1. Να σχεδιάσετε τις γραμμές τις οποίες παριστάνουν οι εξισώσεις:

(i) $x^2 - y^2 + 4y - 4 = 0$ (ii) $x^2 - y^2 - 4x + 2y + 3 = 0$.

i) $x^2 - (y^2 - 4y + 4) = 0$

$x^2 - (y-2)^2 = 0$

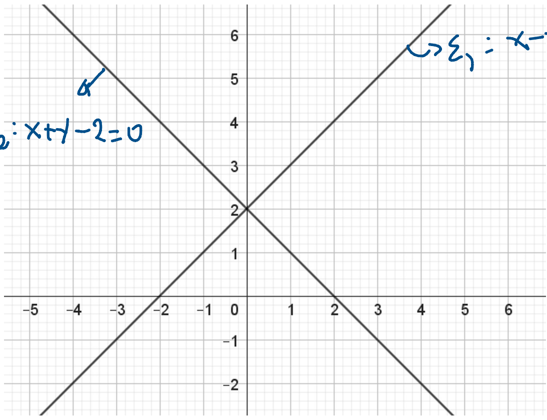
$(x - y + 2)(x + y - 2) = 0$

$x - y + 2 = 0$ ή $x + y - 2 = 0$

ε_1 ε_2

x	0	-2
y	2	0

x	0	2
y	2	0



ii) $x^2 - 4x + 4 - (y^2 - 2y + 1) = 0$

$(x-2)^2 - (y-1)^2 = 0$

$(x - 2 + y - 1)(x - 2 - y + 1) = 0$

$(x + y - 3)(x - y - 1) = 0$

$x + y - 3 = 0$ ή $x - y - 1 = 0$

ε_1 ε_2

2ος τρόπος

$x^2 - 4x - y^2 + 2y + 3 = 0$

τριώνυμο ως προς x

$\alpha = 1,\ \beta = -4,\ \gamma = -y^2 + 2y + 3$

$\Delta = \beta^2 - 4\alpha\gamma = 16 - 4(-y^2 + 2y + 3) =$
$= 16 + 4y^2 - 8y - 12 = 4y^2 - 8y + 4 =$
$= (2y - 2)^2$

$x = \dfrac{4 \pm \sqrt{(2y-2)^2}}{2 \cdot 1} = \begin{cases} \dfrac{4 + 2y - 2}{2} = \dfrac{2y + 2}{2} = y + 1 \\ \dfrac{4 - 2y + 2}{2} = \dfrac{6 - 2y}{2} = 3 - y \end{cases}$

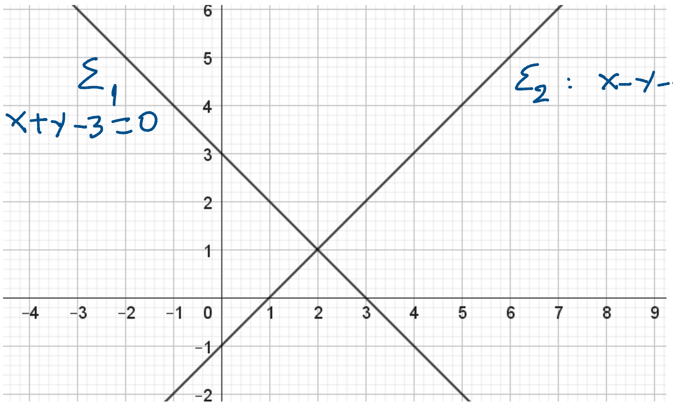
Συνεπώς $x = y + 1$ ή $x = 3 - y$

$x - y - 1 = 0$ $\quad$ $x + y - 3 = 0$

ε_2 ε_1

x	0	1
y	-1	0

x	0	3
y	3	0



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6. Να δείξετε ότι:

(i) Η εξίσωση $x^2 - 4xy + y^2 = 0$ παριστάνει δύο ευθείες.

(ii) Καθεμιά σχηματίζει με την $x - y = 0$ γωνία 30^0.

i) $x^2 - 4xy + y^2 = 0$

i) $x^2-4xy+y^2=0$

$x^2-4xy+4y^2-3y^2=0 \iff (x-2y)^2-(\sqrt{3}y)^2=0 \iff$

$(x-2y-\sqrt{3}y)(x-2y+\sqrt{3}y)=0$

$x-2y-\sqrt{3}y=0$ ή $x-2y+\sqrt{3}y=0$

$\underset{\varepsilon_1}{x-(2+\sqrt{3})y=0}$ $\qquad$ $\underset{\varepsilon_2}{x-(2-\sqrt{3})y=0}$

ii) $\varepsilon_1: x-(2+\sqrt{3})y=0 \longrightarrow \vec{\delta}_1=(2+\sqrt{3},1) \parallel \varepsilon_1$

$\varepsilon: x-y=0 \longrightarrow \vec{\delta}=(1,1) \parallel \varepsilon$

$$\text{συν}(\widehat{\vec{\delta}_1,\vec{\delta}})=\frac{\vec{\delta}_1\cdot\vec{\delta}}{|\vec{\delta}_1|\cdot|\vec{\delta}|}=\frac{2+\sqrt{3}+1}{\sqrt{(2+\sqrt{3})^2+1^2}\cdot\sqrt{1^2+1^2}}=\frac{3+\sqrt{3}}{\sqrt{8+4\sqrt{3}}\cdot\sqrt{2}}=$$

$$=\frac{3+\sqrt{3}}{\sqrt{2(4+2\sqrt{3})}\cdot\sqrt{2}}=\frac{3+\sqrt{3}}{\sqrt{2}\cdot\sqrt{4+2\sqrt{3}}\cdot\sqrt{2}}=\frac{3+\sqrt{3}}{\sqrt{2}^2\cdot\sqrt{(\sqrt{3})^2+1+2\sqrt{3}}}=$$

$$=\frac{3+\sqrt{3}}{2\cdot\sqrt{(\sqrt{3}+1)^2}}=\frac{\sqrt{3}^2+\sqrt{3}}{2(\sqrt{3}+1)}=\frac{\sqrt{3}(\sqrt{3}+1)}{2(\sqrt{3}+1)}=\frac{\sqrt{3}}{2} \Rightarrow (\widehat{\vec{\delta}_1,\vec{\delta}})=30^\circ \Rightarrow$$

$$(\widehat{\varepsilon_1,\varepsilon})=30^\circ$$