

Σελ. 88 ΣΧΟΛΙΚΟΥ

10. Να λύσετε τις εξισώσεις

i) $2\eta\mu^2\omega + \eta\mu\omega - 1 = 0$ ii) $2\sigma\upsilon\nu^2x + 3\sigma\upsilon\nu x - 2 = 0$ iii) $3\epsilon\varphi^2t = 3 + 2\sqrt{3}\epsilon\varphi t$

i) Θετω $\eta\mu\omega = x$

$$2x^2 + x - 1 = 0$$

$$\Delta = 9, \quad x_1 = -1, \quad x_2 = \frac{1}{2}$$

για $x = -1$

$$\eta\mu\omega = -1$$

$$\boxed{\omega = 2k\pi - \frac{\pi}{2}}$$

$$k \in \mathbb{Z}$$

για $x = \frac{1}{2}$

$$\eta\mu\omega = \frac{1}{2}$$

$$\boxed{\omega = 2k\pi + \frac{\pi}{6}}$$

$$\omega = 2k\pi + \frac{\pi}{6}$$

$$\boxed{\omega = 2k\pi + \frac{5\pi}{6}}$$

ii) Θετω $\sigma\upsilon\nu x = w$

$$2w^2 + 3w - 2 = 0$$

$$\Delta = 25$$

$$w_1 = -2 \quad \text{ή} \quad w_2 = \frac{1}{2}$$

$$\sigma\upsilon\nu x = -2$$

αδύνατο

$$\sigma\upsilon\nu x = \frac{1}{2}$$

$$\sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{3}$$

$$\boxed{x = 2k\pi + \frac{\pi}{3}}$$

$$\boxed{x = 2k\pi - \frac{\pi}{3}}$$

$$k \in \mathbb{Z}$$

iii) $3\epsilon\varphi^2t - 2\sqrt{3}\epsilon\varphi t - 3 = 0$

Θετω $\epsilon\varphi t = x$

$$3x^2 - 2\sqrt{3}x - 3 = 0$$

$$\Delta = (-2\sqrt{3})^2 - 4 \cdot 3 \cdot (-3) = 12 + 36 = 48$$

$$x_{1,2} = \frac{-(-2\sqrt{3}) \pm \sqrt{48}}{2 \cdot 3} =$$

$$= \frac{2\sqrt{3} \pm 4\sqrt{3}}{6} = \begin{cases} \frac{6\sqrt{3}}{6} = \sqrt{3} \\ \frac{-2\sqrt{3}}{6} = -\frac{\sqrt{3}}{3} \end{cases}$$

για $x = \sqrt{3}$

$$\epsilon\varphi t = \sqrt{3}$$

$$\epsilon\varphi t = \epsilon\varphi \frac{\pi}{3}$$

$$\boxed{t = k\pi + \frac{\pi}{3}}$$

$$k \in \mathbb{Z}$$

για $x = -\frac{\sqrt{3}}{3}$

$$\epsilon\varphi t = -\frac{\sqrt{3}}{3}$$

$$\epsilon\varphi t = \epsilon\varphi \left(-\frac{\pi}{6}\right)$$

$$\boxed{t = k\pi - \frac{\pi}{6}}$$

11. Να λύσετε τις εξισώσεις

i) $\eta\mu^2x + 5\sigma\upsilon\nu^2x = 4$

$$1 - \sigma\upsilon\nu^2x + 5\sigma\upsilon\nu^2x = 4$$

$$4\sigma\upsilon\nu^2x = 3$$

$$\sigma\upsilon\nu^2x = \frac{3}{4}$$

$$\sigma\upsilon\nu x = \frac{\sqrt{3}}{2} \quad \text{ή} \quad \sigma\upsilon\nu x = -\frac{\sqrt{3}}{2}$$

$$\sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{6}$$

$$\boxed{x = 2k\pi \pm \frac{\pi}{6}}$$

$$\sigma\upsilon\nu x = \sigma\upsilon\nu \left(\frac{5\pi}{6}\right)$$

$$\boxed{x = 2k\pi \pm \frac{5\pi}{6}}$$

$$k \in \mathbb{Z}$$

ii) $\epsilon\varphi x \cdot \sigma\varphi 2x = 1$

Αν $\sigma\varphi 2x = 0$ Ατομο
Αν $\sigma\varphi 2x \neq 0$ και $\epsilon\varphi x$

$$\epsilon\varphi x = \frac{1}{\sigma\varphi 2x}$$

$$\epsilon\varphi x = \epsilon\varphi 2x$$

$$x = k\pi + 2x$$

$$-x = k\pi$$

$$x = -k\pi \rightarrow \text{απορρίπτεται}$$

$$k \in \mathbb{Z}$$

Τελικά η εξίσωση

είναι αδύνατη

$$2x \neq k\pi + \frac{\pi}{2}$$

$$x \neq \frac{k\pi}{2} + \frac{\pi}{4}$$

$$\frac{\pi}{3} + 3x \neq k\pi$$

$$3x \neq k\pi - \frac{\pi}{3}$$

$$x \neq \frac{k\pi}{3} - \frac{\pi}{9}$$

1. Να λύσετε τις εξισώσεις

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i) $\eta\mu x + \sigma\upsilon\nu\left(\frac{\pi}{4} - x\right) = 0$

ii) $\epsilon\varphi 2x - \sigma\varphi\left(\frac{\pi}{3} + 3x\right) = 0$

Handwritten notes above equation ii):
 $x \neq \frac{\pi}{2} + \frac{\pi}{4}$ (pointing to $\frac{\pi}{4}$)
 $3x \neq \frac{\pi}{3} - \frac{\pi}{9}$ (pointing to $\frac{\pi}{3}$)

$$\sigma\upsilon\nu\left(\frac{\pi}{4} - x\right) = -\eta\mu x$$

$$\sigma\upsilon\nu\left(\frac{\pi}{4} - x\right) = \sigma\upsilon\nu\left(\frac{\pi}{2} + x\right)$$

$$\frac{\pi}{4} - x = 2k\pi + \frac{\pi}{2} + x \quad \vee \quad \frac{\pi}{4} - x = 2k\pi - \frac{\pi}{2} - x$$

$$-x - x = 2k\pi + \frac{\pi}{2} - \frac{\pi}{4}$$

$$-2x = 2k\pi + \frac{\pi}{4}$$

$$\boxed{x = -k\pi - \frac{\pi}{8}} \quad k \in \mathbb{Z}$$

$$-x + x = 2k\pi - \frac{\pi}{2} - \frac{\pi}{4}$$

$$0 = 2k\pi - \frac{3\pi}{4}$$

Αδύνατο

$$\epsilon\varphi 2x = \sigma\varphi\left(\frac{\pi}{3} + 3x\right)$$

$$\sigma\varphi\left(\frac{\pi}{2} - 2x\right) = \sigma\varphi\left(\frac{\pi}{3} + 3x\right)$$

$$\frac{\pi}{2} - 2x = k\pi + \frac{\pi}{3} + 3x$$

$$-2x - 3x = k\pi + \frac{\pi}{3} - \frac{\pi}{2}$$

$$-5x = k\pi - \frac{\pi}{6}$$

$$\boxed{x = -\frac{k\pi}{5} + \frac{\pi}{30}} \quad k \in \mathbb{Z}$$