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I. UNCERTAINTY AND ERROR IN MEASUREMENT

Physics is an experimental science. All physical laws, theories, and formulae were developed based on measurements of things. All measurements in science suffer from **uncertainty** which results from unavoidable **errors**. No matter how hard we try to control things, some level of experimental error is unavoidable. A source of error may be one of a number of things, or any number of those things. No measuring instrument is perfect! Reporting experimental results with appropriate uncertainty and following these through in calculations is a critical part of IB Physics.

CRUCIAL THINGS TO KNOW:

- An **error** is the difference between the measured value and the expected value of something (unavoidable).
- An **uncertainty** is a way of expressing or summarizing the error (unavoidable).
- A **mistake** is simply not doing something correctly through carelessness (avoidable).
- Thus, an error is not the same as an uncertainty, though both are unavoidable. They are often used in common language to mean the same thing, but they are not!

Our strategy in dealing with uncertainties and errors is to minimize them, but at the same time be honest about them by reporting them truthfully. You will never lose marks by quoting a large final uncertainty if your procedure and method of collecting data are correct and justified, and all sources of error discussed.

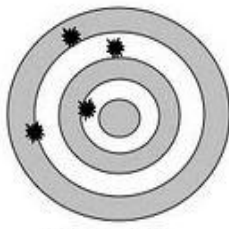
When we make a measurement, we generally assume that some exact or true value exists based on how we define what is being measured. While we may never know this true value exactly, we attempt to find this ideal quantity to the best of our ability with the time and resources available.

A. PRECISION AND ACCURACY

Accuracy is the closeness of agreement between a measured value and a true or accepted value (measurement error reveals the amount of inaccuracy).

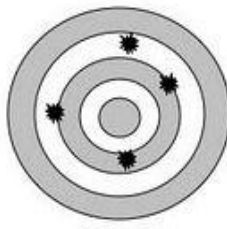
Precision is a measure of the degree of consistency and agreement among independent measurements of the same quantity (also the reliability or reproducibility of the result).

Think of it while you are playing darts, like this:



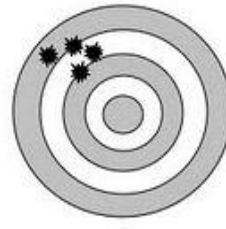
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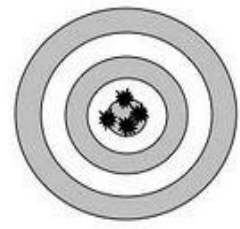
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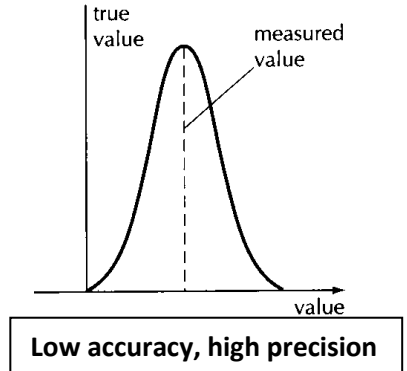
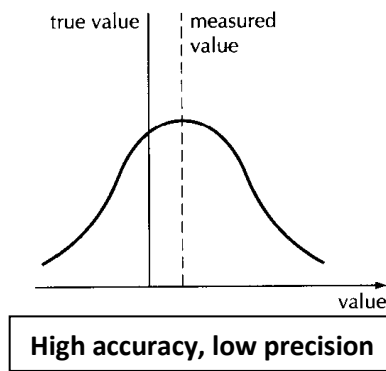
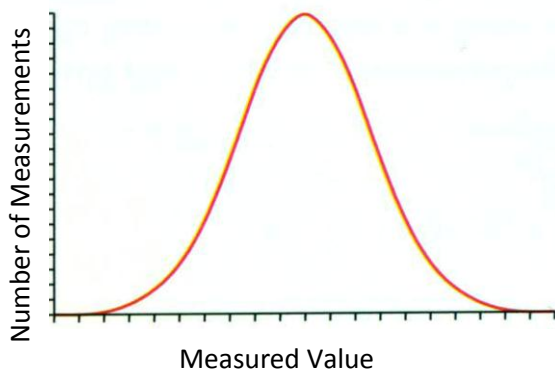


ACCURATE

PRECISE

Precision is really about detail. It has nothing to do with accuracy. Accuracy is about giving true readings, not detailed readings.

Imagine that you have many measured values of the same thing. If you and your instruments were perfect, every single measurement would be exactly the same. But in reality, you end up with a **normal distribution** of data. The area under the curve between any two measured values gives the number of measurements in that range of values.



- An ammeter has a zero offset error. This fault will affect
- neither the precision nor the accuracy of the readings.
 - only the precision of the readings.
 - only the accuracy of the readings.**
 - both the precision and the accuracy of the readings.

The reading of a constant potential difference is made four times by a student. The readings are
1.176 V, 1.178 V, 1.177 V, 1.176 V

The student averages these readings but does not take into account the zero error on the voltmeter. The average measurement of the potential difference is

- precise and accurate.
- precise but not accurate.**
- accurate but not precise.
- not accurate and not precise.

B. RANDOM AND SYSTEMATIC ERRORS

There are 2 types of errors in measured data. It is important to understand which you are dealing with, and how to handle them.

RANDOM ERRORS refer to random fluctuations in the measured data due to:

- The readability of the instrument
- The effects of something changing in the surroundings between measurements
- The observer being less than perfect ☺

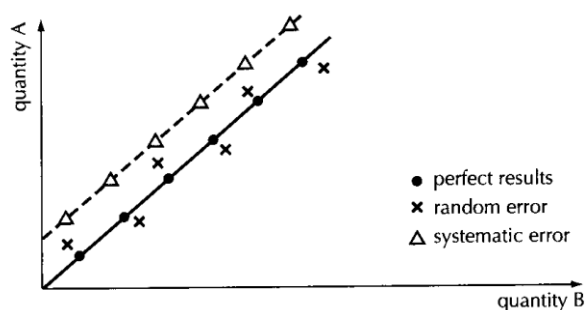
Random errors can be reduced by averaging. A precise experiment has small random error.

SYSTEMATIC ERRORS refer to reproducible fluctuations consistently in the same direction due to:

- An instrument being wrongly calibrated
- An instrument with zero error (it does not read zero when it should – to correct for this, the value should be subtracted from every reading)
- The observer being less than perfect in the same way during each measurement. ☺

Systematic errors cannot be detected or reduced by taking more measurements. An accurate experiment has small systematic error.

When graphing experimental data, you can see immediately if you are dealing with random or systematic errors (if you can compare with theoretical or expected results).



Both random and systematic errors are present in the measurement of a particular quantity. What changes, if any, would repeated measurements of this quantity have on the random and systematic errors?

	Random	Systematic
A.	reduced	reduced
B.	reduced	unchanged
C.	unchanged	reduced
D.	unchanged	unchanged

C. REPORTING A SINGLE MEASUREMENT

You would be surprised at how few people actually know how to take a proper reading of something. Most people try to report a measured value with a degree of certainty that is too generous – expressing more certainty in a reported value than really exists. You should avoid this! It is bad practice.

Generally we report the measured value of something with the decimal place or precision **going not beyond the smallest graduation (called the 'least count') on the instrument**. In cases where the least counts are wide enough to estimate beyond them *with certainty*, you may do so. It is ultimately up to the experimenter to determine how to report a measured value, but be conservative and do not overestimate the precision of the instrument.

Sometimes you hear that uncertainties should generally be reported as $\frac{1}{2}$ the least count; this is technically correct. But since they should be reported with the same # of decimal places as the instrument, in practice this amounts to stating them as $\pm$ the least count.

Instrument uncertainties should be stated as $\pm$ the least count, and match the # of decimal places of the reported measurement. They should also have only one significant figure if possible.

If the number of decimal places in your uncertainty does not match the measurement, **round your uncertainty up** so that it does. In most cases, you do this anyway – only when you are reporting a value beyond the least count would you not do this. Decimal uncertainties always have 1 significant figure.

Measurement = measured value $\pm$ uncertainty (unit of measurement)

For example, with a vertical meterstick (with 1 mm as the least count), you might measure the height of a chair as 41.2 cm. You should report the value as 41.2 ± 0.1 cm. There are some exceptions:

USING STANDARD MASSES (mass)

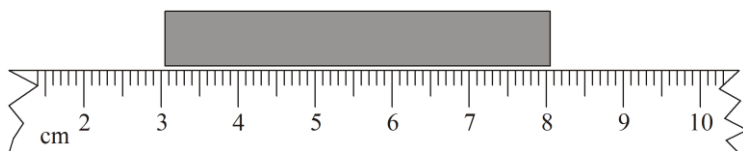
So-called ‘standard masses’ are less accurate than you would think, and vary considerably. Generally consider the uncertainty to be 1%. So, 100 g = 100 ± 1 g, 1.0 kg = 1000 ± 10 g, etc.

USING STOPWATCHES (time)

Most stopwatches report to the 0.01 s. However, taking this least count as the uncertainty is ridiculous since you could never, ever move your thumb that fast! A more realistic uncertainty, which should be used for all time measurements with stopwatches, is ± 0.3 s. In reality, the uncertainty in timing with a stopwatch is asymmetric but we’ll simplify things and always use this value.

Unfortunately, there is no general rule for determining the uncertainty in all measurements. The experimenter is the one who can best evaluate and quantify the uncertainty of a measurement based on all the possible factors that affect the result. Therefore, the person making the measurement has the obligation to make the best judgment possible and report the uncertainty in a way that clearly explains what the uncertainty represents.

The length of a rod is measured using part of a metre rule that is graduated in millimetres, as shown below.



Which **one** of the following is the measurement, with its uncertainty, of the length of the rod?

A. 5 ± 0.1 cm

B. 5 ± 0.2 cm

C. 5.0 ± 0.1 cm

D. 5.0 ± 0.2 cm

D. REPORTING YOUR BEST ESTIMATE OF A MEASUREMENT

The best way to come up with a good measurement of something is to take several measurements and average them all together. Each individual measurement has uncertainty, but the reported uncertainty in your average value is different than the uncertainty in your instrument. You do not use the instrument uncertainty in your final stated uncertainty – **the precision of the instrument is not the same as the uncertainty in the measurement.**

If you take several measurements of something, you will get a range of values. The ‘real’ value should be within this range, and the uncertainty is determined by dividing the range of values by two. Always round your stated uncertainty up to match the number of decimal places of your measurement, if necessary. Your stated uncertainty should have only one significant figure if possible. In the IB Physics laboratory, you should take **3 to 5 measurements** of everything. Five is always best if you can manage it.

Six students measure the resistance of a lamp. Their answers in Ω are: 609; 666; 639; 661; 654; 628. What should the students report as the resistance of the lamp?

Average resistance = 643 Ω Largest - smallest resistance 666 - 609 = 57 Ω

Dividing the range by 2 = 29 Ω

*So, the resistance of the lamp is reported as: **643 $\pm$ 30 Ω***

When taking several measurements, it should be clear if you have a value with a large error. **Do not be afraid to throw out any measurement that is clearly a mistake.** You will never be penalized for this if you explain your rationale for doing so. In fact, it is permissible, if you have many measurements, to throw out the maximum and minimum values.

Three pairs of perpendicular measurements are made of the diameter of a cylinder. Results, in mm, are: 21.8; 21.6; 22.1; 26.1; 21.9; 22.1. What should be the reported value of the cylinder's diameter?

The datum 26.1 mm is very different from the others and can be discarded as a mistake. Thus, present your results, in mm, as 21.8; 21.6; 22.1; 24.4; 21.9; 22.1

The average of the "good" five is 21.9 mm. The difference between the largest and smallest measurements is 0.5 mm. Hence, the uncertainty is $\pm 0.5 / 2 = \pm 0.25$ mm.

*The cylinder has a diameter of **21.9 $\pm$ 0.3 mm.***

When taking time measurements, the stated uncertainty cannot be unreasonably small – not smaller than 0.3 s, no matter what the range. When taking time measurements (such as the period of a pendulum), we can improve the accuracy of our data by measuring the time taken for 20 oscillations (20T). In this case, you can divide the uncertainty for 20T by 20 to get the uncertainty in T.

For a pendulum, 20 oscillations (20T) are timed (in seconds) at 14.73; 14.69; 14.75 . What is T?

*The range in values is 0.06, but you cannot report the value for 20T as 14.72 ± 0.03 s. You must report it as **14.72 ± 0.30 s** but $T = 0.7360 \pm 0.0150$ s or better stated as **$T = 0.7360 \pm 0.0200$ s***

Remember that the manner in which you report measured and calculated values is entirely up to you, as the experimenter. However, be realistic in your precision and be able to fully justify reported measurements and calculated values, showing all of your work in doing so.

II. UNCERTAINTY IN CALCULATED RESULTS

A. ABSOLUTE AND PERCENTAGE UNCERTAINTIES

Whether included as part of a measurement or at the end of a calculation, uncertainties can be expressed in two ways:

1. **Absolute uncertainties** are expressed as $\pm$ the number of units in the measurement ($\pm \Delta x$).

$$\text{Length} = 234 \pm 2 \text{ mm}$$

$$\text{Period} = 1.6 \pm 0.3 \text{ s}$$

This tells you immediately the maximum and minimum experimental values of a measurement. Absolute uncertainties have the same units as the stated measurement. All uncertainties begin as an absolute uncertainty, stated according to the uncertainty in the precision of the instrument.

2. **Percentage uncertainties** are expressed as $\pm$ [the fractional uncertainty in the measurement $\times 100$] ($\pm [(\Delta x/x)100]\%$).

$$\begin{array}{llll} \text{Length} = 234 \pm 2 \text{ mm} & \text{or} & 234 \pm (2/234) \times 100 & = & 234 (\pm 0.85 \%) \text{ mm} \\ \text{Period} = 1.6 \pm 0.3 \text{ s} & \text{or} & 1.6 \pm (0.3/1.6) \times 100 & = & 1.6 (\pm 18.8 \%) \text{ s} \end{array}$$

Percentage uncertainties are unitless and can save lots of time when making calculations, even though it seems cumbersome to express uncertainty this way

B. DETERMINING FINAL UNCERTAINTIES IN STATED RESULTS

Physics is an experimental science, so we need to be concerned with how we treat uncertainties in calculated values using experimental data. The general rules are:

If data are to be **added** or **subtracted**, add the *absolute uncertainty*:

$$(A \pm \Delta A) + (B \pm \Delta B) = (A + B) \pm (\Delta A + \Delta B) \quad (A \pm \Delta A) - (B \pm \Delta B) = (A - B) \pm (\Delta A + \Delta B)$$

If data are to be **multiplied** or **divided**, add the *percentage uncertainty*:

$$(A \pm \Delta A) \times (B \pm \Delta B) = (A \times B) \pm \left[\left(\frac{\Delta A}{A} 100 \right) \% + \left(\frac{\Delta B}{B} 100 \right) \% \right]$$

$$\frac{(A \pm \Delta A)}{(B \pm \Delta B)} = \frac{A}{B} \pm \left[\left(\frac{\Delta A}{A} 100 \right) \% + \left(\frac{\Delta B}{B} 100 \right) \% \right]$$

If data are **raised to a power**, multiply the *percentage uncertainty* by that power:

$$(A \pm \Delta A)^n = A^n \pm n \left(\frac{\Delta A}{A} 100 \right) \% = A^n \pm n \Delta A \%$$

It is good form to leave all final calculated answers with an absolute uncertainty. Therefore, you need to be able to convert from absolute uncertainties to percentage and back again. Constants such as π do not affect the uncertainty calculation.

When doing calculations involving percentage uncertainties, it is easier to leave out the ($\times 100$) step and simply multiply using the decimal form.

A cylinder has a radius of 1.60 ± 0.01 cm and a height of 11.5 ± 0.1 cm. Find the volume.

$$V = \pi r^2 h = \pi (1.60)^2 \times 11.5 = 92.488 \text{ cm}^3 = 92 \text{ cm}^3$$

The percentage uncertainty in the radius is $(0.01/1.60) \times 100 = 0.625\%$ (but leave as 0.00625).

The percentage uncertainty in r^2 is thus $2 \times 0.00625 = 0.01250$

The percentage uncertainty in the height is $(0.1/11.5) \times 100 = 0.87\%$ (but leave as 0.00870).

So, the percentage uncertainty in the volume is $0.01250 + 0.00870$ since we are multiplying h and r^2 .
 $= 0.02120$

And the absolute uncertainty in V is $0.02120 \times 92.488 \text{ cm}^3 = 1.96075 \text{ cm}^3$

The final answer, with sig figs, is therefore $V = 92 \pm 2 \text{ cm}^3$

For all other mathematical functions (trigonometric, logarithmic, etc), you need to determine the |max value - mean value| and the |min value – mean value|. Whichever is greater, that becomes the ± reported uncertainty value.

This can be time-consuming and cumbersome as it does not involve percentage uncertainties in the final uncertainty determination.

If $k = 4.78 \pm 0.35 \text{ cm}^{-1}$ and $x = 23.5 \pm 0.1 \text{ cm}$, find $\sin(kx)$.

The value of kx is 112.33

The maximum value of kx is $(5.13) \times (23.6) = 121.068$

The minimum possible value of kx is $(4.43) \times (23.4) = 103.662$

$\sin(112.33) = 0.92501$ (mean value)

$\sin(121.068) = 0.85656$ (min value)

$\sin(103.662) = 0.97171$ (max value)

$|\text{max value} - \text{mean value}| = |0.97171 - 0.92501| = 0.04670$

$|\text{min value} - \text{mean value}| = |0.85656 - 0.92501| = 0.06845$

The final answer is therefore 0.925 ± 0.068 , better stated as 0.925 ± 0.070 (no units)

A student measures a distance several times. The readings lie between 49.8 cm and 50.2 cm. This measurement is best recorded as

A. $49.8 \pm 0.2 \text{ cm}$.

B. $49.8 \pm 0.4 \text{ cm}$

C. $50.0 \pm 0.2 \text{ cm}$

D. $50.0 \pm 0.4 \text{ cm}$.

The power dissipated in a resistor of resistance R carrying a current I is equal to I^2R . The value of I has an uncertainty of $\pm 2\%$ and the value of R has an uncertainty of $\pm 10\%$. The value of the uncertainty in the calculated power dissipation is

A. $\pm 8\%$.

B. $\pm 12\%$.

C. $\pm 14\%$.

D. $\pm 20\%$.

When a force F of $(10.0 \pm 0.2) \text{ N}$ is applied to a mass m of $(2.0 \pm 0.1) \text{ kg}$, the percentage uncertainty attached to the value of the calculated acceleration is

A. 2%.

B. 5%.

C. 7%.

D. 10%.

A student measures two lengths as follows:

$T = 10.0 \pm 0.1 \text{ cm}$

$S = 20.0 \pm 0.1 \text{ cm}$.

The student calculates:

FT, the fractional uncertainty in T

FS, the fractional uncertainty in S

FT–S, the fractional uncertainty in $(T - S)$

FT+S, the fractional uncertainty in $(T + S)$.

Which of these uncertainties has the largest magnitude?

A. FT

B. FS

C. FT–S

D. FT+S

The volume V of a cylinder of height h and radius r is given by the expression $V = \pi r^2 h$. In a particular experiment, r is to be determined from measurements of V and h . The uncertainties in V and in h are :

$V \pm 7\%$, $h \pm 3\%$

The approximate uncertainty in r is

A. 10%.

B. 5%.

C. 4%.

D. 2%.

III. UNCERTAINTIES IN GRAPHS

In many cases, the best way to present and analyze data is to make a graph. A graph is a visual representation of 2 things and shows nicely how they are related. A graph is the **visual display of quantitative information** and allows us to recognize trends in data. Graphs also let you display uncertainties nicely.

A. MAKING GRAPHS

You need to be able to make graphs by hand (always on graph paper!), even though in most cases in IB Physics you will be using computer software to create graphs using spreadsheets of data.

When making graphs:

1. The **independent variable** is on the x-axis and the **dependent variable** is on the y-axis.
2. Every graph should have a **title** that is concise but descriptive, in the form 'Graph of (dependent variable) vs. (independent variable)'.
3. The **scales** of the axes should suit the data ranges.
4. The axes should be **labeled** with the variable, units, and uncertainties.
5. Ample paper area should be used.
6. The **data points** should be clear.
7. **Error bars** should be shown correctly (using a straight-edge)
8. Data points should not be connected dot-to-dot fashion. A **line of best fit** should be drawn instead.
9. Each point that does not fit with the best fit line should be identified.
10. Think about whether the **origin** should be included in your graph (what is the physical significance of that point?)

When using software to create graphs, make sure that each graph has all of the above. The software will do some of it for you but you are responsible for making sure it is complete!

Note that the independent variable is often *evenly spaced* because it is controlled by the experimenter (for example, time at 0, 1, 2, 3, 4, 5, 6 s). When titling a graph, the dependent variable goes first.

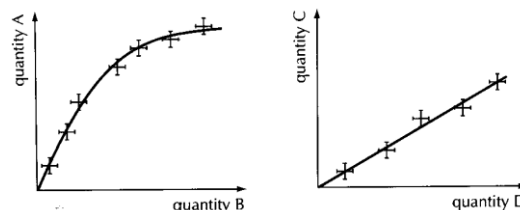
B. ERROR BARS

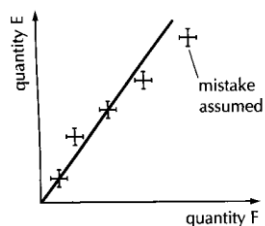
A nice way to show uncertainty in data is with **error bars**. These are bars in the x and y directions around each data point that show immediately how big or small the uncertainty is for that value. Uncertainties can be constant values for every data point or percentage values (in which case the length will vary).

Either way, an **error box** is created when there are error bars in both x and y directions around a data point. It is usually a rectangle and often varies in size around every point.

C. BEST FIT LINES

The **line of best fit** (which can be a curve OR a straight line) represents the trend shown on a graph. If you are doing this by hand, it is an estimation based on what the trend appears to be. For example, the data might suggest a linear relationship – if so, your job is to draw a line (with a straight-edge) that goes through as many data points as possible. Approximately the same number of data points should be above your line as below it.

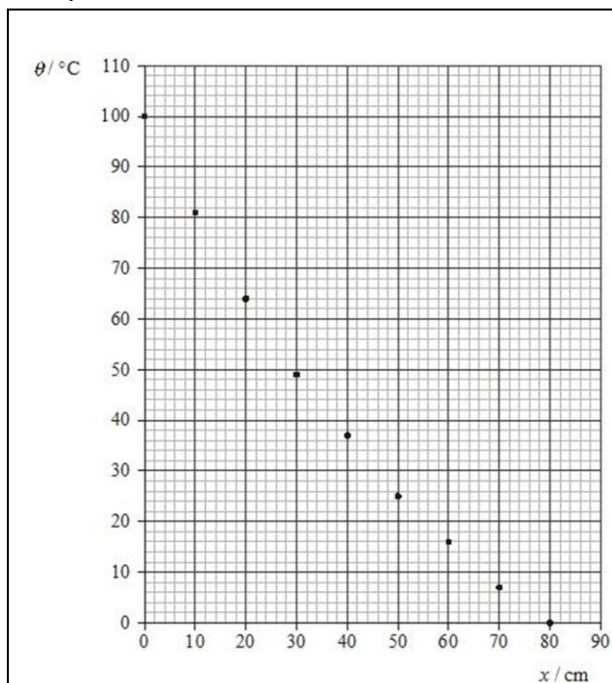
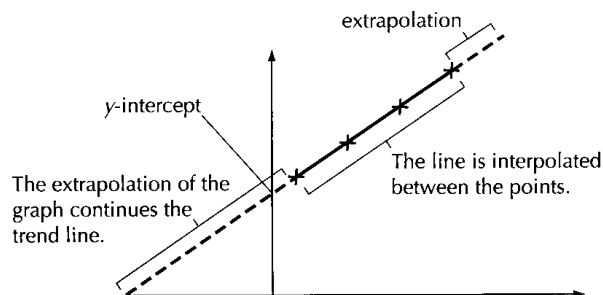




Your best fit line should also go through every single error box. If it is not possible to draw a line that goes through every single box, then your data is suspect OR you have chosen the wrong kind of line to fit.

Almost always, however, you will be using computer software to create graphs in IB Physics. The program will put the line of best fit on the graph, but it is up to you to determine what kind of fit you need.

Your line of best fit is a crucial bit of information in determining the relationship between the two variables being graphed. For example, if the relationship is linear, the slope of the line often gives the experimental value of a physical constant, often the whole point of your experiment! Sometime you may be asked to **interpolate** (estimate values in between data points) or even **extrapolate** (estimate values outside of the available data).



Data points are plotted for temperatures (θ) at certain distances along a copper rod, as shown in the graph.

The uncertainty in the measurement of θ is $\pm 2^\circ\text{C}$. The uncertainty in the measurement of x is negligible.

- On the graph, draw the uncertainty in the data points (error bars) for $x = 10$ cm, $x = 40$ cm and $x = 70$ cm.
- On the graph, draw the line of best-fit for the data.
- Use your graph to estimate the temperature of the rod at $x = 55$ cm.

We use graphs to analyze data all the time in Physics. Linear graphs are easier to work with without calculus.

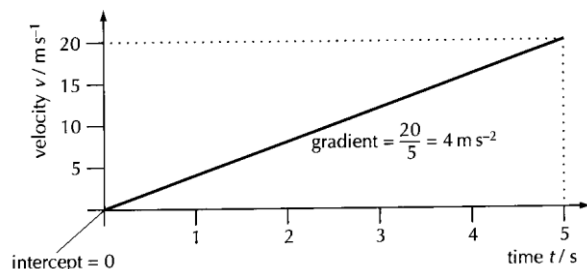
The equation of a linear graph is of the form: $y = mx + b$ where m is the slope of the line and b is the y -intercept. For any line, m and c are constants. We often have to choose carefully what to graph that will give us a straight line, and interpret that slope intelligently.

For example, an equation of motion is $v = u + at$ (where v = speed, u = initial speed, a = acceleration, and t = time). Clearly this is of the form:

$$y = mx + b$$

$$v = at + u$$

So, in graphing v vs. t we get:

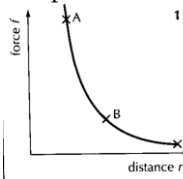


The slope gives the acceleration, in units of ms^{-2} ! And if the y-intercept is 0, then the initial speed was 0. What if the y-intercept had been 5 ms^{-1} ?

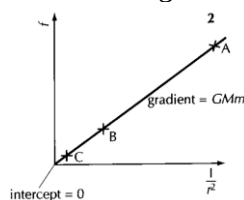
Being able to choose what to graph that will give you a straight line is very important. To do this, you must algebraically rearrange the equation, keeping in mind your constants and your variables. You can plot any mathematical combinations of your original readings on any one axis (this is still a variable), and eventually write your equation in the form $y = mx + b$. Sometimes this can really take some thinking!

The gravitational force F that acts on an object at a distance r away from the center of a planet is given by the equation $F = \frac{GMm}{r^2}$ where M is the mass of the planet, m is the mass of the object, and G is a constant.

1. Graph F and r .



2. Make a straight line graph of this data and interpret the meaning of the slope of the line.



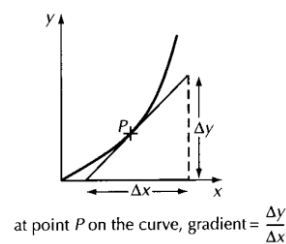
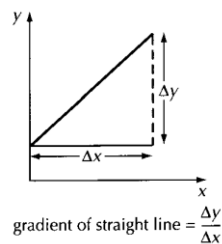
The 3 most useful things we use graphs for are:

1. y-intercept

- A linear graph can only cross the y axis once
- In experimental data, this value has physical significance.
- If a line goes through the origin, then the two quantities are **proportional** to one another.

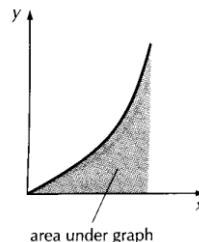
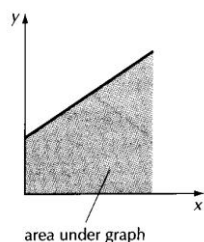
2. gradient (slope)

- The slope is the 'rise over run' or change in y over change in x or $\Delta y/\Delta x$.
- A linear graph has a constant slope.
- Slope has units. They are the y-units / x-units.
- When calculating the slope, **always** use the biggest triangle you can manage from your graph.
- Only if the x-axis is time does the slope give the **rate of change** ('rate' always refers to the change in something per unit time and is often misused.)
- The slope of a curve at a particular point is the slope of the tangent line at that point.



3. area between the line and the x-axis

- The area under a line often represents a physical quantity.
- The area can be found by finding the area of a trapezoid (for linear data, easy).
- The area has units. They are (y-units) x (x-units).
- The area under a curve can be found by counting all the small squares under it and adding them all up (provided you know the area of one square – difficult!)



The time period T of oscillation of a mass m suspended from a vertical spring is given by the expression

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{where } k \text{ is a constant.}$$

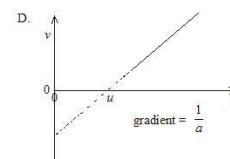
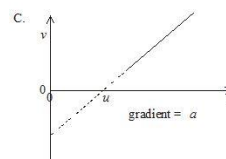
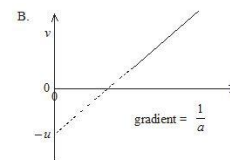
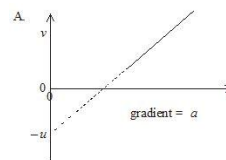
Which **one** of the following plots will give rise to a straight-line graph?

- A. T^2 against m B. $\sqrt{T}$ against $\sqrt{m}$
 C. T against m D. $\sqrt{T}$ against m

The variation with time t of the speed v of an object is given by the expression $v = u + at$ where u and a are constants.

A graph of the variation with time t of speed v is plotted.

Which one of the following correctly shows how the constants may be determined from this graph?



A particle is moving in a circular path of radius r . The time taken for one complete revolution is T . The

acceleration a of the particle is given by the expression $a = \frac{4\pi^2 r}{T^2}$.

Which of the following graphs would produce a straight-line?

- A. a against T B. a against T^2 C. a against $\frac{1}{T}$ D. a against $\frac{1}{T^2}$

E. WHAT TO DO WITH MAX AND MIN LINES

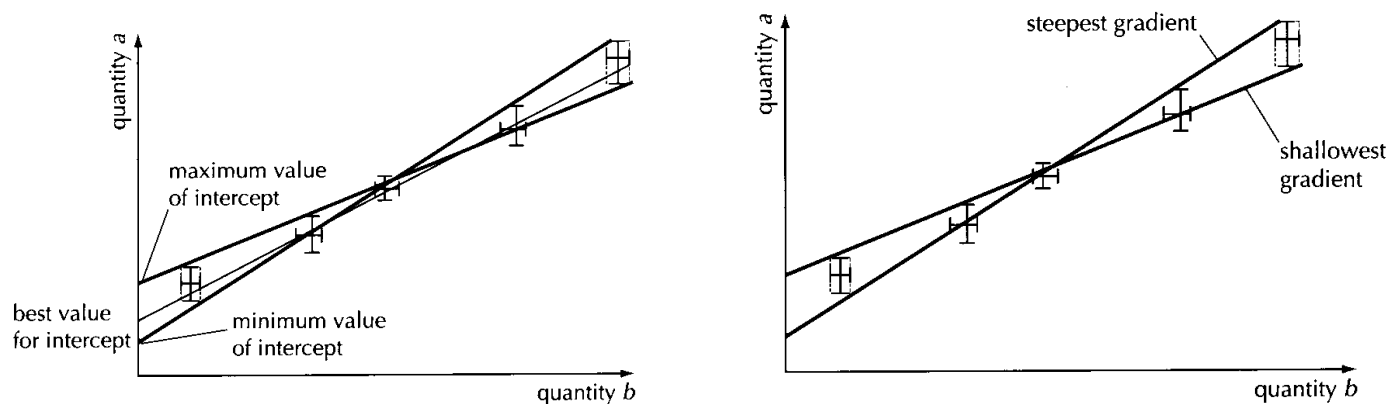
Remember that most Physics experiments lead to graphical analysis of data. We can use graphs to express and find uncertainties. You must include graphs of your data in aspect 3 in DCP.

Let's say you have gotten to the point where you have:

- taken data
- decided which variables to graph on the x and y axes, and done so with proper error bars shown
- found the line of best fit and the slope of this line (in Graphical Analysis or another suitable graphing program)

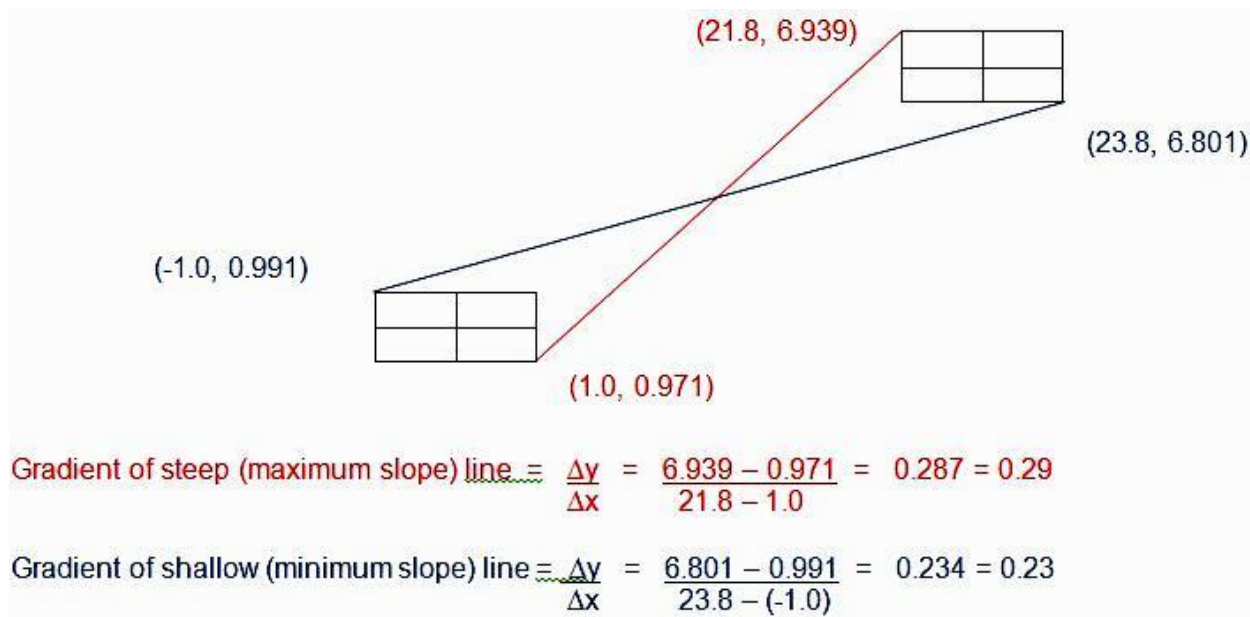
Therefore, you have found a value of the slope that corresponds to some physical quantity. Now you must use the **maximum** and **minimum best-fit lines** to determine the final uncertainty in the stated value of the slope of your best-fit line. Here's how:

1. Draw a straight line with the **least** slope possible (minimum best-fit line) that connects corners of your first and last error boxes.
2. Draw a straight line with the **greatest** slope possible (maximum best-fit line) that connects corners of your first and last error boxes.
3. Determine the slopes of these two lines.
4. Your final uncertainty in the stated value of the slope of your best fit line is: $(\text{max slope} - \text{min slope})/2$.



Lines of Best Fit and Max/Min Lines for 'Graph of Quantity a vs. Quantity b'

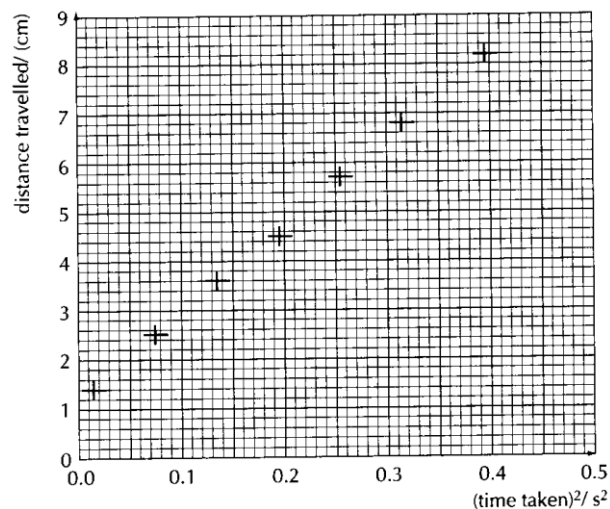
Note that by using this technique, you may get max and min lines that do not go through the error boxes of every data point. This is ok and you will not be penalized for it. Another way to think of it:



The graph of distance travelled vs. time² for an object rolling down an inclined plane is shown.

1. What quantity is given by the gradient of the graph?

2. Explain why the graph suggests that the collected data is valid, but includes a systematic error.



3. Does the data suggest that distance is proportional to time? Explain your answer.

4. Making allowance for the systematic error, calculate the acceleration of the object.

5. The following graph shows the same data after the uncertainty ranges have been calculated and drawn as error bars. Draw the best-fit line, and add two more lines to show the range of the possible acceptable values for the gradient of the graph.

