

1. Να βρείτε τους τριγωνομετρικούς αριθμούς της γωνίας:

i) 1200°

ii) -2850°

$$\begin{array}{r} 1200^\circ \\ 1080^\circ \\ \hline 120^\circ \end{array} \bigg| \begin{array}{r} 360^\circ \\ 3 \end{array}$$

$$1200^\circ = 3 \cdot 360^\circ + 120^\circ$$

$$\psi 1200^\circ = \psi (3 \cdot 360^\circ + 120^\circ) = \psi 120^\circ = \psi 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 1200^\circ = \dots = \sin 120^\circ = -\sin 60^\circ = -\frac{1}{2}$$

$$\cos 1200^\circ = \dots = \cos 120^\circ = -\cos 60^\circ = -\frac{1}{2}$$

$$\sec 1200^\circ = \dots = \sec 120^\circ = -\sec 60^\circ = -\frac{2}{\sqrt{3}}$$

$$\begin{array}{r} 2850^\circ \\ 2520^\circ \\ \hline 330^\circ \end{array} \bigg| \begin{array}{r} 360^\circ \\ 7 \end{array}$$

$$2850^\circ = 7 \cdot 360^\circ + 330^\circ$$

$$-2850^\circ = -7 \cdot 360^\circ - 330^\circ$$

$$-2850^\circ = -7 \cdot 360^\circ - 360^\circ + 30^\circ$$

$$-2850^\circ = -8 \cdot 360^\circ + 30^\circ$$

$$\psi (-2850^\circ) = \psi (-8 \cdot 360^\circ + 30^\circ) = \psi 30^\circ = \frac{1}{2}$$

$$\sin (-2850^\circ) = \sin (-8 \cdot 360^\circ + 30^\circ) = \sin 30^\circ = \frac{1}{2}$$

2ος τρόπος

$$\psi (-2850^\circ) = -\psi 2850^\circ =$$

$$-\psi (7 \cdot 360^\circ + 330^\circ) = -\psi 330^\circ =$$

$$= -\psi (360^\circ - 30^\circ) = -(-\psi 30^\circ) = \psi 30^\circ = \frac{1}{2}$$

$$\cos (-2850^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sec (-2850^\circ) = \sec 30^\circ = \frac{2}{\sqrt{3}}$$

2. Να βρείτε τους τριγωνομετρικούς αριθμούς της γωνίας

i) $\frac{187\pi}{6}$ rad

ii) $\frac{21\pi}{4}$ rad.

$$\begin{array}{r} 187 \\ -180 \\ \hline 7 \end{array} \bigg| \begin{array}{r} 6 \\ 31 \end{array}$$

$$187 = 31 \cdot 6 + 1$$

$$\frac{187}{6} = \frac{31 \cdot 6}{6} + \frac{1}{6}$$

$$\frac{187}{6} = 31 + \frac{1}{6}$$

$$\frac{187 \cdot \pi}{6} = 31\pi + \frac{\pi}{6}$$

$$\begin{aligned} \psi \frac{187\pi}{6} &= \psi \left(31\pi + \frac{\pi}{6} \right) = \\ &= \psi \left(30\pi + \pi + \frac{\pi}{6} \right) = \psi \left(\pi + \frac{\pi}{6} \right) = \end{aligned}$$

$$= -\psi \frac{\pi}{6} = -\frac{1}{2}$$

$$\sin \frac{187\pi}{6} = \sin \left(31\pi + \frac{\pi}{6} \right) =$$

$$= \sin (30\pi + \pi + \frac{\pi}{6}) = \sin \left(\pi + \frac{\pi}{6} \right) =$$

$$= -\sin \frac{\pi}{6} = -\frac{1}{2}$$

$$\cos \frac{187\pi}{6} = \frac{\psi \frac{187\pi}{6}}{\sin \frac{187\pi}{6}} = \frac{-\frac{1}{2}}{-\frac{1}{2}} = \frac{2}{2} = \frac{1}{1} = \frac{\sqrt{3}}{3}$$

$$\sec \frac{187\pi}{6} = \frac{1}{\cos \frac{187\pi}{6}} = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$$

3. Σε κάθε τρίγωνο $AB\Gamma$ να αποδείξετε ότι:

i) $\eta\mu A = \eta\mu(B+\Gamma)$

ii) $\sigma\upsilon\nu A + \sigma\upsilon\nu(B+\Gamma) = 0$

iii) $\eta\mu \frac{A}{2} = \sigma\upsilon\nu \frac{B+\Gamma}{2}$

iv) $\sigma\upsilon\nu \frac{A}{2} = \eta\mu \frac{B+\Gamma}{2}$.

$$\hat{A} + \hat{B} + \hat{\Gamma} = \pi \Rightarrow \boxed{\hat{B} + \hat{\Gamma} = \pi - \hat{A}} \quad (1) \Rightarrow \boxed{\frac{\hat{B} + \hat{\Gamma}}{2} = \frac{\pi}{2} - \frac{\hat{A}}{2}} \quad (2)$$

i) $\psi(\hat{B} + \hat{\Gamma}) \stackrel{(1)}{=} \psi(\pi - \hat{A}) = \psi \hat{A}$

ii) $\sigma\upsilon\nu \hat{A} + \sigma\upsilon\nu(\hat{B} + \hat{\Gamma}) \stackrel{(1)}{=} \sigma\upsilon\nu \hat{A} + \sigma\upsilon\nu(\pi - \hat{A}) = \sigma\upsilon\nu \hat{A} + (-\sigma\upsilon\nu \hat{A}) = 0$

iii) $\sigma\upsilon\nu \frac{\hat{B} + \hat{\Gamma}}{2} \stackrel{(2)}{=} \sigma\upsilon\nu \left(\frac{\pi}{2} - \frac{\hat{A}}{2} \right) = \psi \frac{\hat{A}}{2}$

iv) $\psi \frac{\hat{B} + \hat{\Gamma}}{2} \stackrel{(2)}{=} \psi \left(\frac{\pi}{2} - \frac{\hat{A}}{2} \right) = \sigma\upsilon\nu \frac{\hat{A}}{2}$