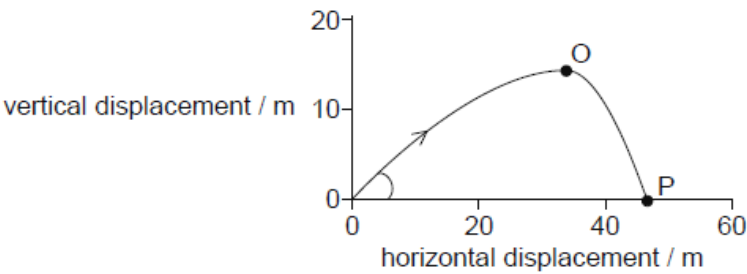


Mechanics motion HL *[169 marks]*

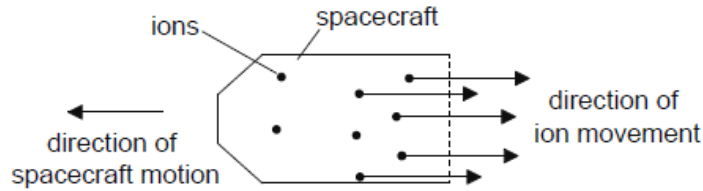
1. A projectile is fired at an angle to the horizontal. The path of the projectile *[1 mark]*
is shown.



Which gives the magnitude of the horizontal component and the magnitude of the vertical component of the velocity of the projectile between O and P?

	Magnitude of horizontal component of velocity	Magnitude of vertical component of velocity
A.	decreases	increases
B.	decreases	constant
C.	constant	increases
D.	constant	constant

Ion-thrust engines can power spacecraft. In this type of engine, ions are created in a chamber and expelled from the spacecraft. The spacecraft is in outer space when the propulsion system is turned on. The spacecraft starts from rest.



The mass of ions ejected each second is $6.6 \times 10^{-6} \text{ kg}$ and the speed of each ion is $5.2 \times 10^4 \text{ m s}^{-1}$. The initial total mass of the spacecraft and its fuel is 740 kg . Assume that the ions travel away from the spacecraft parallel to its direction of motion.

2a. Determine the initial acceleration of the spacecraft.

[2 marks]

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An initial mass of 60 kg of fuel is in the spacecraft for a journey to a planet. Half of the fuel will be required to slow down the spacecraft before arrival at the destination planet.

- 2b. (i) Estimate the maximum speed of the spacecraft. *[3 marks]*
(ii) Outline why the answer to (i) is an estimate.

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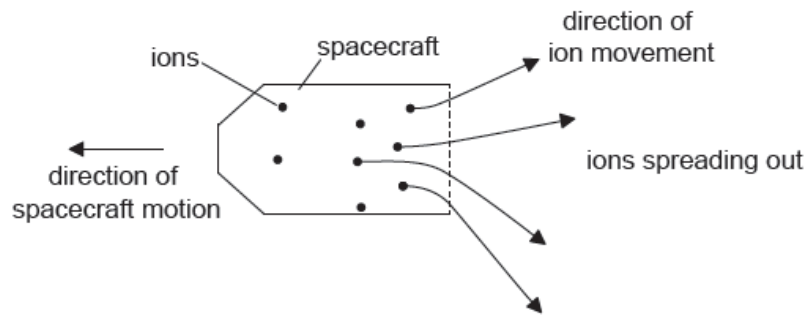
- 2c. Outline why scientists sometimes use estimates in making calculations. *[1 mark]*

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In practice, the ions leave the spacecraft at a range of angles as shown.



2d. Outline why the ions are likely to spread out.

[2 marks]

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2e. Explain what effect, if any, this spreading of the ions has on the acceleration of the spacecraft.

[2 marks]

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An electron with total energy 1.50 MeV collides with a positron at rest. As a result two photons are produced. One photon moves in the same direction as the electron and the other in the opposite direction.

3a. Show that the momentum of the electron is 1.41 MeV c^{-1} .

[1
mark]

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The momenta of the photons produced have magnitudes p_1 and p_2 . A student writes the following correct equations.

$$p_1 - p_2 = 1.41 \text{ MeV c}^{-1}$$

$$p_1 + p_2 = 2.01 \text{ MeV c}^{-1}$$

3b. Explain the origin of each equation.

[2 marks]

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3c. Calculate, in MeV c^{-1} , p_1 and p_2 .

[2 marks]

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A probe launched from a spacecraft moves towards the event horizon of a black hole.

4a. State what is meant by the event horizon of a black hole.

[1
mark]

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4b. The mass of the black hole is $4.0 \times 10^{36} \text{ kg}$. Calculate the Schwarzschild radius of the black hole.

[1
mark]

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- 4c. The probe is stationary above the event horizon of the black hole in (a). [3 marks]
The probe sends a radio pulse every 1.0 seconds (as measured by clocks on the probe). The spacecraft receives the pulses every 2.0 seconds (as measured by clocks on the spacecraft). Determine the distance of the probe from the centre of the black hole.

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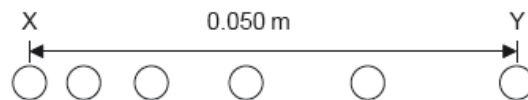
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5. A parachutist of total mass 70 kg is falling vertically through the air at a constant speed of 8 m s^{-1} . [1 mark]

What is the total upward force acting on the parachutist?

- A. 0 N
- B. 70 N
- C. 560 N
- D. 700 N

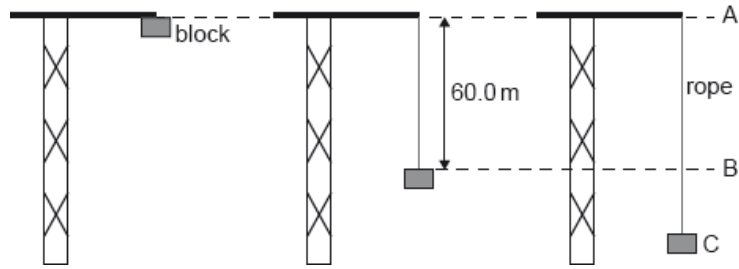
6. A ball starts from rest and moves horizontally. Six positions of the ball are shown at time intervals of 1.0 ms. The horizontal distance between X, the initial position, and Y, the final position, is 0.050 m. [1 mark]



What is the average acceleration of the ball between X and Y?

- A. 2000 m s^{-2}
- B. 4000 m s^{-2}
- C. 5000 m s^{-2}
- D. 8000 m s^{-2}

An elastic climbing rope is tested by fixing one end of the rope to the top of a crane. The other end of the rope is connected to a block which is initially at position A. The block is released from rest. The mass of the rope is negligible.



The unextended length of the rope is 60.0 m. From position A to position B, the block falls freely.

- 7a. At position B the rope starts to extend. Calculate the speed of the block at *[2 marks]* position B.

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At position C the speed of the block reaches zero. The time taken for the block to fall between B and C is 0.759 s. The mass of the block is 80.0 kg.

- 7b. Determine the magnitude of the average resultant force acting on the *[2 marks]* block between B and C.

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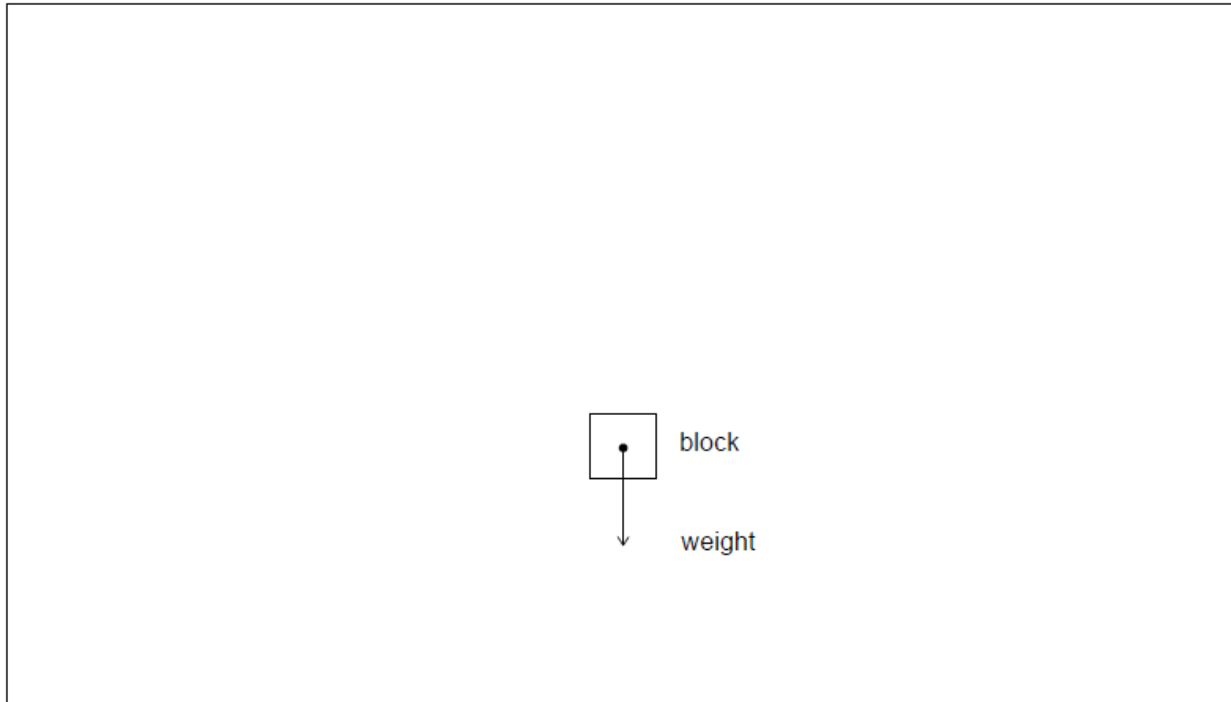
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- 7c. Sketch on the diagram the average resultant force acting on the block between B and C. The arrow on the diagram represents the weight of the block. *[2 marks]*



- 7d. Calculate the magnitude of the average force exerted by the rope on the block between B and C. *[2 marks]*

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For the rope and block, describe the energy changes that take place

7e. between A and B.

[1
mark]

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7f. between B and C.

[1
mark]

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7g. The length reached by the rope at C is 77.4 m. Suggest how energy considerations could be used to determine the elastic constant of the rope. [2 marks]

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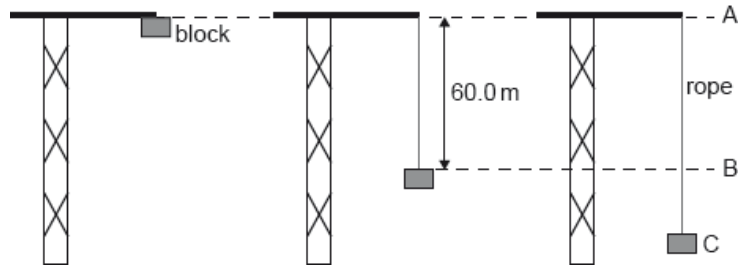
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An elastic climbing rope is tested by fixing one end of the rope to the top of a crane. The other end of the rope is connected to a block which is initially at position A. The block is released from rest. The mass of the rope is negligible.



The unextended length of the rope is 60.0 m. From position A to position B, the block falls freely.

In another test, the block hangs in equilibrium at the end of the same elastic rope. The elastic constant of the rope is 400 Nm^{-1} . The block is pulled 3.50 m vertically below the equilibrium position and is then released from rest.

- 7h. Calculate the time taken for the block to return to the equilibrium position *[2 marks]* for the first time.

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- 7i. Calculate the speed of the block as it passes the equilibrium position. *[2 marks]*

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A positive pion decays into a positive muon and a neutrino.

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

The momentum of the muon is measured to be 29.8 MeV c^{-1} in a laboratory reference frame in which the pion is at rest. The rest mass of the muon is 105.7 MeV c^{-2} and the mass of the neutrino can be assumed to be zero.

For the laboratory reference frame

8a. write down the momentum of the neutrino.

[1
mark]

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8b. show that the energy of the pion is about 140 MeV.

[2 marks]

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8c. State the rest mass of the pion with an appropriate unit.

[1
mark]

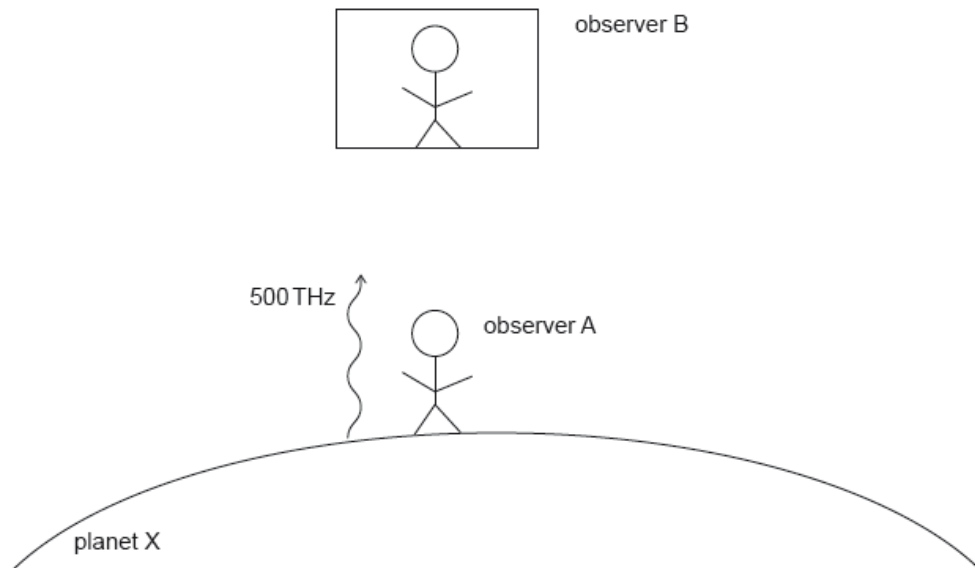
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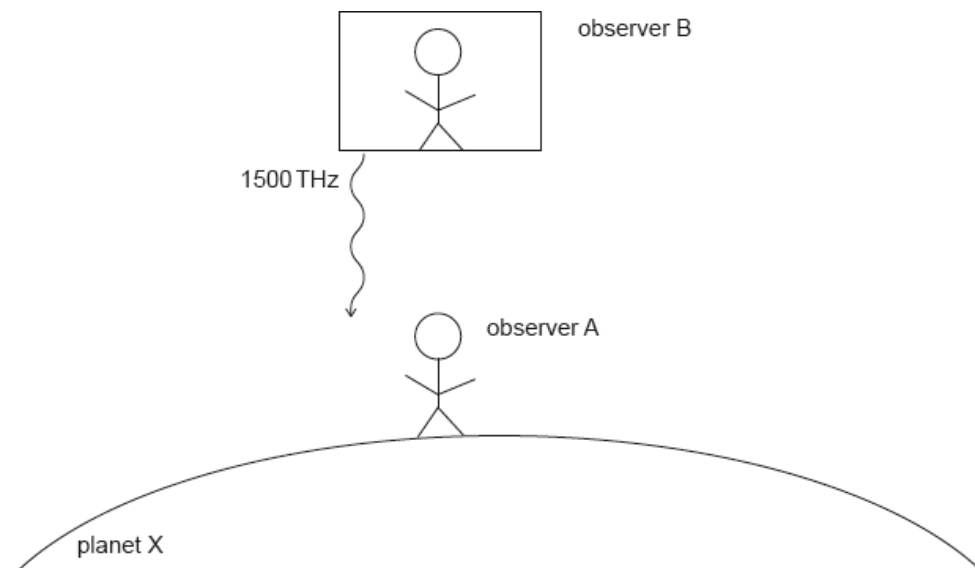
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An observer A is on the surface of planet X. Observer B is in a stationary spaceship above the surface of planet X.

Observer A sends a beam of light with a frequency 500 THz towards observer B. When observer B receives the light he observes that the frequency has changed by Δf .



Observer B then sends a signal with frequency 1500 THz towards observer A.



9a. Calculate the shift in frequency observed by A in terms of Δf . [2 marks]

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9b. Calculate the gravitational field strength on the surface of planet X. [2 marks]

The following data is given:

$$f = 170 \text{ Hz.}$$

The distance between observer A and B is 10 km.

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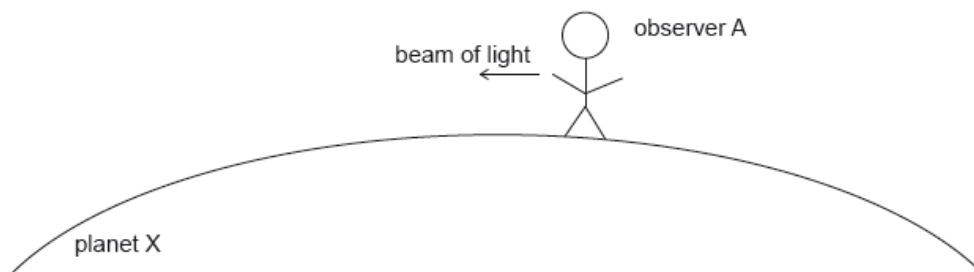
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9c. Observer A now sends a beam of light initially parallel to the surface of the planet. [2 marks]



Explain why the path of the light is curved.

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Two protons, travelling in opposite directions, collide. Each has a total energy of 3.35 GeV.

10a. Calculate the gamma (γ) factor for one of the protons.

[1
mark]

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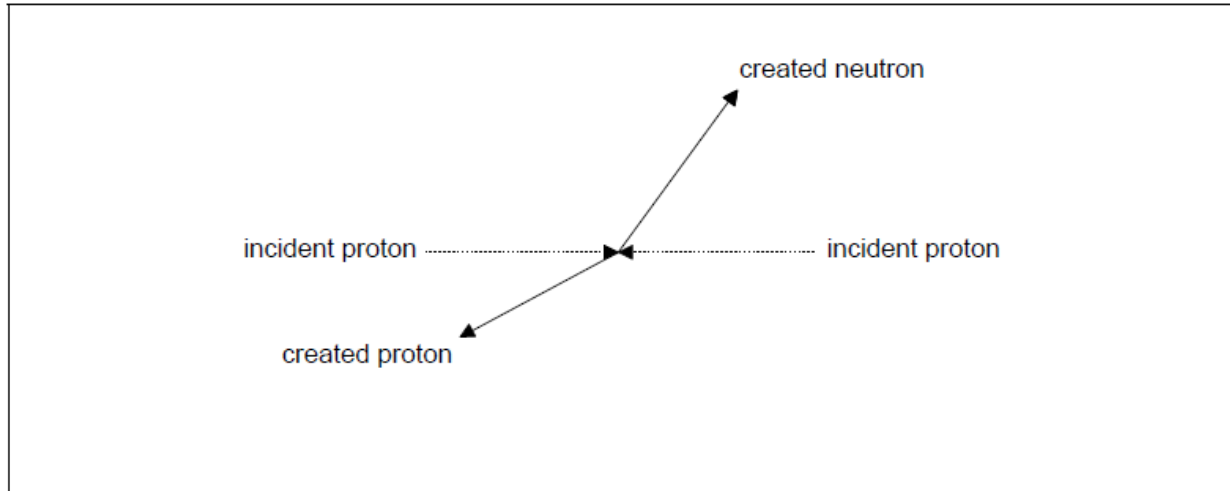
As a result of the collision, the protons are annihilated and three particles, a proton, a neutron, and a pion are created. The pion has a rest mass of $140 \text{ MeV } c^{-2}$. The total energy of the emitted proton and neutron from the interaction is 6.20 GeV.

10b. Determine, in terms of $\text{MeV } c^{-1}$, the momentum of the pion.

[3 marks]

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- 10c. The diagram shows the paths of the incident protons together with the proton and neutron created in the interaction. On the diagram, draw the path of the pion. [1 mark]



It is believed that a non-rotating supermassive black hole is likely to exist near the centre of our galaxy. This black hole has a mass equivalent to 3.6 million times that of the Sun.

- 11a. Outline what is meant by the event horizon of a black hole. [1 mark]

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- 11b. Calculate the distance of the event horizon of the black hole from its centre. [2 marks]

$$\text{Mass of Sun} = 2 \times 10^6 \text{ kg}$$

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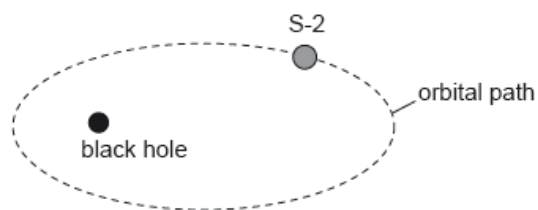
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- 11c. Star S-2 is in an elliptical orbit around a black hole. The distance of S-2 from the centre of the black hole varies between a few light-hours and several light-days. A periodic event on S-2 occurs every 5.0 s. [2 marks]



Discuss how the time for the periodic event as measured by an observer on the Earth changes with the orbital position of S-2.

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The Λ^0 (Lambda) particle decays spontaneously into a proton and a negatively charged pion of rest mass 140 MeV c^{-2} . After the decay, the particles are moving in the same direction with a proton momentum of 630 MeV c^{-1} and a pion momentum of 270 MeV c^{-1} .

12a. Determine the rest mass of the Λ^0 particle.

[4 marks]

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12b. Determine, using your answer to (a), the initial speed of the Λ^0 particle. [2 marks]

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The Schwarzschild radius of a black hole is 6.0×10^5 m. A rocket is 7.0×10^8 m from the black hole and has a clock. The proper time interval between the ticks of the clock on the rocket is 1.0 s. These ticks are transmitted to a distant observer in a region free of gravitational fields.

- 13a. Outline why the clock near the black hole runs slowly compared to a clock close to the distant observer. *[2 marks]*

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- 13b. Calculate the number of ticks detected in 10 ks by the distant observer. *[2 marks]*

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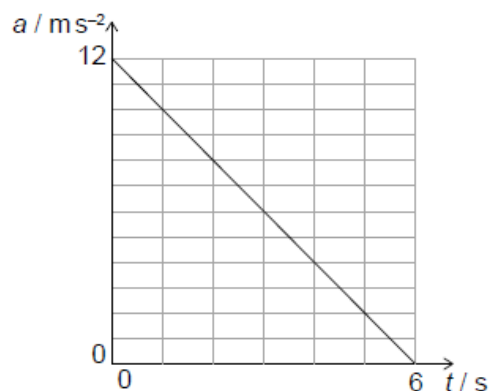
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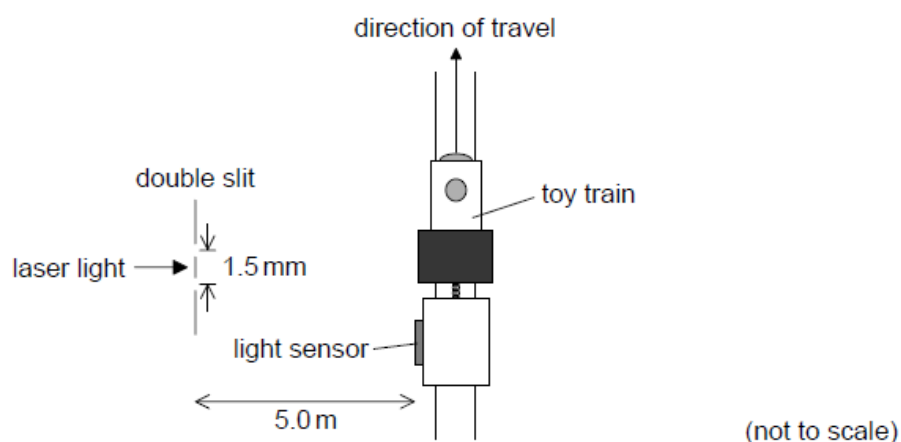
14. The graph shows the variation of the acceleration a of an object with time t . [1 mark]



What is the change in speed of the object shown by the graph?

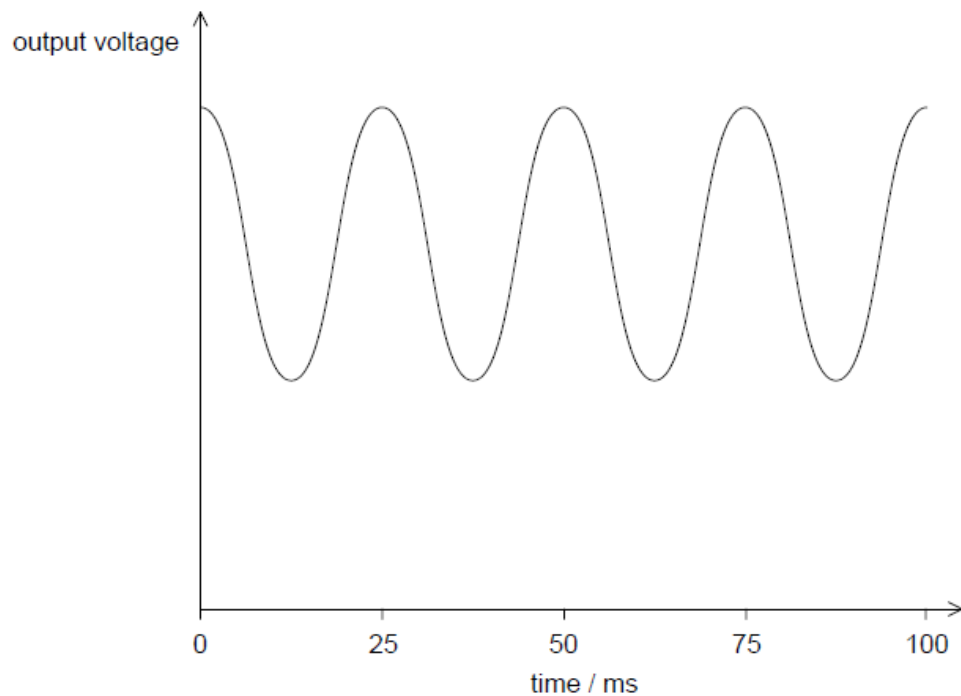
- A. 0.5 m s^{-1}
- B. 2.0 m s^{-1}
- C. 36 m s^{-1}
- D. 72 m s^{-1}

A student investigates how light can be used to measure the speed of a toy train.



Light from a laser is incident on a double slit. The light from the slits is detected by a light sensor attached to the train.

The graph shows the variation with time of the output voltage from the light sensor as the train moves parallel to the slits. The output voltage is proportional to the intensity of light incident on the sensor.



15a. Explain, with reference to the light passing through the slits, why a series *[3 marks]* of voltage peaks occurs.

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15b. The slits are separated by 1.5 mm and the laser light has a wavelength of 6.3×10^{-7} m. The slits are 5.0 m from the train track. Calculate the *[1 mark]* separation between two adjacent positions of the train when the output voltage is at a maximum.

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15c. Estimate the speed of the train.

[2 marks]

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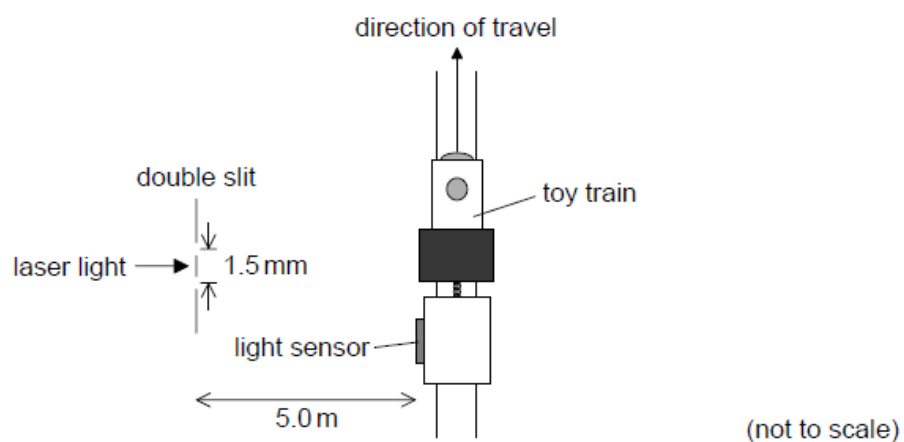
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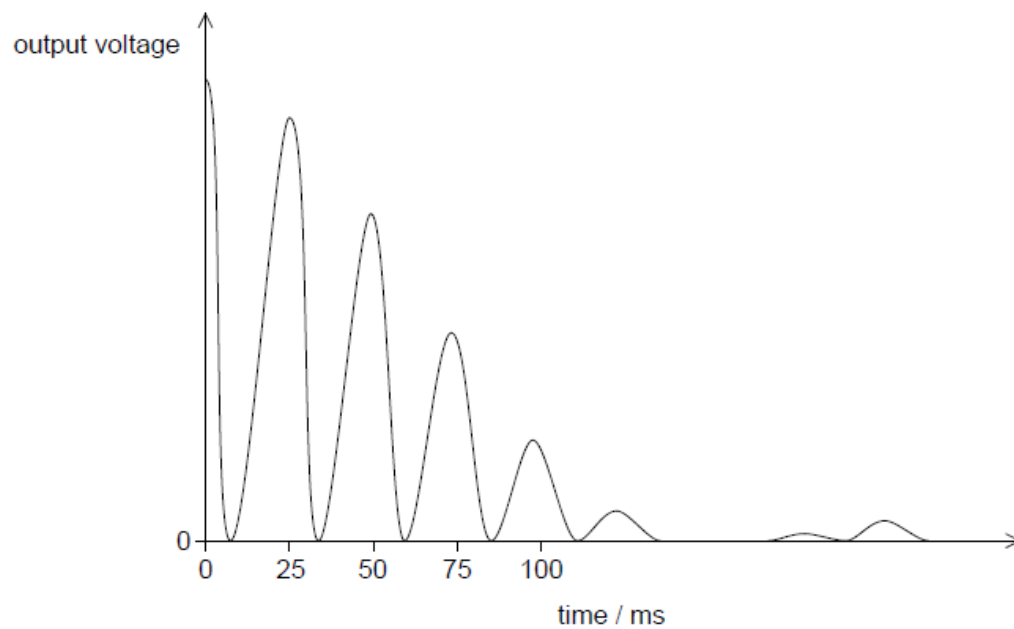
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A student investigates how light can be used to measure the speed of a toy train.

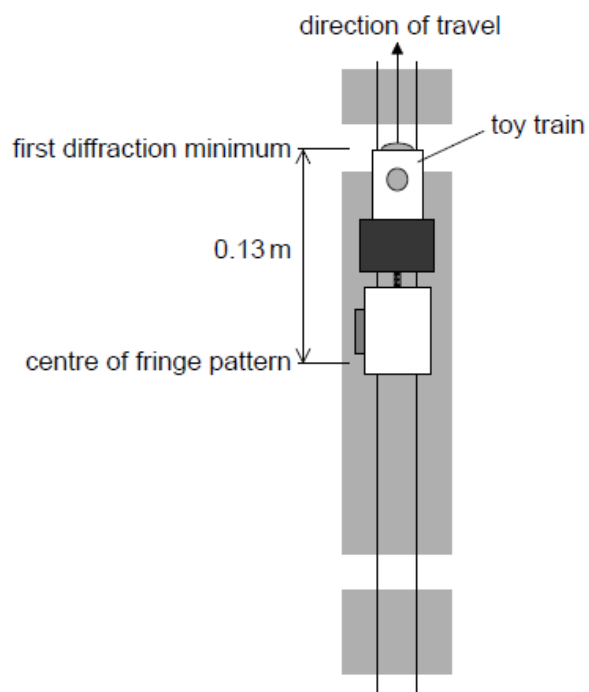


Light from a laser is incident on a double slit. The light from the slits is detected by a light sensor attached to the train.

The graph shows the variation with time of the output voltage from the light sensor as the train moves parallel to the slits. The output voltage is proportional to the intensity of light incident on the sensor.



As the train continues to move, the first diffraction minimum is observed when the light sensor is at a distance of 0.13 m from the centre of the fringe pattern.



(not to scale)

15d. Determine the width of one of the slits.

[2 marks]

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15e. Suggest the variation in the output voltage from the light sensor that will *[2 marks]* be observed as the train moves beyond the first diffraction minimum.

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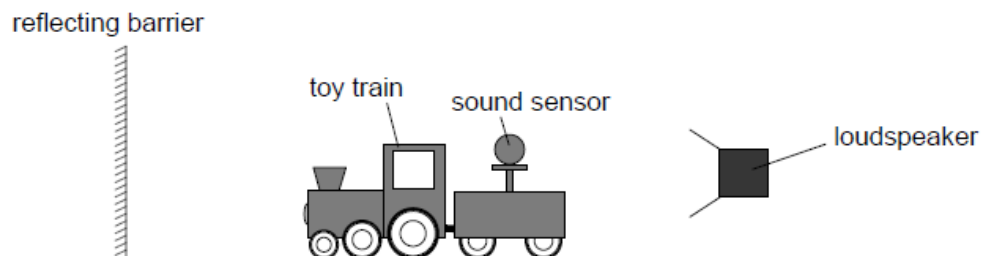
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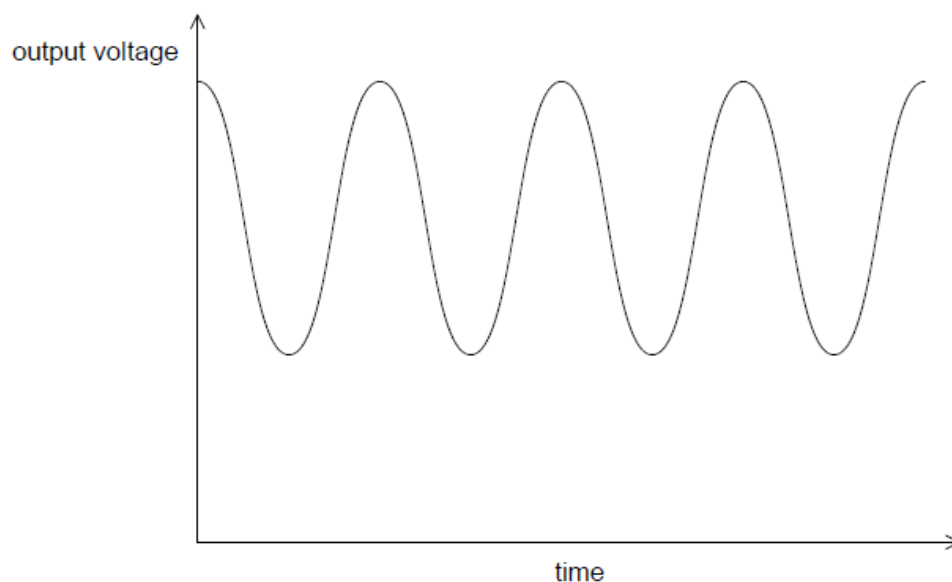
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- 15f. In another experiment the student replaces the light sensor with a sound sensor. The train travels away from a loudspeaker that is emitting sound waves of constant amplitude and frequency towards a reflecting barrier. [2 marks]



The graph shows the variation with time of the output voltage from the sound sensor.



Explain how this effect arises.

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A proton is accelerated from rest through a potential difference V to a speed of $0.86c$.

16a. Calculate the potential difference V .

[3 marks]

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16b. The proton collides with an antiproton moving with the same speed in the opposite direction. As a result both particles are annihilated and two photons of equal energy are produced.

Determine the momentum of one of the photons.

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In an experiment a source of iron-57 emits gamma rays of energy 14.4 keV. A detector placed 22.6 m vertically above the source measures the frequency of the gamma rays.

- 17a. Calculate the expected shift in frequency between the emitted and the detected gamma rays. [2 marks]

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- 17b. Explain whether the detected frequency would be greater or less than the emitted frequency. [2 marks]

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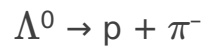
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18. A lambda Λ^0 particle at rest decays into a proton p and a pion π^- according to the reaction [4 marks]



where the rest energy of p = 938 MeV and the rest energy of π^- = 140 MeV.

The speed of the pion after the decay is $0.579c$. For this speed $\gamma = 1.2265$.
Calculate the speed of the proton.

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- 19a. State what is meant by the event horizon of a black hole. [1 mark]

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- 19b. Show that the surface area A of the sphere corresponding to the event horizon is given by [1
mark]

$$A = \frac{16\pi G^2 M^2}{c^4}.$$

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- 19c. Suggest why the surface area of the event horizon can never decrease. [1
mark]

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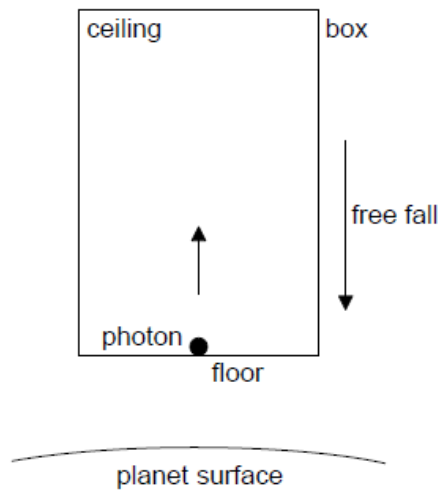
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19d. The diagram shows a box that is falling freely in the gravitational field of a planet. [3 marks]



A photon of frequency f is emitted from the floor of the box and is received at the ceiling. State and explain the frequency of the photon measured at the ceiling.

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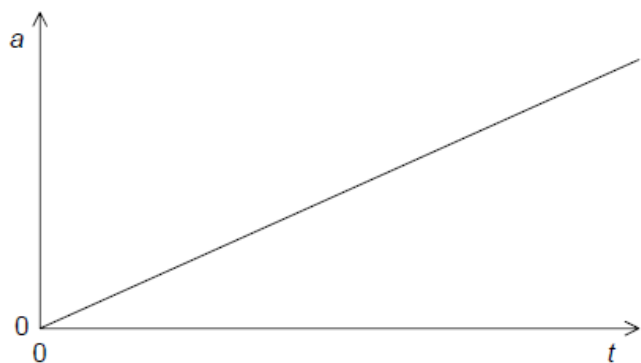
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20. A student draws a graph to show the variation with time t of the acceleration a of an object.

[1
mark]

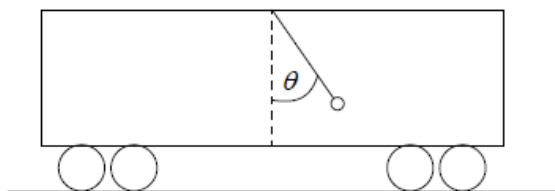


What can the student deduce from this graph only, and what quantity from the graph is used to make this deduction?

	Deduction	Quantity used
A.	change in velocity	gradient of graph
B.	change in velocity	area under line
C.	change in displacement	gradient of graph
D.	change in displacement	area under line

21. A mass is suspended from the ceiling of a train carriage by a string. The string makes an angle θ with the vertical when the train is accelerating along a straight horizontal track.

[4 marks]



What is the acceleration of the train?

- A. $g \sin \theta$
B. $g \cos \theta$
C. $g \tan \theta$
D. $\frac{g}{\tan \theta}$

An electron and a positron have identical speeds but are travelling in opposite directions. Their collision results in the annihilation of both particles and the production of two photons of identical energy. The initial kinetic energy of the electron is 2.00 MeV.

- 22a. Explain, in terms of a conservation law, why two photons need to be created. *[1 mark]*

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- 22b. Determine the speed of the incoming electron. *[3 marks]*

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- 22c. Calculate the energy and the momentum for each photon after the collision. *[2 marks]*

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The global positioning system (GPS) uses satellites that orbit the Earth. The satellites transmit information to Earth using accurately known time signals derived from atomic clocks on the satellites. The time signals need to be corrected due to the gravitational redshift that occurs because the satellites are at a height of 20 Mm above the surface of the Earth.

- 23a. The gravitational field strength at 20 Mm above the surface of the Earth [3 marks] is about 0.6 N kg^{-1} . Estimate the time correction per day needed to the time signals, due to the gravitational redshift.

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- 23b. Suggest, whether your answer to (a) underestimates **or** overestimates the [1 mark] correction required to the time signal.

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An electron is emitted from a nucleus with a total energy of 2.30MeV as observed in a laboratory.

24a. Show that the speed of the electron is about 0.98c.

[3 marks]

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[7 marks]

(i) For the reference frame of the electron, calculate the distance travelled by the detector.

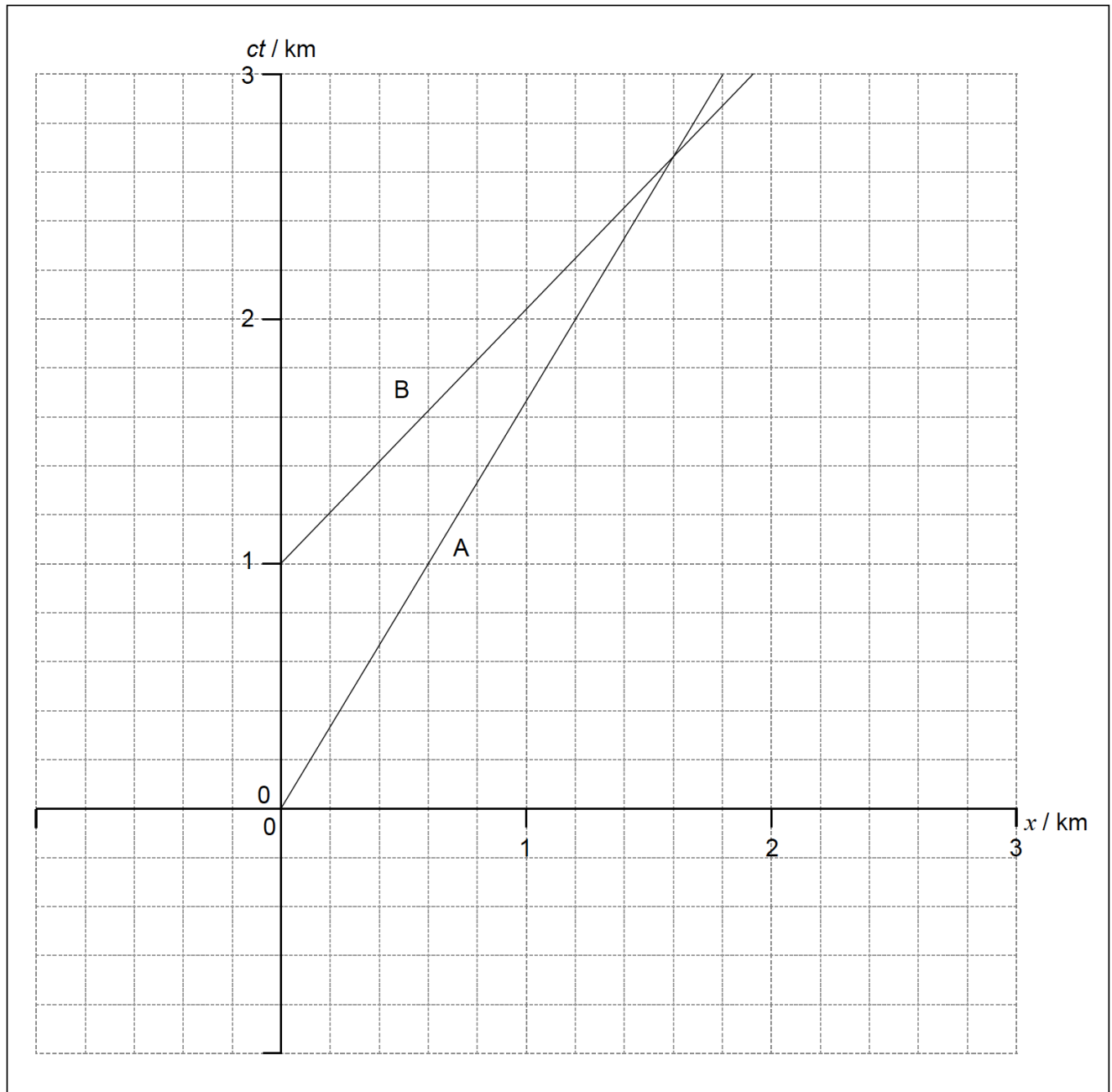
(ii) For the reference frame of the laboratory, calculate the time taken for the electron to reach the detector after its emission from the nucleus.

(iii) For the reference frame of the electron, calculate the time between its emission at the nucleus and its detection.

(iv) Outline why the answer to (b)(iii) represents a proper time interval.

[illegible]

An observer on Earth watches two rockets, A and B. The spacetime diagram shows part of the motion of A and B in the reference frame of the Earth observer. A and B are travelling in the same direction.



25a. For the reference frame of the Earth observer, calculate the speed of rocket A in terms of the speed of light c . [2 marks]

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25b. One rocket passes the other at event E. For the reference frame of the Earth observer, estimate *[2 marks]*
Earth observer, estimate

- (i) the space coordinate of E, in kilometres.
- (ii) the time coordinate of E, in seconds.

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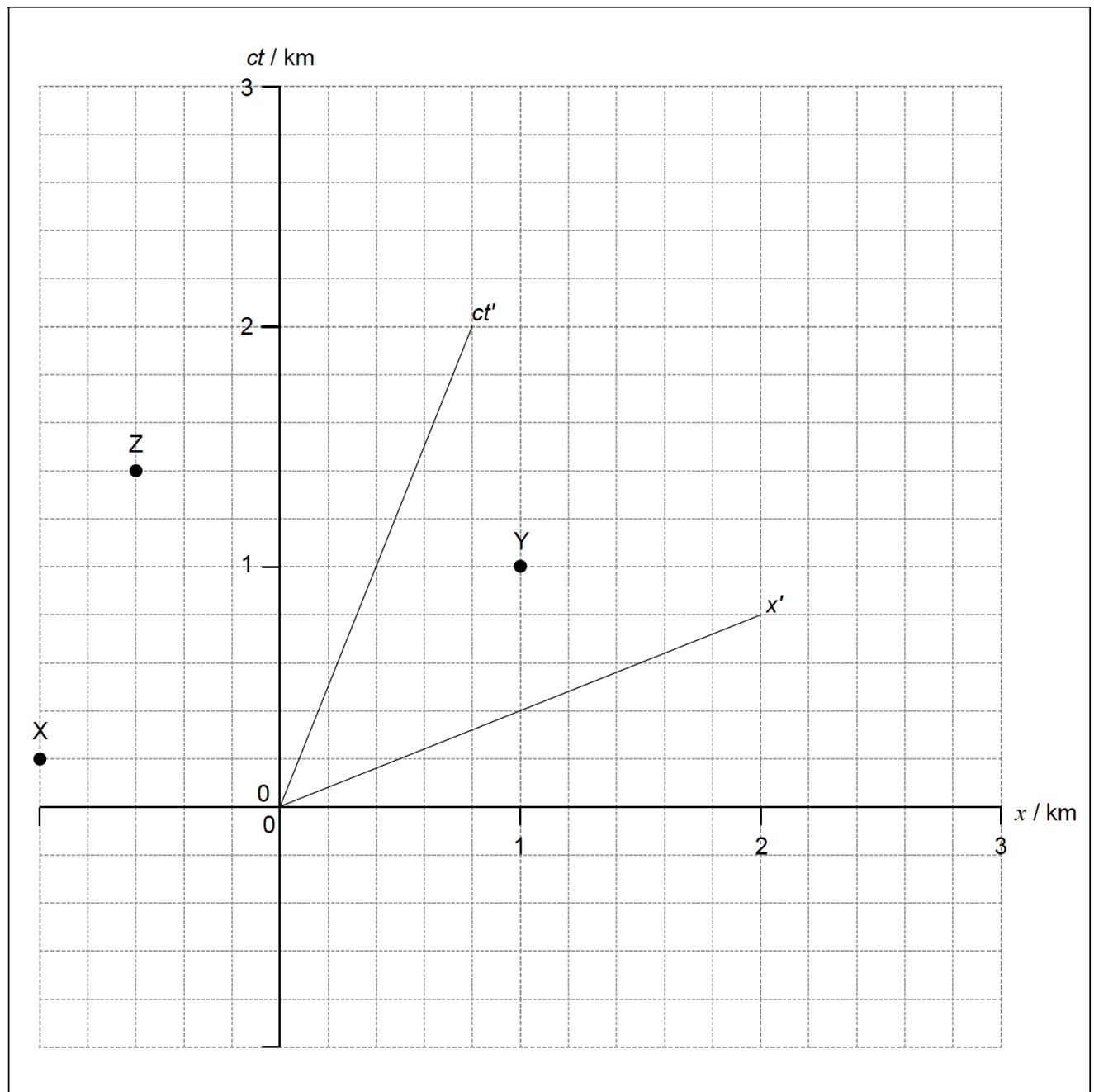
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25c. Three flashing light beacons, X, Y and Z, are used to guide another rocket *[4 marks]*
C. The flash events are shown on the spacetime diagram. The diagram shows the axes for the reference frames of Earth and of rocket C. The Earth observer is at the origin.



- Using the graph opposite, deduce the order in which
- (i) the beacons **flash** in the reference frame of rocket C.
 - (ii) the Earth observer **sees** the beacons flash.

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26a. Outline what is meant by a black hole.

[2 marks]

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- 26b. An observer views a distant spacecraft that is 23.0 km from the centre of [3 marks]
a black hole. The spacecraft contains a clock that ticks once every
second and the ticks can be detected by the distant observer. In 2.00 minutes the
observer counts 112 ticks of the clock.

Determine the mass of the black hole.

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27. An object is dropped from rest. Air resistance is **not** negligible. What is the [1
acceleration of the object at the start of the motion? mark]

- A. Zero
- B. Increasing
- C. Decreasing
- D. Constant

28. A student throws a stone with velocity v at an angle θ to the vertical from [1
the surface of a lake. Air resistance can be ignored. The acceleration due to mark]
gravity is g .



What is the time taken for the stone to hit the surface of the lake?

- A. $\frac{v \sin \theta}{g}$
- B. $\frac{v \cos \theta}{g}$
- C. $\frac{2v \sin \theta}{g}$
- D. $\frac{2v \cos \theta}{g}$

This question is about relativistic kinematics.

29a. State what is meant by an inertial frame of reference.

[2 marks]

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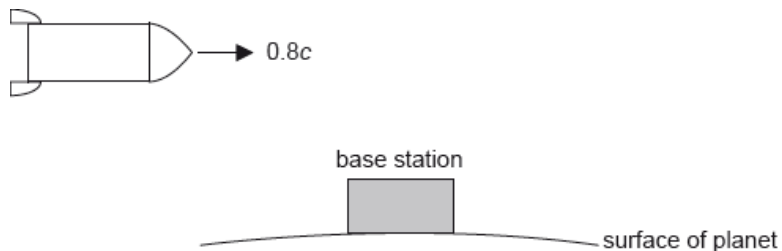
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A spacecraft is flying in a straight line above a base station at a speed of $0.8c$.



Suzanne is inside the spacecraft and Juan is on the base station.

29b. A light on the base station flashes regularly. According to Suzanne, the light flashes every 3 seconds. Calculate how often the light flashes according to Juan.

[2 marks]

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29c. While moving away from the base station, Suzanne observes another spacecraft travelling towards her at a speed of $0.8c$. Suzanne measures the other spacecraft to have a length of 8.00 m. Calculate the proper length of the other spacecraft. *[3 marks]*

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29d. Suzanne's spacecraft is on a journey to a star. According to Juan, the distance from the base station to the star is 11.4 ly. Show that Suzanne measures the time taken for her to travel from the base station to the star to be about 9 years. *[2 marks]*

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- 29e. Suzanne then returns to the base station at the same speed. The total time since leaving the base station as measured by Suzanne is around 18 years but the total time according to Juan is around 29 years. Explain how it is possible for Suzanne and Juan to have aged by different amounts. [2 marks]

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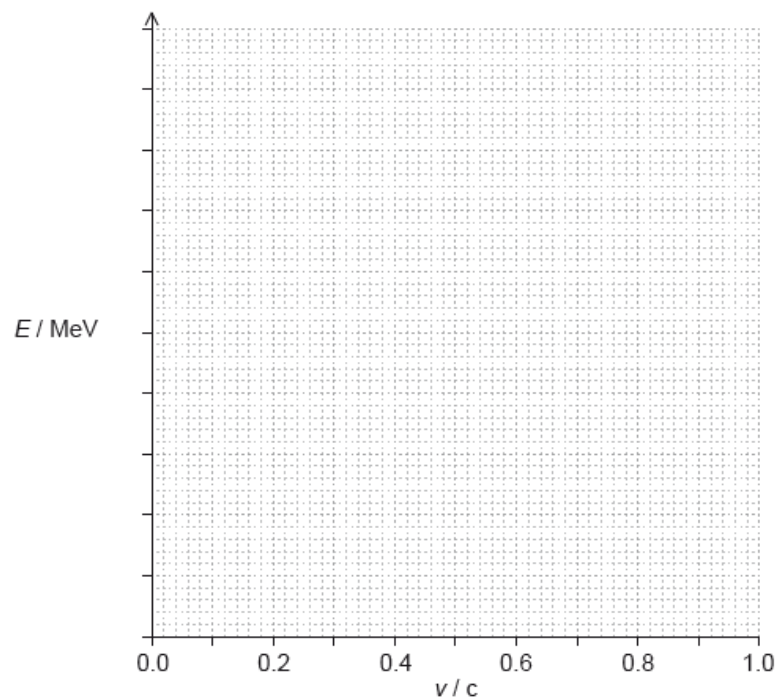
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This question is about mass and energy.

The positive kaon K^+ has a rest mass of 494 MeV c^{-2} .

- 30a. Using the grid, sketch a graph showing how the energy of the kaon increases with speed. [2 marks]



- 30b. The kaon is accelerated from rest through a potential difference so that its energy becomes three times its rest energy. Calculate the potential difference through which the kaon was accelerated. [2 marks]

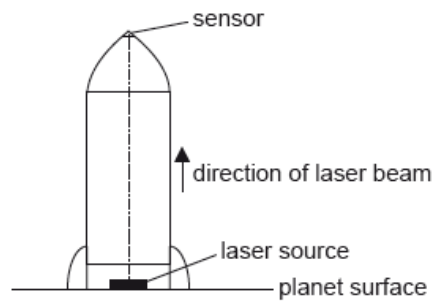
- 30c. The neutral kaon is unstable and one of its possible modes of decay is [2 marks]

$$K^0 \rightarrow \pi^0 + \pi^0.$$

The π^0 has a rest mass of 135 MeV c^{-2} . The K^0 has a rest mass of 498 MeV c^{-2} . The K^0 is at rest before it decays. The two π^0 particles move apart in opposite directions along a straight line. Determine the momentum of **one** of the π^0 particles.

This question is about general relativity.

A spacecraft is at rest on the surface of a distant planet. A laser beam is fired from the base of the spacecraft to a sensor at the top of the spacecraft.



The spacecraft has a height of 112 m. The laser beam emitted from the source has a frequency of 4.52×10^{14} Hz and the sensor detects a shift in the frequency of the laser beam of 3.20 Hz.

31a. Show that the gravitational field strength at the surface of the planet is [2 marks]
about 5.7 N kg^{-1} .

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31b. Discuss the shift in frequency of the laser beam.

[2 marks]

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31c. The spacecraft leaves the planet with an acceleration of 5.7 m s^{-2} . The same experiment is carried out by firing the laser beam from the base of the spacecraft to the sensor at the top of the spacecraft. Compare the shift in frequency of the laser beam with that detected in (a). [2 marks]

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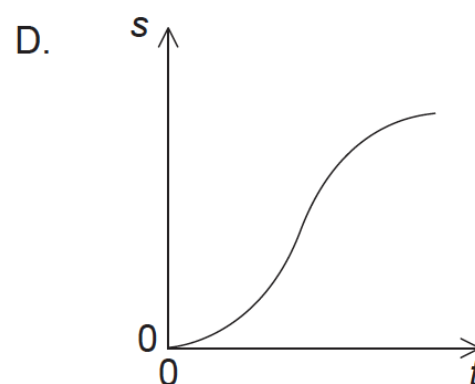
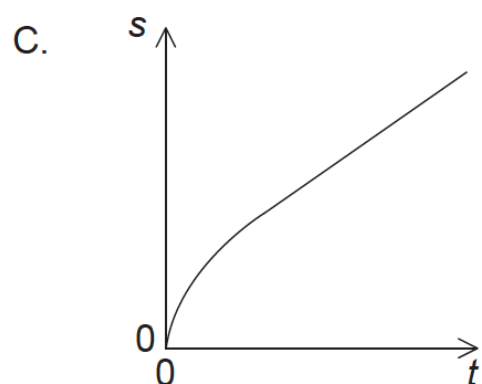
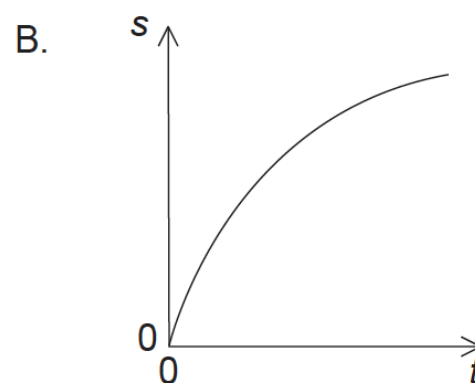
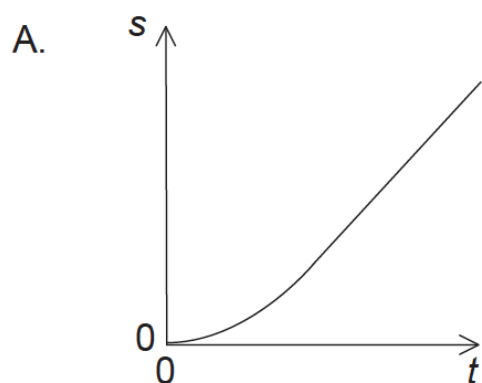
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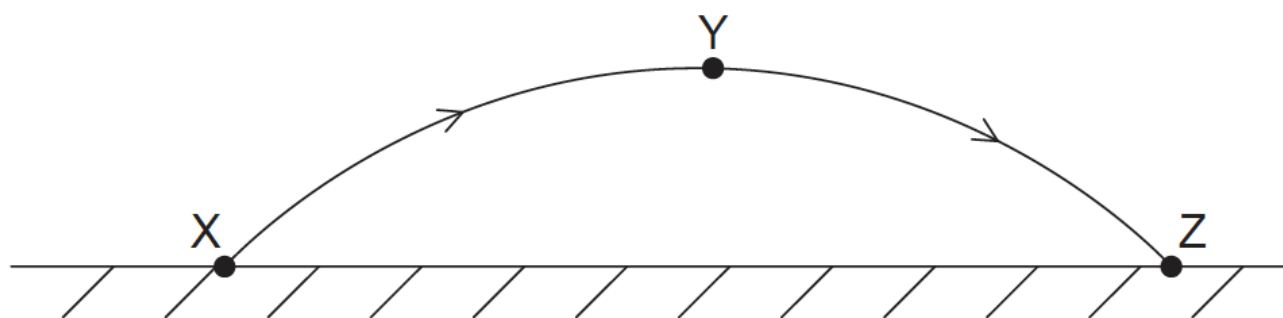
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32. A tennis ball is dropped from the top of a tall building. Air resistance is **not** negligible. Which graph shows the variation with time t of the displacement s of the ball? [1 mark]



33. A ball is thrown from point X and follows path XYZ. Air resistance is negligible. [1 mark]

[1 mark]



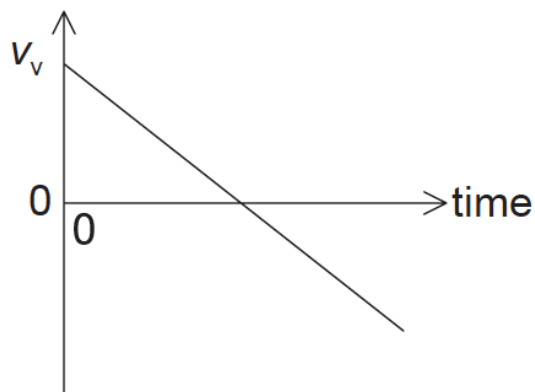
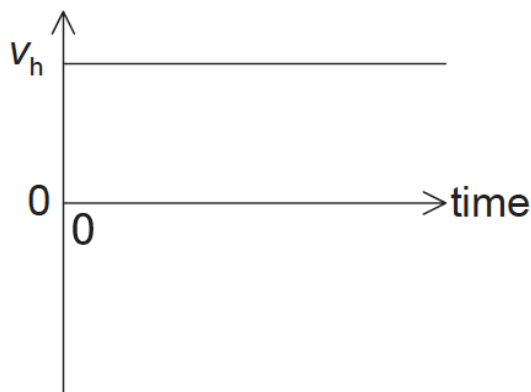
Which quantity is zero when the ball is at the highest point Y of the path?

- A. The horizontal component of the ball's acceleration
- B. The vertical component of the ball's acceleration
- C. The horizontal component of the ball's velocity
- D. The kinetic energy of the ball

34. Two isolated spherical planets have the same gravitational potential at their surfaces. Which ratio must also be the same for the two planets? [1 mark]

- A. $\frac{\text{radius}^3}{\text{mass}}$
- B. $\frac{\text{radius}^2}{\text{mass}}$
- C. $\frac{\text{radius}}{\text{mass}}$
- D. radius

35. The horizontal component v_h and the vertical component v_v of velocity of an object are shown on the graphs. Air resistance is negligible. [1 mark]



These graphs could represent the motion of an object fired from a cliff

- A. vertically upwards.
- B. at an angle above the horizontal.
- C. horizontally.
- D. at an angle below the horizontal.