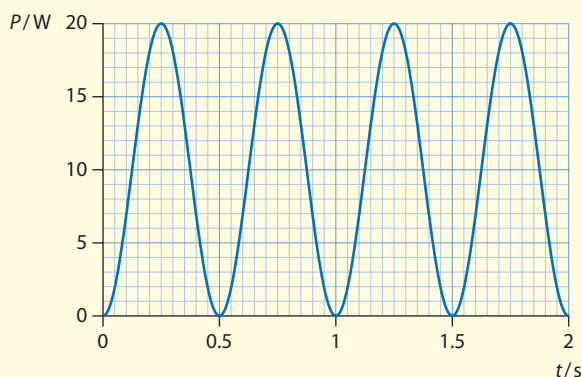


Test yourself

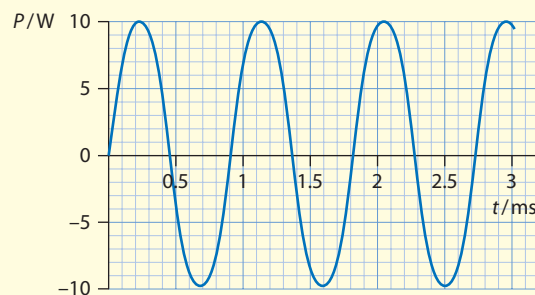
- 14 The graph shows the variation of the flux in a coil as it rotates in a magnetic field with the angle between the magnetic field and the normal to the coil.



- Draw a graph to show the variation of the induced emf with angle.
The same coil is now rotated at double the speed in the same magnetic field. Draw graphs to show:
 - the variation of the flux with angle
 - the variation of the induced emf with angle.
- 15 The graph shows the variation with time of the power dissipated in a resistor when an alternating voltage from a generator is established at its ends. Assume that the resistance is constant at $2.5\ \Omega$.
- Find the rms value of the current.
 - Find the rms value of the voltage.
 - Find the period of rotation of the coil.
 - The coil is now rotated at double the speed. Draw a graph to show the variation with time of the power dissipated in the resistor.



- 16 A transformer has 500 turns in its primary coil and 200 in the secondary coil.
- If an ac voltage of 220 V and frequency 50 Hz is established in the primary coil, find the voltage and frequency induced in the secondary coil.
 - If the primary current is 6.0 A, find the current in the secondary coil, assuming an efficiency of 70%.
- 17 A 300 MW power station produces electricity at 80 kV, which is then supplied to consumers along cables of total resistance $5.0\ \Omega$.
- What percentage of the produced power is lost in the cables?
 - What does the percentage become if the electricity is produced at 100 kV?
- 18 The rms voltage output of a generator is 220 V. The coil is a square of side 20.0 cm, has 300 turns of wire and rotates at 50 revolutions per second. What is the magnetic field?
- 19 The graph shows the variation, with time, of the magnetic flux linkage through a loop. What is the rms value of the emf produced in the loop?



- 20 A power station produces 150 kW of power, which is transmitted along cables of total resistance $2.0\ \Omega$. What percentage of the power is lost if it is transmitted at:
- 1000 V
 - 5000 V?
- 21 Calculate the average power dissipated in a $24\ \Omega$ resistor that is connected in series to a source of a.c voltage of peak value 140 V.



- a** The peak voltage is 4.0 V.
- b** The period is 20 ms (we look from the peak at 5 ms to that at 25 ms; the peak at 15 ms is a 'rectified trough') and so the frequency is:

$$f = \frac{1}{20 \times 10^{-3}} = 50 \text{ Hz}$$

- c** The voltage changes by 0.6 V due to the discharge. So:

$$\Delta q = C \Delta V = 12 \times 10^{-6} \times 0.6 = 7.2 \mu\text{C}$$

- d** The discharge lasted for 7.0 ms, and so:

$$I = \frac{\Delta q}{\Delta t} = \frac{7.2 \times 10^{-6}}{7.0 \times 10^{-3}} = 1.0 \text{ mA}$$

The average voltage across the resistor is 3.7 V and so an estimate of the resistance is:

$$R = \frac{V}{I} = \frac{3.7}{1.0 \times 10^{-3}} \approx 3.7 \text{ M}\Omega$$

Nature of science

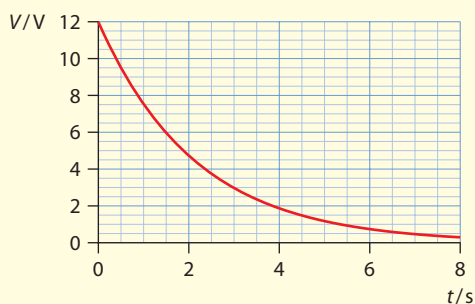
Common formalism

The mathematics of RC circuits follows closely the mathematics of radioactive decay. Therefore the same techniques of analysis that have been used in one area can also be used in the other. This happens countless times in physics – examples include oscillations in mechanics and oscillations in electrical circuits; electrostatics and gravitation; and thermodynamics and the physics of black hole event horizons.

? Test yourself

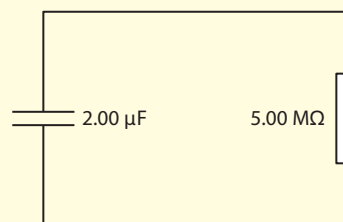
- 22** A parallel plate capacitor in a vacuum has a capacitance of 1.0 F. The plates are separated by a distance of 1 cm. Calculate the area of one of the capacitor plates. Comment on your answer.
- 23** Calculate the charge on one of the plates of a parallel plate capacitor of area 0.25 m^2 . The plates are separated by a distance of 8.0 mm, in a vacuum. The potential difference across the plates is 24 V.
- 24** A $12 \mu\text{F}$ capacitor is charged to a potential difference of 220 V in 15 ms. Estimate the average current needed to charge the capacitor.
- 25** A 9.0 V battery is used to charge a 20 mF capacitor. Calculate:
- a** the charge on the capacitor
 - b** the energy stored in the capacitor.
- The capacitor discharges in a time of 50 ms.
- c** Estimate the power released during the discharge.
- 26** Two capacitors of capacitance $120 \mu\text{F}$ and $240 \mu\text{F}$ are connected in parallel. The two are then connected to a source of potential difference 6.0 V. Calculate:
- a** the total capacitance of the arrangement
 - b** the charge stored on each capacitor
 - c** the energy stored in each capacitor.

- 27 Repeat question 26 where now the capacitors are connected in series.
- 28 A capacitor of capacitance 25 pF is connected to a battery of emf 24 V for a long time. The battery is then removed and the capacitor is connected to an uncharged capacitor of capacitance 75 pF . Calculate:
- the charge on each capacitor
 - the change in total energy stored before and after the battery was disconnected.
 - Comment on the answer to **b**.
- 29 A 250 mF capacitor is charged by a battery of emf 12 V .
- Calculate the energy stored in the capacitor.
 - Estimate the time for which the lamp is lit, listing any assumptions you make.
The capacitor discharges through a lamp rated $12\text{ V}, 6.0\text{ W}$.
- 30 **a** Sketch a graph to show how the potential difference V across a parallel plate capacitor varies with charge q on one of the plates.
b Suggest what the area under the graph represents.
- 31 A capacitor of capacitance $25.0\text{ }\mu\text{F}$ is charged by a battery of emf 48 V . The battery is removed and the capacitor is connected to a resistor of resistance $15\text{ k}\Omega$ through which it discharges. Determine **a** the charge, **b** the current and **c** the voltage after a time of 0.20 s .
- 32 The graph shows how the voltage V on the plate of a capacitor of capacitance $50.0\text{ }\mu\text{F}$ varies with time t as the capacitor discharges through a resistor of resistance R .



- Use the graph to estimate the time constant of the system.
- Calculate the resistance R .

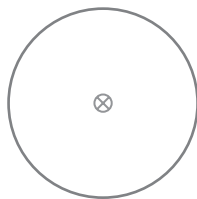
- 33 A capacitor of capacitance $250\text{ }\mu\text{F}$ is charged by a battery of emf 12 V . The battery is removed and the capacitor is connected to a resistor of resistance $75\text{ k}\Omega$ through which it discharges. Determine **a** the charge and **b** the current when the voltage across the capacitor is 6.0 V .
- 34 A capacitor of capacitance $2.00\text{ }\mu\text{F}$ is charged by connecting it to a battery of emf 9.00 V . The capacitor then discharges through a resistor of resistance $5.00\text{ M}\Omega$. Determine at a time of $t = 1.00\text{ s}$:
- the rate at which charge is leaving the capacitor plate
 - the rate at which energy is being dissipated in the resistor
 - the rate at which energy is being lost by the capacitor.



- 35 Refer to Figure 11.47 on page 470. The input voltage is sinusoidal.
- sketch a graph to show how voltage varies with time across
 - diode A
 - diode B
 - suggest how the output can be made even smoother.

Exam-style questions

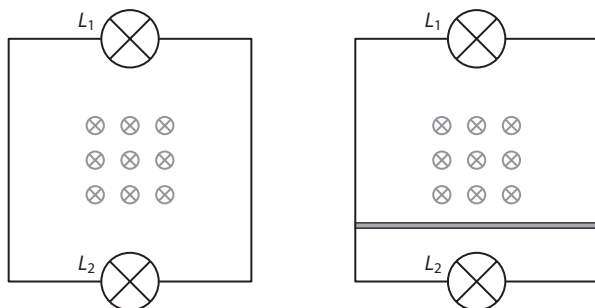
- 1 A magnetic field of uniformly increasing magnitude is directed into the plane of the page as shown. A conducting loop of wire is on the plane of the page.



Which is correct about the direction and magnitude of the induced current in the wire?

	Direction	Magnitude
A	clockwise	constant
B	counter-clockwise	varying
C	clockwise	constant
D	counter-clockwise	varying

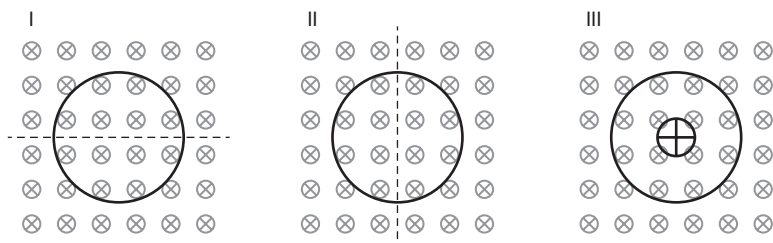
- 2 A loop of wire contains two identical light bulbs, L_1 and L_2 . The region in the loop contains a changing magnetic field whose direction is normal to the plane of the page. Both light bulbs are lit. A copper wire is placed across the loop as shown in the diagram.



What will be the effect of this wire on the brightness of the light bulbs?

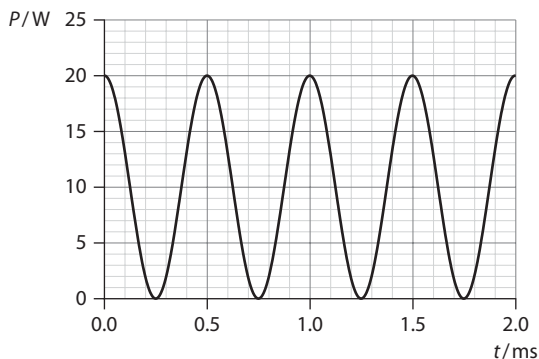
- A** L_1 will go out and L_2 will get dimmer.
- B** L_1 will go out and L_2 will get brighter.
- C** L_2 will go out and L_1 will get brighter.
- D** L_2 will go out and L_1 will get dimmer.

- 3 A conducting loop of wire is in a region of magnetic field directed into the plane of the page. The loop is rotated about axes I, II and III.



In which case or cases will there be an induced current in the loop?

- A** I and II only **B** I only **C** II only **D** I, II and III
- 4 The graph shows the variation with time of the power dissipated in a resistor of resistance $2.0\ \Omega$ in an ac circuit.



What is the rms value of the voltage across the resistor and the period of the current?

	Rms voltage	Period
A	$\sqrt{20}\ \text{V}$	0.5 s
B	$\sqrt{20}\ \text{V}$	1.0 s
C	$\sqrt{40}\ \text{V}$	0.5 s
D	$\sqrt{40}\ \text{V}$	1.0 s

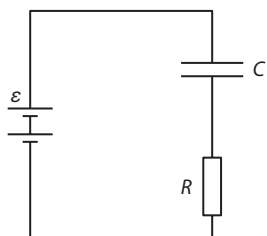
- 5 A parallel plate capacitor is connected to a battery of fixed emf. The energy stored in the capacitor is E and the charge on one of the plates is q . A dielectric is inserted between the plates. Which row in the table gives the correct change(s), if any, in the capacitance and charge stored?

	Capacitance	Charge
A	no change	no change
B	increases	increases
C	decreases	no change
D	no change	increases

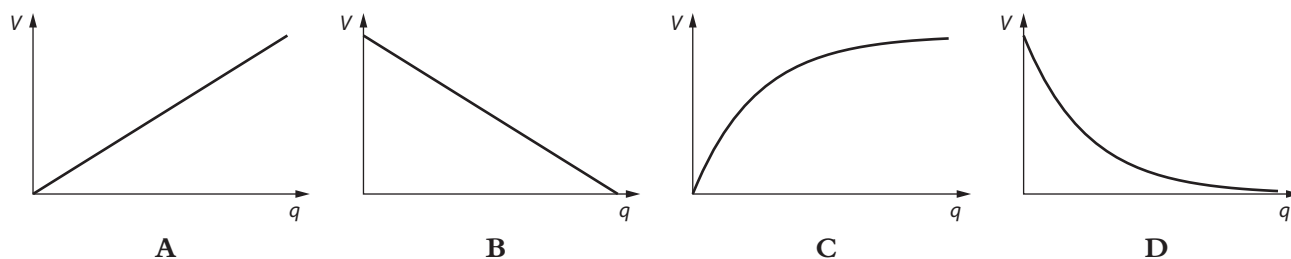
- 6 Capacitor X has capacitance 200 pF and potential difference 100 V. Capacitor Y has capacitance 100 pF and potential difference 200 V. Which row in the table is correct about the energy and charge stored by capacitor Y?

	Energy stored by Y	Charge stored by Y
A	same as X	same as X
B	greater than X	same as X
C	same as X	greater than X
D	greater than X	greater than X

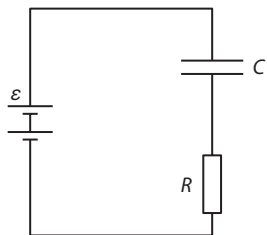
- 7 In the circuit shown a capacitor that is initially uncharged is being charged by a battery.



Which of the following is a correct graph of the variation of the potential difference V across the plates with charge q on one of the capacitor plates?



- 8 What will the initial current be in the circuit below the instant the switch is closed, and what will it be eventually a long time after the switch is closed?



	Initial	Eventual
A	$\frac{\varepsilon}{R}$	0
B	$\frac{\varepsilon}{R}$	$\frac{\varepsilon}{R}$
C	0	0
D	0	$\frac{\varepsilon}{R}$

Nature of science

Quantum physics

The study of emission spectra from flames revealed characteristic lines that could be used to identify different elements. Observations of the spectrum of sunlight showed dark absorption lines. In the 1800s improved instruments allowed for more accurate measurements of the wavelengths of light corresponding to the bright and dark lines in spectra. But what was the reason for these lines? As we saw in [Topic 7](#), a quantum model was needed to explain these patterns. The first of these quantum models was the Bohr model of the hydrogen atom, which could predict the wavelengths of the lines in the spectrum of hydrogen. To explain the puzzling observations seen in the photoelectric effect, Einstein suggested that light existed as packets of energy called photons – a quantum model for light. Accepting the idea of a wave–particle duality revolutionised scientific thinking. Any moving particle could have wave-like characteristics! New ideas opened up new avenues for research, and led to the idea of the wavefunction for electrons in atoms, the uncertainty principle and probability functions. A whole new branch of physics was born – quantum physics.



Test yourself

- 1
 - a Explain what is meant by the **photoelectric effect**.
 - b A photo-surface has a work function of 3.00 eV. Determine the critical frequency.
- 2
 - a What evidence is there for the existence of photons?
 - b A photo-surface has a critical frequency of 2.25×10^{14} Hz. Radiation of frequency 3.87×10^{14} Hz falls on this surface. Deduce the voltage required to stop electrons from being emitted.
- 3 Light of wavelength 5.4×10^{-7} m falls on a photo-surface and causes the emission of electrons of maximum kinetic energy 2.1 eV at a rate of 10^{15} per second. The light is emitted by a 60 W light bulb.
 - a Explain how light causes the emission of electrons.
 - b Calculate the electric current that leaves the photo-surface.
 - c Determine the work function of the surface.
 - d Estimate the maximum kinetic energy of the electrons when the intensity of the light becomes 120 W.
 - e Estimate the current from the photo-surface when the intensity is 120 W.
- 4
 - a State **three** aspects of the photoelectric effect that cannot be explained by the wave theory of light. For each, outline how the photon theory provides an explanation.
 - b Light of wavelength 2.08×10^{-7} m falls on a photo-surface. The stopping voltage is 1.40 V.
 - i Outline what is meant by **stopping voltage**.
 - ii Calculate the largest wavelength of light that will result in emission of electrons from this photo-surface.
- 5
 - a The intensity of the light incident on a photo-surface is doubled while the wavelength of light stays the same. For the emitted electrons, discuss the effect of this, if any, on **i** the energy and **ii** the number.
 - b To determine the work function of a given photo-surface, light of wavelength 2.3×10^{-7} m is directed at the surface and the stopping voltage, V_s , recorded. When light of wavelength 1.8×10^{-7} m is used, the stopping voltage is twice as large as the previous one. Determine the work function.

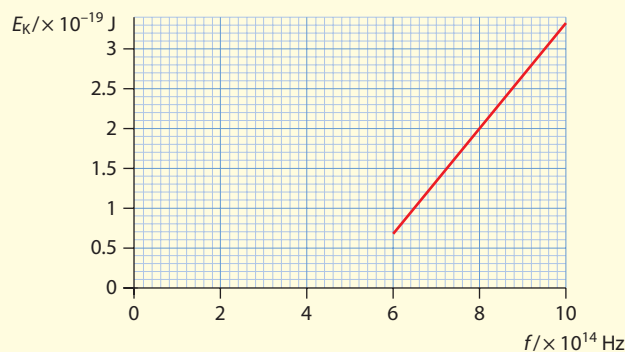


- 6 Light falling on a metallic surface of work function 3.0 eV gives energy to the surface at a rate of $5.0 \times 10^{-4}\text{ W}$ per square metre of the metal's surface. Assume that an electron on the metal surface can absorb energy from an area of about $1.0 \times 10^{-18}\text{ m}^2$.

- Estimate how long it will take the electron to absorb an amount of energy equal to the work function.
- Outline the implication of this.
- Describe how the photon theory of light explains the fact that electrons are emitted almost instantaneously with the incoming photons.

- 7 a From the graph of electron kinetic energy E_K versus frequency of incoming radiation, deduce:
- the critical frequency of the photo-surface
 - the work function.

- What is the kinetic energy of an electron ejected when light of frequency $f = 8.0 \times 10^{14}\text{ Hz}$ falls on the surface?
- Another photo-surface has a critical frequency of $6.0 \times 10^{14}\text{ Hz}$. On a copy of the graph below, sketch the variation with frequency of the emitted electrons' kinetic energy.



- 8 An electron of kinetic energy 11.5 eV collides with a hydrogen atom in its ground state. With what possible kinetic energy can this electron rebound off the atom?
- 9 This question will look at the intensity of radiation in a bit more detail. The intensity of light, I , incident normally on an area A is defined to be $I = \frac{P}{A}$, where P is the power carried by the light.
- Show that $I = \Phi hf$, where Φ is the photon flux density, i.e. the number of photons incident on the surface per second per unit area, and f is the frequency of the light.

- Calculate the intensity of light of wavelength $\lambda = 5.0 \times 10^{-7}\text{ m}$ incident on a surface when the photon flux density is $\Phi = 3.8 \times 10^{18}\text{ m}^{-2}\text{ s}^{-1}$.

- The wavelength of the light is decreased to $\lambda = 4.0 \times 10^{-7}\text{ m}$. Calculate the new photon flux density so that the intensity of light incident on the surface is the same as that found in **b**.

- Hence explain why light of wavelength $\lambda = 4.0 \times 10^{-7}\text{ m}$ and of the same intensity as that of light of wavelength $\lambda = 5.0 \times 10^{-7}\text{ m}$ will result in fewer electrons being emitted from the surface per second.

- State **one** assumption made in reaching this conclusion.

- 10 a What is the evidence for the existence of energy levels in atoms?

- Electrons of kinetic energy **i** 10.10 eV , **ii** 12.80 eV and **iii** 13.25 eV collide with hydrogen atoms and can excite these to higher states. In each case, find the largest n corresponding to the state the atom can be excited to. Assume that the hydrogen atoms are in their ground state initially.

- 11 a What do you understand by the term **ionisation energy**?

- What is the ionisation energy for a hydrogen atom in the state $n = 3$?

- 12 a Find the smallest wavelength that can be emitted in a transition in atomic hydrogen.

- What is the minimum speed an electron must have so that it can ionise an atom of hydrogen in its ground state?

- 13 Consider a brick of mass 0.250 kg moving at 10 m s^{-1} .

- Estimate its de Broglie wavelength.

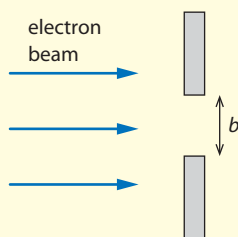
- Comment on whether it makes sense to treat the brick as a wave.

- 14 a Describe an experiment in which the de Broglie wavelength of an electron can be measured directly.

- Determine the speed of an electron whose de Broglie wavelength is equal to that of red light (680 nm).

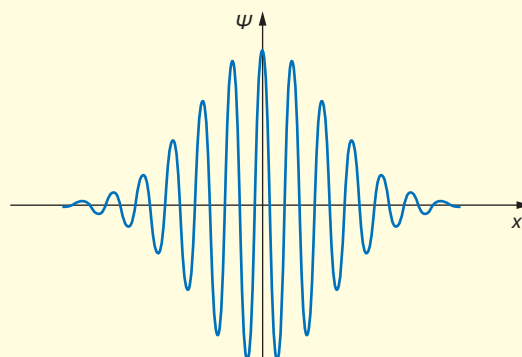
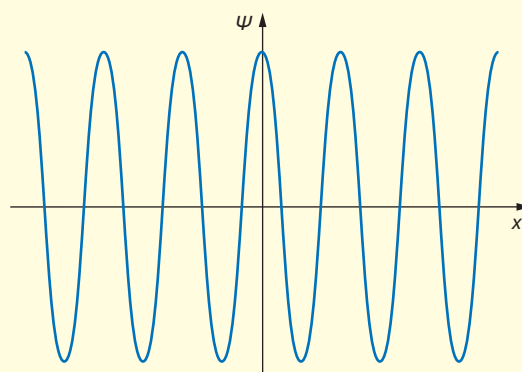
- 15 **a** Show that the de Broglie wavelength of an electron that has been accelerated from rest through a potential difference V is given by:

$$\lambda = \frac{h}{\sqrt{2meV}}$$
- b** Calculate the ratio of the de Broglie wavelength of a proton to that of an alpha particle when both have been accelerated from rest by the same potential difference.
- c** Calculate the de Broglie wavelength of an electron accelerated from rest through a potential difference of 520 V.
- 16 **a** Find the de Broglie wavelength of a proton (mass 1.67×10^{-27} kg) whose kinetic energy is 200.0 MeV.
- b** What is the de Broglie wavelength of an electron in the $n=2$ state of hydrogen?
- 17 Using the uncertainty principle, show that an electron in a hydrogen atom will have a kinetic energy of a few eV.
- 18 **a** State the de Broglie hypothesis.
- b** Calculate the de Broglie wavelength of an electron that has been accelerated by a potential difference of 5.0 V.
- c** Explain why precise knowledge of the wavelength of an electron implies imprecise knowledge of its position.
- 19 An experimenter wishes to make a very narrow beam of electrons. To do that, she suggests the arrangement shown in the diagram. She expects that the beam can be made as narrow as possible by reducing the size b of the aperture through which the electrons will pass.



- a** Explain why in principle it is not possible to make a perfectly narrow beam.
- b** Are her chances of producing a narrow beam better with slow or fast electrons?

- 20 A tennis ball is struck so that it moves with momentum 6.0 N s straight through an open square window of side 1.0 m. Because of the uncertainty principle, the tennis ball may deviate from its original path after going through the window. Estimate the angle of deviation of the path of the tennis ball. Comment on your answer.
- 21 Theoretically it is possible in principle to balance a pencil on its tip so that it stands vertically on a horizontal table. Explain why in quantum theory this is impossible in principle. (You can turn this problem into a good theoretical extended essay if you try to estimate the time the pencil will stay up after it has been momentarily balanced!)
- 22 The graphs represent the wavefunctions of two electrons. Identify the electron with:
- a** the least uncertainty in momentum
- b** the least uncertainty in position.
- Explain your answers.



- 23 Assume that an electron can exist within a nucleus (size 10^{-15} m) such that its associated wave forms a fundamental mode standing wave with nodes at the edges of the nucleus.
- Estimate the wavelength of this electron.
 - Calculate the kinetic energy of the electron in MeV.

- Using your answer in **b**, comment on whether the electron emitted in beta-minus decay could have existed within the nucleus before the decay.

12.2 Nuclear physics

In this section we will examine ideas we met earlier in some more detail. These include Rutherford scattering (and more importantly, deviations from it), nuclear energy levels, the neutrino and the mathematics of radioactive decay.

Rutherford scattering

In scattering experiments such as Rutherford's (see [Section 7.3](#)), simple energy considerations can be used to calculate the **distance of closest approach** of the incoming particle to the target. Consider, as an example, an alpha particle (of charge $q = 2e$) that is projected head-on toward a stationary nucleus of charge $Q = Ze$ ([Figure 12.22](#)).

Initially the system has a total energy consisting of the alpha particle's kinetic energy $E = E_K$. We take the separation of the alpha particle and the nucleus to be large so no potential energy exists. At the point of closest approach, a distance d from the centre of the nucleus, the alpha particle stops and is about to turn back. Thus, the total energy now is the electric potential energy of the alpha and the nucleus, given by:

$$E = k \frac{Qq}{d}$$

$$E = k \frac{(2e)(Ze)}{d}$$

$$E = k \frac{2Ze^2}{d}$$

(We are assuming that the nucleus does not recoil, so its kinetic energy is ignored.)

Then, by conservation of energy:

$$E_K = k \frac{2Ze^2}{d}$$

$$\Rightarrow d = k \frac{2Ze^2}{E_K}$$

Learning objectives

- Understand how the Rutherford scattering experiment led to the idea of the nucleus.
- Discuss how scattering experiments may be used to determine nuclear radii.
- Understand the nature of nuclear energy levels.
- Understand the nature of the neutrino.
- Solve problems involving the radioactive decay law.

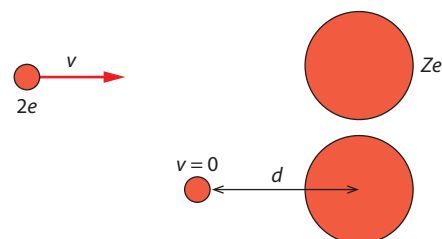


Figure 12.22 The closest approach of an alpha particle happens in a head-on collision.

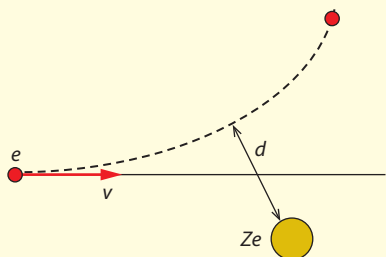
the number of neutrinos was about one-third of what the theoretical calculation predicted. This created the ‘solar neutrino problem’. There were three ways out: either the Davis experiment was wrong or the theory was wrong, or there was new physics involved.

Happily, it turned out that it was the last possibility that actually was in play. The number of neutrinos predicted by theory was based on the assumption that the neutrino was massless. If the neutrino had mass, then the theory would have to be modified because in that case ‘neutrino oscillations’ would take place. This is a rare quantum phenomenon, in which the three types of neutrinos could turn into each other. The Davis and subsequent experiments all measured electron neutrinos. Much later, when advances in instrumentation and computing power allowed experiments to detect all three types of neutrinos, the number was in agreement with the theory. But the neutrinos produced in the Sun were only electron neutrinos!

By this time, experiments in Japan and elsewhere provided convincing evidence that neutrinos had a tiny mass. So, because of neutrino oscillations, by the time the electron neutrinos reached Earth some of them had turned into muon neutrinos and some into tau neutrinos. On the average, about one-third would be electron neutrinos, in agreement with Davis’s results! Ray Davis shared the 2002 Nobel prize in Physics.

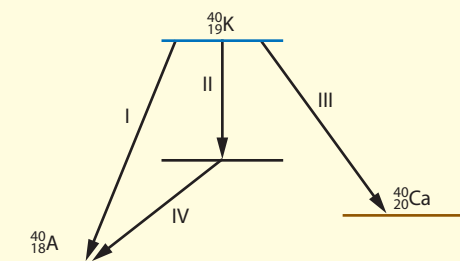
? Test yourself

- 24 An alpha particle is fired head-on at a stationary gold nucleus from far away. Calculate the initial speed of the particle so that the distance of closest approach is 8.5×10^{-15} m. (Take the mass of the alpha particle to be 6.64×10^{-27} kg.)
- 25 A particle of mass m and charge e is directed from very far away toward a massive ($M \gg m$) object of charge $+Ze$ with a velocity v , as shown in the diagram. The distance of closest approach is d . Sketch (on the same axes) a graph to show the variation with separation of:
- the particle’s kinetic energy
 - the particle’s electric potential energy.



- 26 a Deviations from Rutherford scattering are expected when the alpha particles reach large energies. Suggest an explanation for this observation.
- b Some alpha particles are directed at a thin foil of gold ($Z=79$) and some others at a thin foil of aluminium ($Z=13$). Initially, all alpha particles have the same energy. This energy is gradually increased. Predict in which case deviations from Rutherford scattering will first be observed.
- 27 Show that the nuclear density is the same for all nuclei. (Take the masses of the proton and neutron to be the same.)
- 28 a State the evidence in support of nuclear energy levels.
- Radium’s first excited nuclear level is 0.0678 MeV above the ground state.
- b Write down the reaction that takes place when radium decays from the first excited state to the ground state.
- c Find the wavelength of the photon emitted.

- 29 Plutonium ($^{242}_{94}\text{Pu}$) decays into uranium ($^{238}_{92}\text{U}$) by alpha decay. The energy of the alpha particles takes four distinct values: 4.90 MeV, 4.86 MeV, 4.76 MeV and 4.60 MeV. In all cases a gamma ray photon is also emitted except when the alpha energy is 4.90 MeV. Use this information to suggest a possible nuclear energy level diagram for uranium.
- 30 The diagram shows a few nuclear energy levels for $^{40}_{18}\text{Ar}$, $^{40}_{19}\text{K}$ and $^{40}_{20}\text{Ca}$.



Identify the **four** indicated transitions.

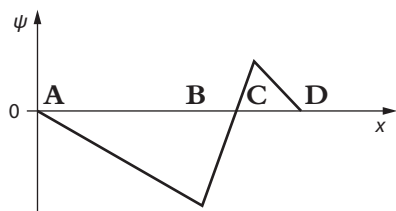
- 31 **a** Find the decay constant for krypton-92, whose half-life is 3.00 s.
b Suppose that you start with $\frac{1}{100}$ mol of krypton. Estimate how many undecayed atoms of krypton there are after **i** 1 s, **ii** 2 s, **iii** 3 s.
- 32 **a** State the probability that a radioactive nucleus will decay during a time interval equal to a half-life.
b Calculate the probability that it will have decayed after the passage of three half-lives.
c A nucleus has not decayed after the passage of four half-lives. State the probability it will decay during the next half-life.
- 33 Estimate the activity of 1.0 g of radium-226 (molar mass = $226.025 \text{ g mol}^{-1}$). The half-life of radium-226 is 1600 yr.
- 34 The half-life of an unstable element is 12 days. Find the activity of a given sample of this element after 20 days, given that the initial activity was 3.5 MBq.
- 35 A radioactive isotope of half-life 6.0 days used in medicine is prepared 24 h prior to being administered to a patient. The activity must be 0.50 MBq when the patient receives the isotope. Estimate the number of atoms of the isotope that should be prepared.
- 36 The age of very old rocks can be found from uranium dating. Uranium is suitable because of its very long half-life: 4.5×10^9 yr. The final stable product in the decay series of uranium-238 is lead-206. Find the age of rocks that are measured to have a ratio of lead to uranium atoms of 0.80. You must assume that no lead was present in the rocks other than that due to uranium decaying.
- 37 The isotope $^{40}_{19}\text{K}$ of potassium is unstable, with a half-life of 1.37×10^9 yr. It decays into the stable isotope $^{40}_{18}\text{Ar}$. Moon rocks were found to contain a ratio of potassium to argon atoms of 1 : 7. Find the age of the Moon rocks.
- 38 Two unstable isotopes are present in equal numbers (initially). Isotope A has a half-life of 4 min and isotope B has a half-life of 3 min. Calculate the ratio of the activity of A to that of B after: **a** 0 min, **b** 4 min, **c** 12 min.
- 39 A sample contains two unstable isotopes. A counter placed near it is used to record the decays. Discuss how you would determine each of the half-lives of the isotopes from the data.
- 40 The half-life of an isotope with a very long half-life cannot be measured by observing its activity as a function of time, since the variation in activity over any reasonable time interval would be too small to be observed. Let m be the mass in grams of a given isotope of long half-life.
a Show that the number of nuclei present in this quantity is $N_0 = \frac{m}{\mu} N_A$ where μ is the molar mass of the isotope in g mol^{-1} and N_A is the Avogadro constant.
b From $A = -\frac{dN}{dt} = N_0 \lambda e^{-\lambda t}$, show that the initial activity is $A_0 = \frac{m N_A}{\mu} \lambda$ and hence that the half-life can be determined by measuring the initial activity (in Bq) and the mass of the sample (in grams).

Exam-style questions

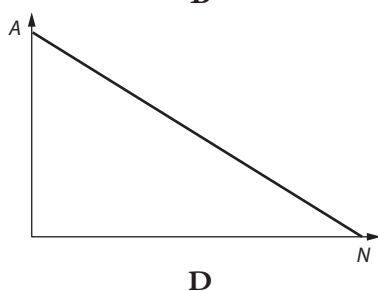
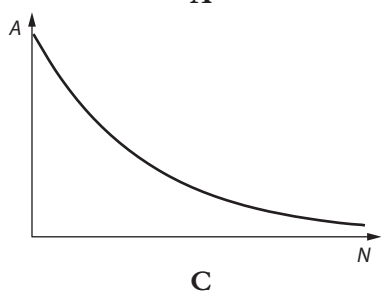
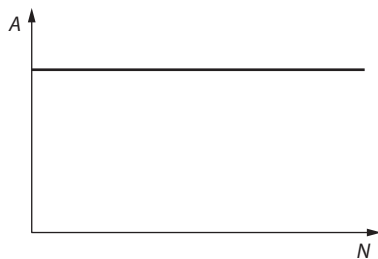
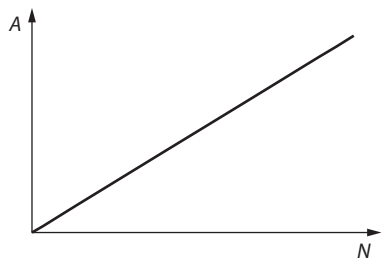
- Light of wavelength λ is incident on a surface. The stopping voltage is V_s . The wavelength of the light is halved. What is the stopping voltage now?
A V_s **B** between V_s and $2V_s$ **C** $2V_s$ **D** greater than $2V_s$
- Light of wavelength λ and intensity I is incident on a metallic surface and electrons are emitted. Which of the following change will, by itself, result in a greater number of electrons being emitted?
A increase λ **B** decrease λ **C** increase I **D** decrease I
- The absorption spectra of hydrogen atoms at a low temperature and at a high temperature are compared. What is the result of such a comparison?
A The spectra are identical.
B The spectra are identical but the high temperature spectrum is more prominent.
C There are more lines in the absorption spectrum at low temperature.
D There are more lines in the absorption spectrum at high temperature.
- An electron orbiting a proton in the $n=2$ state of hydrogen has speed v . The electron is put in the $n=3$ state of hydrogen. What is the speed of the electron in the new orbit?
A $\frac{v}{9}$ **B** $\frac{v}{2}$ **C** $\frac{3v}{2}$ **D** $\frac{2v}{3}$
- A particle of mass m is confined within a region of linear size L . Which is an order of magnitude **estimate** of the particle's kinetic energy?
A $\frac{mh^2}{L^2}$ **B** $\frac{h^2}{mL^2}$ **C** $\frac{mL^2}{h}$ **D** $\frac{L^2}{mh^2}$
- Two beams of electrons of the same energy approach two potential barriers of the same height. One barrier has width w and the other width $2w$. Which is a correct comparison of the tunnelling probability and the de Broglie wavelength of the transmitted electrons in the two beams?

	Probability	Wavelength
A	same	same
B	same	different
C	different	same
D	different	different

- 7 The graph shows the wavefunction of a particle. Near which position is the particle most likely to be found?



- 8 The activity of a pure radioactive sample is A when the number of nuclei present in the sample is N . Which graph shows the variation of A with N ?



- 9 Alpha particles of energy E are directed at a thin metallic foil made out of atoms of atomic number Z . For what values of E and Z are deviations from Rutherford scattering likely to be observed?

	E	Z
A	low	low
B	high	low
C	low	high
D	high	high

- 10 Which is evidence for the existence of nuclear energy levels?

- A** The short range of the nuclear force.
- B** The energies of alpha and gamma particles in radioactive decay.
- C** The energies of beta particles in radioactive decay.
- D** The emission spectra of gases at low pressure.

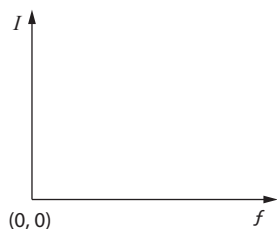
- 11 a In a photoelectric effect experiment a constant number of photons is incident on a photo-surface.

i Outline what is meant by **photons**.

[2]

- ii On a copy of the axes below, sketch a graph to show the variation of the electric current I that leaves the photo-surface, with photon frequency f .

[2]



iii Explain the features of the graph you drew in ii.

[2]

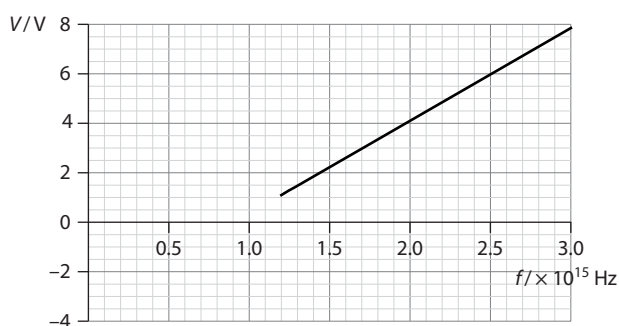
- b i State **one** feature of the photoelectric effect that cannot be explained by the wave theory of light.

[1]

ii Describe how the feature stated in i is explained by the photon theory of light.

[2]

- c In another experiment, a source of constant intensity and variable frequency f is incident on a metallic surface. The graph shows the variation of the stopping potential V with photon frequency f , for a particular value of intensity.



Use the graph to estimate:

i the work function of the metallic surface

[2]

ii the Planck constant obtained from this experiment

[3]

iii the longest wavelength of light that will result in electron being emitted from the surface.

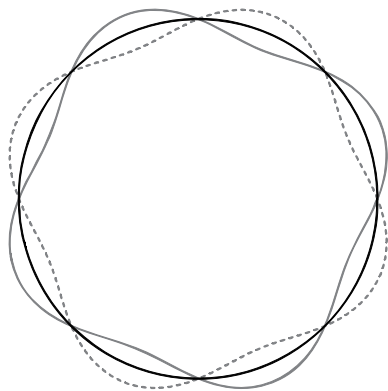
[2]

- d The intensity of the source in c is doubled. Discuss how the graph in c will change, if at all.

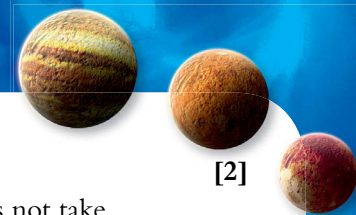
[2]

- 12 In a hydrogen atom an electron of mass m orbits the proton with speed v in a circular orbit of radius r .
- By writing down an expression for the net force on the electron, deduce that $v^2 = \frac{ke^2}{mr}$, where k is the Coulomb constant. [2]
 - Using the Bohr condition show that $r = \frac{h^2}{4\pi ke^2 m} \times n^2$. [3]
 - Hence deduce that the total energy of the electron is given by $E = -\frac{1}{2} \frac{ke^2}{r}$. [2]
 - State the significance of the negative sign in the total energy. [1]
 - Demonstrate that the Bohr condition is equivalent to $2\pi r = n\lambda$ where λ is the de Broglie wavelength of the electron. [2]

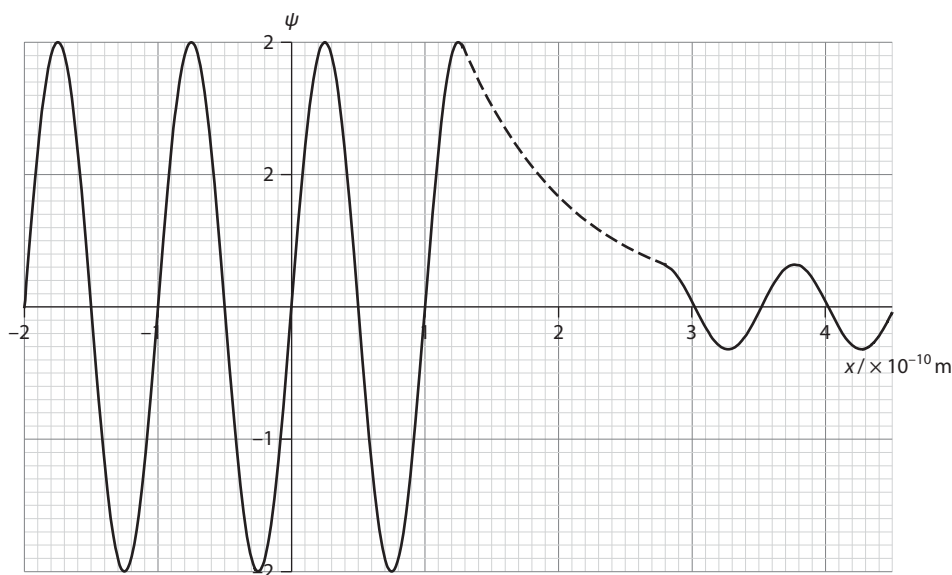
The diagram shows an electron wave in hydrogen.



- State what is meant by an **electron wave**. [1]
 - Determine the radius of the circular orbit of this electron. [3]
 - Predict the energy that must be supplied for this electron to become free. [1]
 - According to the Bohr model, the electron in the hydrogen atom in **f** has a well-defined circular orbit radius. Discuss, by reference to the wave-like properties of the electron, why this is not quite correct in quantum theory. [3]
- 13
- Outline what is meant by the **de Broglie hypothesis**. [2]
 - Show that the de Broglie wavelength of an electron that has been accelerated from rest by a potential difference V is given by $\lambda = \frac{h}{\sqrt{2mqV}}$. [2]
 - Calculate the de Broglie wavelength of an electron that has been accelerated from rest by a potential difference of 120 V.
 - Outline an experiment in which the de Broglie hypothesis is tested. [3]
 - A bullet of mass 0.080 kg leaves a gun with speed 420 m s^{-1} . The gun is in perfect condition and has been fired by an expert marksman. The bullet must pass through a circular hole of diameter 5.0 cm on its way to its target. A student says that the bullet will miss its mark because of de Broglie's hypothesis. By suitable calculations determine whether the student is correct. [4]



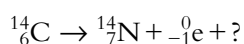
- 14 a State what is meant by **tunnelling**. [2]
 b The graph shows the wavefunction of electrons that undergo tunnelling. (The graph does not take reflected electrons into account.)
 The values on the vertical axis are arbitrary.



Use the graph to determine:

- i the width of the barrier [1]
 ii the ratio of the kinetic energy of the electrons after tunnelling to the kinetic energy before tunnelling [2]
 iii the fraction of the incident electrons that tunnel through the barrier. [2]
 c Outline how your answer to **b iii** would change, if at all, if protons with the same energy were directed at the barrier. [1]

- 15 Carbon-14 is unstable and decays to nitrogen by beta minus emission according to the reaction equation:



- a State, for the missing particle in the reaction equation:
 i its name [1]
 ii **two** of its properties. [2]
 b Outline the evidence that made the presence of this particle in beta decay necessary. [2]

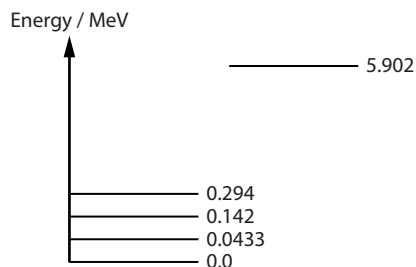
In a living tree, the ratio of carbon-14 to carbon-12 atoms is constant at 1.3×10^{-12} .

- c Suggest why this ratio will decrease after the tree dies. [2]

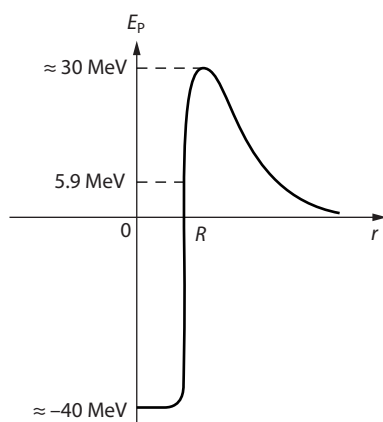
A 15 g piece of charcoal is found in an archaeological site.

- d Calculate the number of atoms of carbon-12 in the piece of charcoal. [2]
 e The piece of charcoal has an activity of 1.40 Bq. Deduce that the ratio of carbon-14 to carbon-12 atoms in the charcoal is 4.85×10^{-13} . [3]
 f The half-life of ${}^{14}_6\text{C}$ is 5730 years. Deduce the age of the charcoal. [3]

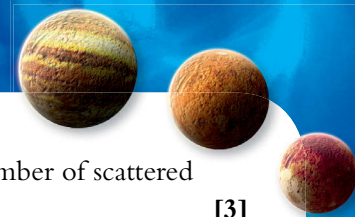
- 16 a Outline the evidence in support of nuclear energy levels. [2]
- b The diagram shows nuclear energy levels for $^{244}_{96}\text{Cm}$ and $^{240}_{94}\text{Pu}$.
- On a copy of the diagram, indicate the alpha decay of $^{244}_{96}\text{Cm}$ into $^{240}_{94}\text{Pu}$ that is followed by the emission of a photon of energy 0.043 MeV. [1]
 - Deduce the energy of the emitted alpha particle. [1]



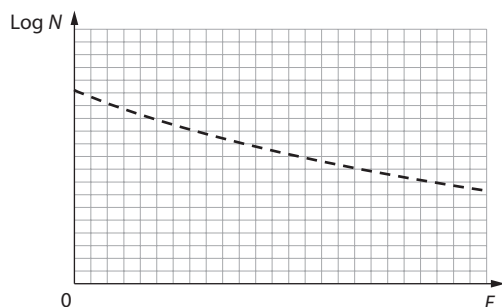
- c The diagram shows the variation of the potential energy of an alpha particle with distance from the nuclear centre. The nuclear radius is R . The total energy of an alpha particle within the nucleus is 5.9 MeV.



- The potential energy for distances $r > R$ is entirely electric potential energy. Suggest why, for these distances, there is no contribution to the potential energy from the strong nuclear force. [2]
 - Explain why the alpha particle cannot leave the nucleus according to the laws of classical mechanics. [2]
 - The alpha particle does in fact leave the nucleus. By reference to the laws of quantum mechanics explain how this is possible. [2]
- d The lifetimes of nuclei decaying by alpha decay can vary from 10^{-7} s to 10^{10} years . Suggest, by reference to the diagram in c and your answer to c iii, what might cause such great variation in lifetime. [2]
- e Use the uncertainty principle to deduce that the kinetic energy of an alpha particle confined within a nucleus is of order 1 MeV. [3]



- 17 a i Neutrons of energy 54 MeV are directed at lead nuclei. A strong minimum in the number of scattered neutrons is observed at a scattering angle of 16° . Estimate the diameter of lead nuclei. [3]
- ii Show that the density of all nuclei is about $2 \times 10^{17} \text{ kg m}^{-3}$. [3]
- b In Rutherford scattering, alpha particles of energy 5.2 MeV are directed head-on at lead nuclei. Estimate the distance of closest approach between the alpha particles and the centre of a lead nucleus. [3]
- c The graph shows how the number of alpha particles that are observed at a fixed scattering angle depends on alpha particle energy according to Rutherford's scattering formula.



- i State **one** assumption the Rutherford scattering formula is based on. [1]
- ii On a copy of the diagram above, indicate deviations from the Rutherford scattering. Explain your answer. [2]