$2^x = 4$

$2^x = 2^2$

$x = 2$

$2^x = 5$

$x = \log_2 5$

$2^x = 8$

$2^x = 2^3$

$x = 3$

$2^x = -3$

αδύνατη

$$a^x = \theta \iff x = \log_a \theta \quad , \quad \begin{matrix} \theta > 0 \\ a > 0, \ a \neq 1 \end{matrix}$$

$\log_2 16 = 4$ αφού $2^4 = 16$

$\log_3 27 = 3$ αφού $3^3 = 27$

$\log_4 64 = 3$ αφού $4^3 = 64$

$\log_2 \frac{1}{2} = -1$ αφού $2^{-1} = \frac{1}{2}$

$\log_3 \sqrt{3} = \frac{1}{2}$ αφού $3^{\frac{1}{2}} = \sqrt{3}$

$\log_2 \sqrt[3]{4} = x$

$2^x = \sqrt[3]{4}$

$2^x = \sqrt[3]{2^2} = 2^{\frac{2}{3}}$

$x = \frac{2}{3}$

$\log_{10} \theta = \log \theta$

$\log 100 = 2, \ 10^2 = 100$

$\log 1000 = 3, \ 10^3 = 1000$

$\log 0{,}01 = -2, \ 10^{-2} = 0{,}01$

$\log \sqrt{10} = \frac{1}{2}, \ 10^{\frac{1}{2}} = \sqrt{10}$

$\log_e \theta = \ln \theta$

$\ln \sqrt{e} = \frac{1}{2}, \ e^{\frac{1}{2}} = \sqrt{e}$

$\ln \sqrt[3]{e^2} = \frac{2}{3}, \ e^{\frac{2}{3}} = \sqrt[3]{e^2}$

$\ln \frac{1}{e} = -1, \ e^{-1} = \frac{1}{e}$

$\log_2 2 = 1$

$\log_{\sqrt{2}} \sqrt{2} = 1$

$\log 10 = 1$

$\ln e = 1$

$$\log_a a = 1$$

$\log_3 1 = 0$

$\log_{\frac{1}{2}} 1 = 0$

$\log 1 = 0$

$\ln 1 = 0$

$$\log_a 1 = 0$$

$a^x = \theta \ (1) \iff x = \log_a \theta \ (2)$

$(2) \overset{(1)}{\Longrightarrow} \log_a a^x = x$

$\log 10^6 = 6$

$\log_2 2^3 = 3$

$7 = \log_5 5^7$

$3 = \log 10^3$

$\ln e^{\sqrt{2}} = \sqrt{2}$

$\log_3 3^{-5} = -5$

$-2 = \ln e^{-2}$

$\log_a a = 1$ | $\log_a 1 = 0$

$1 = \log_5 5$, $3 = \log 10^3$, $-2 = \ln e$

(1) $\overset{(2)}{\Rightarrow}$ $a^{\log_a \theta} = \theta$

$10^{\log 5} = 5$, $e^{\ln 3} = 3$

$10^{\log \theta} = \theta$, $e^{\ln \theta} = \theta$

1. Να υπολογισθούν, χωρίς τη χρήση υπολογιστή τσέπης, οι λογάριθμοι:

i) $\log_{10} 0,001$ ii) $\log_{\frac{1}{10}} \sqrt{10}$ iii) $\log_{\frac{1}{2}} 32$

iv) $\log_9 \frac{\sqrt{27}}{3}$ v) $\log_{\sqrt{2}} 16$ vi) $\log_{\frac{3}{2}} \sqrt{\frac{8}{27}}$

$a^{\frac{\nu}{\nu}} = \sqrt[\nu]{a^\nu}$

i) $\log 0,001 = -3$ αφού $10^{-3} = 0,001$

ii) $\log_{\frac{1}{10}} \sqrt{10} = x \Leftrightarrow \left(\frac{1}{10}\right)^x = \sqrt{10} \Leftrightarrow 10^{-x} = 10^{\frac{1}{2}} \Leftrightarrow -x = \frac{1}{2} \Leftrightarrow x = -\frac{1}{2}$

iii) $\log_{\frac{1}{2}} 32 = x \Leftrightarrow \left(\frac{1}{2}\right)^x = 32 \Leftrightarrow 2^{-x} = 2^5 \Leftrightarrow -x = 5 \Leftrightarrow x = -5$

iv) $\log_9 \frac{\sqrt{27}}{3} = x \Leftrightarrow 9^x = \frac{\sqrt{27}}{3} \Leftrightarrow 3^{2x} = \frac{\sqrt{3^3}}{3} \Leftrightarrow 3^{2x} = \frac{3^{\frac{3}{2}}}{3} \Leftrightarrow 3^{2x} = 3^{\frac{3}{2}-1}$

$\Leftrightarrow 3^{2x} = 3^{1/2} \Leftrightarrow 2x = \frac{1}{2} \Leftrightarrow x = \frac{1}{4}$

v) $\log_{\sqrt{2}} 16 = x \Leftrightarrow \sqrt{2}^x = 16 \Leftrightarrow 2^{\frac{x}{2}} = 2^4 \Leftrightarrow \frac{x}{2} = 4 \Leftrightarrow x = 8$

vi) $\log_{\frac{3}{2}} \sqrt{\frac{8}{27}} = x \Leftrightarrow \left(\frac{3}{2}\right)^x = \sqrt{\frac{8}{27}} \Leftrightarrow \left(\frac{3}{2}\right)^x = \sqrt{\left(\frac{2}{3}\right)^3} \Leftrightarrow \left(\frac{3}{2}\right)^x = \left(\frac{2}{3}\right)^{3/2}$

$\left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^{-3/2} \Leftrightarrow x = -\frac{3}{2}$