

Μάθημα 22ο - Αναγωγή στο 1ο τεταρτημόριο

Θέμα

1. Το $\eta\mu 120^\circ$ ισούται με	$\eta\mu(180^\circ - 60^\circ) = \eta\mu 60^\circ = \frac{\sqrt{3}}{2}$
2. Το $\sigma\upsilon\nu 210^\circ$ ισούται με	$\sigma\upsilon\nu(180^\circ + 30^\circ) = -\sigma\upsilon\nu 30^\circ = -\frac{\sqrt{3}}{2}$
3. Η $\epsilon\varphi 300^\circ$ ισούται με	$\epsilon\varphi(360^\circ - 60^\circ) = -\epsilon\varphi 60^\circ = -\sqrt{3}$
4. Η $\sigma\varphi 225^\circ$ ισούται	$\sigma\varphi(180^\circ + 45^\circ) = \sigma\varphi 45^\circ = 1$
5. Το $\eta\mu 240^\circ$ ισούται με	$\eta\mu(180^\circ + 60^\circ) = -\eta\mu 60^\circ = -\frac{\sqrt{3}}{2}$
6. Το $\sigma\upsilon\nu 150^\circ$ ισούται με	$\sigma\upsilon\nu(180^\circ - 30^\circ) = -\sigma\upsilon\nu 30^\circ = -\frac{\sqrt{3}}{2}$
7. Η $\epsilon\varphi 210^\circ$ ισούται με	$\epsilon\varphi(180^\circ + 30^\circ) = \epsilon\varphi 30^\circ = \frac{\sqrt{3}}{3}$
8. Η $\sigma\varphi 135^\circ$ ισούται με	$\sigma\varphi(180^\circ - 45^\circ) = -\sigma\varphi 45^\circ = -1$
9. Το $\eta\mu 330^\circ$ ισούται με	$\eta\mu(360^\circ - 30^\circ) = -\eta\mu 30^\circ = -\frac{1}{2}$
10. Το $\sigma\upsilon\nu 300^\circ$ ισούται με	$\sigma\upsilon\nu(360^\circ - 60^\circ) = \sigma\upsilon\nu 60^\circ = \frac{1}{2}$
11. Η $\epsilon\varphi 120^\circ$ ισούται με	$\epsilon\varphi(180^\circ - 60^\circ) = -\epsilon\varphi 60^\circ = -\sqrt{3}$
12. Η $\sigma\varphi 315^\circ$ ισούται με	$\sigma\varphi(360^\circ - 45^\circ) = -\sigma\varphi 45^\circ = -1$
13. Το $\eta\mu 150^\circ$ ισούται με	$\eta\mu(180^\circ - 30^\circ) = \eta\mu 30^\circ = \frac{1}{2}$
14. Το $\sigma\upsilon\nu 240^\circ$ ισούται με	$\sigma\upsilon\nu(270^\circ - 30^\circ) = -\eta\mu 30^\circ = -\frac{1}{2}$
15. Η $\epsilon\varphi 330^\circ$ ισούται με	$\epsilon\varphi(360^\circ - 30^\circ) = -\epsilon\varphi 30^\circ = -\frac{\sqrt{3}}{3}$
16. Η $\sigma\varphi 120^\circ$ ισούται με	$\sigma\varphi(180^\circ - 60^\circ) = -\sigma\varphi 60^\circ = -\frac{\sqrt{3}}{3}$

$$\eta\mu\left(\frac{5\pi}{2} - \theta\right) = \eta\mu\left(2\pi + \frac{\pi}{2} - \theta\right) = \eta\mu\left(\frac{\pi}{2} - \theta\right) = \sigma\upsilon\nu\theta$$

$$\sigma\upsilon\nu\left(\frac{7\pi}{2} - \theta\right) = \sigma\upsilon\nu\left(2\pi + \frac{3\pi}{2} - \theta\right) = \sigma\upsilon\nu\left(\frac{3\pi}{2} - \theta\right) = -\eta\mu\theta$$

$$\epsilon\varphi\left(\frac{9\pi}{2} - \theta\right) = \epsilon\varphi\left(4\pi + \frac{\pi}{2} - \theta\right) = \epsilon\varphi\left(\frac{\pi}{2} - \theta\right) = \sigma\varphi\theta$$

$$\sigma\varphi\left(\frac{11\pi}{2} - \theta\right) = \sigma\varphi\left(4\pi + \frac{3\pi}{2} - \theta\right) = \sigma\varphi\left(\frac{3\pi}{2} - \theta\right) = \epsilon\varphi\theta$$

$$\eta\mu\left(\frac{13\pi}{2} - \theta\right) = \eta\mu\left(6\pi + \frac{\pi}{2} - \theta\right) = \eta\mu\left(\frac{\pi}{2} - \theta\right) = \sigma\upsilon\nu\theta$$

$$\sigma\upsilon\nu\left(\frac{19\pi}{2} - \theta\right) = \sigma\upsilon\nu\left(8\pi + \frac{3\pi}{2} - \theta\right) = \sigma\upsilon\nu\left(\frac{3\pi}{2} - \theta\right) = -\eta\mu\theta$$

4. Να απλοποιήσετε την παράσταση $\frac{\sigma\upsilon\nu(-\alpha) \cdot \sigma\upsilon\nu(180^\circ + \alpha)}{\eta\mu(-\alpha) \cdot \eta\mu(90^\circ + \alpha)} =$

$$\frac{\sigma\upsilon\nu\alpha \cdot (-\sigma\upsilon\nu\alpha)}{-\eta\mu\alpha \cdot \sigma\upsilon\nu\alpha} = \sigma\phi\alpha$$

6. Να δείξετε ότι έχει σταθερή τιμή η παράσταση:

$$\eta\mu^2(\pi - x) + \sigma\upsilon\nu(\pi - x)\sigma\upsilon\nu(2\pi - x) + 2\eta\mu^2\left(\frac{\pi}{2} - x\right).$$

$$\begin{aligned} &= \eta\mu^2 x + (-\sigma\upsilon\nu x) \cdot \sigma\upsilon\nu x + 2\sigma\upsilon\nu^2 x = \eta\mu^2 x - \sigma\upsilon\nu^2 x + 2\sigma\upsilon\nu^2 x = \\ &= \eta\mu^2 x + \sigma\upsilon\nu^2 x = 1 \end{aligned}$$

5. Να αποδείξετε ότι: $\frac{\epsilon\phi(\pi - x) \cdot \sigma\upsilon\nu(2\pi + x) \cdot \sigma\upsilon\nu\left(\frac{9\pi}{2} + x\right)}{\eta\mu(13\pi + x) \cdot \sigma\upsilon\nu(-x) \cdot \sigma\phi\left(\frac{21\pi}{2} - x\right)} = -1.$

$$\epsilon\phi(\pi - x) = -\epsilon\phi x$$

$$\sigma\upsilon\nu(2\pi + x) = \sigma\upsilon\nu x$$

$$\sigma\upsilon\nu\left(\frac{9\pi}{2} + x\right) = \sigma\upsilon\nu\left(4\pi + \frac{\pi}{2} + x\right) = \sigma\upsilon\nu\left(\frac{\pi}{2} + x\right) = -\eta\mu x$$

$$\eta\mu(13\pi + x) = \eta\mu(12\pi + \pi + x) = \eta\mu(\pi + x) = -\eta\mu x$$

$$\sigma\upsilon\nu(-x) = \sigma\upsilon\nu x$$

$$\sigma\phi\left(\frac{21\pi}{2} - x\right) = \sigma\phi\left(10\pi + \frac{\pi}{2} - x\right) = \sigma\phi\left(\frac{\pi}{2} - x\right) = \epsilon\phi x$$

$$\text{οπότε } \frac{-\epsilon\phi x \cdot \sigma\upsilon\nu x \cdot (-\eta\mu x)}{-\eta\mu x \cdot \sigma\upsilon\nu x \cdot \epsilon\phi x} = -1$$

2. Να αποδείξετε ότι:

$$\frac{\eta\mu(5\pi + \omega) \cdot \sigma\upsilon\nu(7\pi - \omega) \cdot \eta\mu\left(\frac{5\pi}{2} - \omega\right) \cdot \sigma\upsilon\nu\left(\frac{7\pi}{2} + \omega\right)}{\sigma\varphi(5\pi + \omega) \cdot \eta\mu(7\pi - \omega) \cdot \sigma\upsilon\nu\left(\frac{5\pi}{2} - \omega\right) \cdot \sigma\varphi\left(\frac{7\pi}{2} + \omega\right)} = \eta\mu^2\omega - 1.$$

$$\psi(5\pi + \omega) = \psi(4\pi + \pi + \omega) = \psi(\pi + \omega) = -\psi\omega$$

$$\sigma\upsilon\nu(7\pi - \omega) = \sigma\upsilon\nu(6\pi + \pi - \omega) = \sigma\upsilon\nu(\pi - \omega) = -\sigma\upsilon\nu\omega$$

$$\psi\left(\frac{5\pi}{2} - \omega\right) = \psi\left(2\pi + \frac{\pi}{2} - \omega\right) = \psi\left(\frac{\pi}{2} - \omega\right) = \sigma\upsilon\nu\omega$$

$$\sigma\upsilon\nu\left(\frac{7\pi}{2} + \omega\right) = \sigma\upsilon\nu\left(2\pi + \frac{3\pi}{2} + \omega\right) = \sigma\upsilon\nu\left(\frac{3\pi}{2} + \omega\right) = \psi\omega$$

$$\sigma\varphi(5\pi + \omega) = \sigma\varphi(4\pi + \pi + \omega) = \sigma\varphi(\pi + \omega) = \sigma\varphi\omega$$

$$\psi(7\pi - \omega) = \psi(6\pi + \pi - \omega) = \psi(\pi - \omega) = \psi\omega$$

$$\sigma\upsilon\nu\left(\frac{5\pi}{2} - \omega\right) = \sigma\upsilon\nu\left(2\pi + \frac{\pi}{2} - \omega\right) = \sigma\upsilon\nu\left(\frac{\pi}{2} - \omega\right) = \psi\omega$$

$$\sigma\varphi\left(\frac{7\pi}{2} + \omega\right) = \sigma\varphi\left(2\pi + \frac{3\pi}{2} + \omega\right) = \sigma\varphi\left(\frac{3\pi}{2} + \omega\right) = -\sigma\varphi\omega$$

$$\text{οπότε} \quad \frac{-\cancel{\psi\omega}(-\sigma\upsilon\nu\omega) \sigma\upsilon\nu\omega \cdot \cancel{\psi\omega}}{\sigma\varphi\omega \cdot \cancel{\psi\omega} \cdot \cancel{\psi\omega} (-\sigma\varphi\omega)} = -\sigma\upsilon\nu^2\omega = \psi^2\omega - 1$$