

Θέμα 1^ο Να λυθούν οι εξισώσεις:

α) $\frac{x}{x-1} + \frac{x+2}{x+1} = \frac{2}{x^2-1}$

β) $\frac{x^2+2x-4}{x-2} = x^2$

β) $(x-2) \frac{x^2+2x-4}{x-2} = x^2 (x-2) \quad x \neq 2$

$$x^2+2x-4 = x^3-2x^2$$

$$x^3-3x^2-2x+4=0$$

$$\begin{array}{rrrr|l} 1 & -3 & -2 & 4 & 1 \\ & 1 & -2 & -4 & \\ \hline 1 & -2 & -4 & 0 & \end{array}$$

$$(x-1)(x^2-2x-4)=0$$

$$x=1 \text{ ή } x^2-2x-4=0$$

Δεκτή

$$\Delta=20$$

$$x = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = \frac{2(1 \pm \sqrt{5})}{2} = \begin{cases} 1+\sqrt{5} \\ 1-\sqrt{5} \end{cases}$$

Δεκτές

Θέμα 2^ο Να λυθούν οι ανισώσεις:

α) $\frac{x^3+2x-4}{x-2} < 1$

β) $\frac{x^2}{x+1} - \frac{4}{x-1} \leq \frac{2}{x^2-1}$

γ) $\frac{1-x}{-x^2-5x-6} \leq 0$

δ) $\frac{x-3}{2x-1} \geq 1$

ε) $\frac{x+2}{3x+1} \geq \frac{x-2}{2x-1}$

στ) $\frac{x-3}{x+2} + \frac{x+2}{3-x} > \frac{x^2+9}{x^2-x-6}$

ζ) $x^3-4x^2+x+6 > 0$

η) $x^3-3x+2 \geq 0$

α) $\frac{x^3+2x-4}{x-2} < 1 \Leftrightarrow \frac{x^3+2x-4}{x-2} - 1 < 0 \Leftrightarrow$

$$\frac{x^3+2x-4-x+2}{x-2} < 0 \Leftrightarrow \frac{x^3+x-2}{x-2} < 0$$

$$\frac{x^3 + 2x - 4 - x + 2}{x-2} < 0 \iff \frac{x^3 + x - 2}{x-2} < 0 \iff$$

$$\begin{array}{ccc|c|c} 1 & 0 & 1 & -2 & 1 \\ & 1 & 1 & 2 & \\ 1 & 1 & 2 & 0 & \end{array}$$

Teil a

$$x \in (1, 2)$$

$$\frac{(x^2 + x + 2)(x-1)}{x-2} < 0$$

x	$-\infty$	1	2	$+\infty$
$x-1$	-	0	+	+
$x-2$	-	-	0	+
x^2+x+2	+	+	+	+
$p(x)$	+	0	-	+

$$\beta) \frac{x^2}{x+1} - \frac{4}{x-1} \leq \frac{2}{x^2-1} \iff \frac{x-1}{x^2} - \frac{x+1}{4} - \frac{1}{2} \leq 0 \iff$$

$$x \neq -1 \text{ und } x \neq 1$$

$$\frac{x^2(x-1) - 4(x+1) - 2}{(x-1)(x+1)} \leq 0 \iff$$

$$\frac{x^3 - x^2 - 4x - 4 - 2}{(x-1)(x+1)} \leq 0 \iff \frac{x^3 - x^2 - 4x - 6}{(x-1)(x+1)} \leq 0$$

$$\begin{array}{ccc|c|c} 1 & -1 & -4 & -6 & 3 \\ & 3 & 6 & 6 & \\ 1 & 2 & 2 & 0 & \end{array}$$

$$x \in (-\infty, -1) \cup (1, 3]$$

$$\frac{(x^2 + 2x + 2)(x-3)}{(x-1)(x+1)} \leq 0$$

x	$-\infty$	-1	1	3	$+\infty$
$x-1$	-	-	0	+	+
$x+1$	-	0	+	+	+
$x-3$	-	-	-	0	+
x^2+2x+2	+	+	+	+	+
$p(x)$	-	+	-	0	+

$$\delta) \frac{x-3}{2x-1} \geq 1$$

$$2x-1 \neq 0$$

$$x \neq \frac{1}{2}$$

$$\frac{x-3}{2x-1} - 1 \geq 0 \iff \frac{x-3}{2x-1} - \frac{2x-1}{2x-1} \geq 0 \iff \frac{-x-2}{2x-1} \geq 0$$

x	$-\infty$	-2	$\frac{1}{2}$	$+\infty$
$-x-2$	+	0	-	-
$2x-1$	-	-	0	+
$p(x)$	-	0	+	-

$$x \in [-2, \frac{1}{2})$$

