

2. Να λύσετε τις εξισώσεις:

i) $4^{x-1} - 5\sqrt{4^{x-2}} + 1 = 0$

ii) $3^x + 3^{x-1} = \frac{45}{3^{x+2}} + \frac{7}{3^x}$

iii) $21 \cdot 3^x + 5^{x+3} = 3^{x+4} + 5^{x+2}$

iv) $3^{2x} + 9^x = 11 \cdot 4^{x-1} + 4^{x+1}$

v) $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

i) $\frac{4^x}{4} - 5\sqrt{\frac{4^x}{4^2}} + 1 = 0 \Leftrightarrow \frac{4^x}{4} - 5\frac{\sqrt{4^x}}{4} + 1 = 0 \Leftrightarrow$

$4^x - 5\sqrt{4^x} + 4 = 0$ Θέτω $\sqrt{4^x} = w$ οπότε $4^x = w^2$ και έχουμε

$w^2 - 5w + 4 = 0$

$w = 4$	ή	$w = 1$
$\sqrt{4^x} = 4$		$\sqrt{4^x} = 1$
$4^x = 16$		$4^x = 1$
$\boxed{x = 2}$		$\boxed{x = 0}$

ii) $3^x + 3^{x-1} = \frac{45}{3^{x+2}} + \frac{7}{3^x}$

$3^x + \frac{3^x}{3} = \frac{45}{3^x \cdot 3^2} + \frac{7}{3^x}$

$3^x \left(1 + \frac{1}{3}\right) = \frac{5}{3^x} + \frac{7}{3^x} \Leftrightarrow 3^x \cdot \frac{4}{3} = \frac{12}{3^x}$

$3^{2x} = 9 \Leftrightarrow 2x = 2 \Leftrightarrow \boxed{x = 1}$

iii) $21 \cdot 3^x + 5^{x+3} = 3^{x+4} + 5^{x+2}$

$\Leftrightarrow 21 \cdot 3^x + 5^x \cdot 5^3 = 3^x \cdot 3^4 + 5^x \cdot 5^2 \Leftrightarrow$

$21 \cdot 3^x + 125 \cdot 5^x = 81 \cdot 3^x + 25 \cdot 5^x \Leftrightarrow 125 \cdot 5^x - 25 \cdot 5^x = 81 \cdot 3^x - 21 \cdot 3^x \Leftrightarrow$

$100 \cdot 5^x = 60 \cdot 3^x \Leftrightarrow \frac{5^x}{3^x} = \frac{60}{100} \Leftrightarrow \left(\frac{5}{3}\right)^x = \frac{3}{5} \Leftrightarrow \boxed{x = -1}$

iv) $3^{2x} + 9^x = 11 \cdot 4^{x-1} + 4^{x+1}$

$\Leftrightarrow 3^{2x} + 3^{2x} = 11 \cdot \frac{4^x}{4} + 4^x \cdot 4 \Leftrightarrow$

$2 \cdot 3^{2x} = 4^x \left(\frac{11}{4} + 4\right) \Leftrightarrow 2 \cdot 3^{2x} = 4^x \cdot \frac{27}{4} \Leftrightarrow \frac{3^{2x}}{4^x} = \frac{27}{4 \cdot 2} \Leftrightarrow$

$\frac{3^{2x}}{2^{2x}} = \frac{27}{8} \Leftrightarrow \left(\frac{3}{2}\right)^{2x} = \left(\frac{3}{2}\right)^3 \Leftrightarrow 2x = 3 \Leftrightarrow x = \frac{3}{2}$

v) $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

$\Leftrightarrow 2^{2x} - \frac{3^x}{3^{\frac{1}{2}}} = 3^x \cdot 3^{\frac{1}{2}} - \frac{2^{2x}}{2} \Leftrightarrow$

$2^{2x} - \frac{3^x}{\sqrt{3}} = 3^x \cdot \sqrt{3} - \frac{2^{2x}}{2} \Leftrightarrow 2^{2x} + \frac{2^{2x}}{2} = 3^x \cdot \sqrt{3} + \frac{3^x}{\sqrt{3}} \Leftrightarrow$

$2^{2x} \left(1 + \frac{1}{2}\right) = 3^x \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$

$$\begin{aligned}
 2^{2x} \left(1 + \frac{1}{2}\right) &= 3^x \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) \Leftrightarrow 2^{2x} \cdot \frac{3}{2} = 3^x \cdot \frac{4}{\sqrt{3}} \Leftrightarrow \\
 \frac{2^{2x}}{3^x} &= \frac{2 \cdot 4}{3 \cdot \sqrt{3}} \Leftrightarrow \frac{2^{2x}}{\sqrt{3}^{2x}} = \frac{8}{3\sqrt{3}} \Leftrightarrow \left(\frac{2}{\sqrt{3}}\right)^{2x} = \left(\frac{2}{\sqrt{3}}\right)^3 \Leftrightarrow \\
 2x &= 3 \Leftrightarrow \boxed{x = \frac{3}{2}}
 \end{aligned}$$

Άσκηση

Να λυθεί η εξίσωση $27^x + 9^x + 3^{x+1} - 5 = 0$

Λύση

$$3^{3x} + 3^{2x} + 3 \cdot 3^x - 5 = 0 \quad \text{Θέτω } 3^x = w \text{ και έχω}$$

$$w^3 + w^2 + 3w - 5 = 0 \Leftrightarrow (w-1)(w^2 + 2w + 5) = 0$$

1	1	3	-5		1	$w=1$ ή $w^2 + 2w + 5 = 0$
		1	2	5		$\Delta < 0$
1	2	5	0			

$$\boxed{x = 0}$$

Άσκηση

Να λυθεί η εξίσωση $e^{2x} - (e+1)e^x + e = 0 \quad (e \approx 2,71...)$

Λύση

$$\text{Θέτω } e^x = w \text{ και έχω } w^2 - (e+1)w + e = 0$$

$$\begin{aligned}
 S &= e+1 \quad \left| \quad w_1 = e \Leftrightarrow e^x = e \Leftrightarrow x = 1 \right. \\
 P &= e \quad \left| \quad w_2 = 1 \Leftrightarrow e^x = 1 \Leftrightarrow x = 0 \right.
 \end{aligned}$$

