① $v_1' = -\frac{1}{5} v_1$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} \cdot v_1 \Rightarrow -\frac{v_1}{5} = \frac{m_1 - m_2}{m_1 + m_2} \cdot v_1 \Rightarrow$$

$$-m_1 - m_2 = 5m_1 - 5m_2 \Rightarrow 4m_2 = 6m_1 \Rightarrow$$

$$\frac{m_2}{m_1} = \frac{6}{4} = \frac{3}{2}$$ (a)

② $$\alpha\% = \frac{\frac{1}{2} m_2 v_2'^2}{\frac{1}{2} m_1 v_1^2} \cdot 100\% \Rightarrow$$

$$\frac{36}{100} = \frac{\frac{1}{2} m_2 \frac{4m_1^2}{(m_1+m_2)^2} \cdot v_1^2}{\frac{1}{2} m_1 v_1^2} \cdot 100\% \Rightarrow$$

$$0{,}36 = \frac{4 m_1 \cdot m_2}{(m_1+m_2)^2} \Rightarrow$$

$$\frac{4m_1m_2}{(m_1+m_2)^2} = \frac{9}{25} \Rightarrow 9m_2^2 + 9m_1^2 + 18 m_1 m_2 = 100 m_1 m_2$$

$$9m_2^2 - 82 m_1 m_2 + 9 m_1^2 = 0$$

$$\Delta = 6724 m_1^2 - 4 \cdot 9 \cdot 9 m_1^2 \Rightarrow \Delta = 6400 m_1^2$$

$$m_2 = \frac{82 m_1 \pm 80 m_1}{18} \quad \begin{cases} m_2 = \frac{m_1}{9} \\ m_2 = 9 m_1 \end{cases}$$ (b)

③ 

$$v = \frac{m_A v_A}{m_A + m_B}$$

$$\alpha\% = \frac{\frac{1}{2} m_A v_A^2 - \frac{1}{2}(m_A + m_B) v^2}{\frac{1}{2} m_A v_A^2} \Rightarrow$$

$$a\% = \frac{\frac{1}{2}m_A v_A^2 - \frac{1}{2}\frac{m_A^2 v_A^2}{m_A+m_B}}{\frac{1}{2}m_A v_A^2} \Rightarrow$$

$$a\% = 1 - \frac{m_A}{m_A+m_B} \Rightarrow a\% = \frac{m_A+m_B-m_A}{m_A+m_B} \Rightarrow$$

$$\frac{25}{100} = \frac{m_B}{m_A+m_B} \quad (1)$$

$$m_B v_B = (m_A+m_B)\, v$$

$$b\% = \frac{\frac{1}{2}m_B v_B^2 - \frac{1}{2}(m_A+m_B)v^2}{\frac{1}{2}m_B v_B^2} \Rightarrow$$

$$b\% = \frac{m_B v_B^2 - (m_A+m_B)\frac{m_B^2 v_B^2}{(m_A+m_B)^2}}{m_B v_B^2}$$

$$b\% = 1 - \frac{m_B}{m_A+m_B} \Rightarrow b\% = \frac{m_A}{m_A+m_B} \quad (2)$$

Ans (1) $\Rightarrow$ $m_A+m_B = 4m_B \Rightarrow m_A = 3m_B$. (3)

(2) (3) $\Rightarrow$ $b\% = \frac{3m_B}{4m_B} = 0.75$ (3)

(4) $m_1 = 2kg$
$m_2 = 3kg$
$v_1 = 4m/s$
$v_2 = 2m/s$

$P_{sys}^2 = P_1^2 + P_2^2 \Rightarrow$

$P_{sys}^2 = 64 + 36 = 100 \Rightarrow P_{sys} = 10 \Rightarrow$

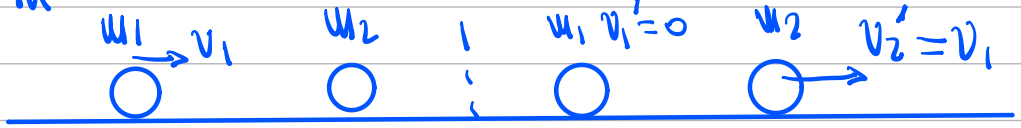
$m_{sys} v_k = 10 \Rightarrow v_k = 2m/s$

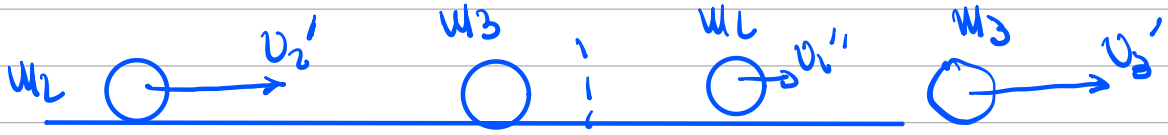
$$K_{\text{кол}} = \frac{p^2_{\text{кол2}}}{2m_0} = \frac{100}{2 \cdot 5} = 10$$ (6)

(5) $m_1 = m$

$m_2 = m$

$m_3 = 3m$





$$v_3' = \frac{2m_2}{m_2 + m_3} \cdot v_2' \Rightarrow v_3 = \frac{2m}{4m} \cdot v_1 \Rightarrow$$

$$\boxed{\frac{v_3}{v_1} = \frac{1}{2}}$$ (6)

(6) $m = 1{,}5$ kg

$R = 1{,}8$ m

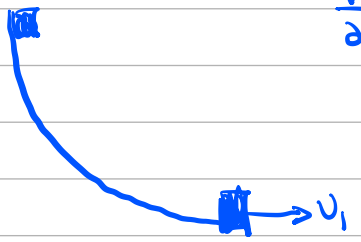
$M = 3$ kg

$L = 1{,}6$ m

а) Вниз: $\Delta K = \Delta W \Rightarrow$

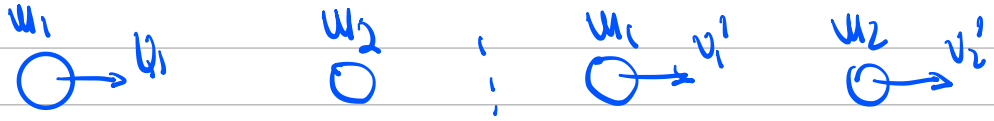
$$\frac{1}{2} m v_1^2 = m g R \Rightarrow$$

$$v_1 = \sqrt{2gR}$$



$$\boxed{v_1 = 6 \text{ м/с}}$$

б) Удар

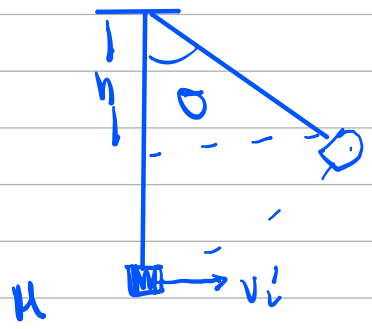


$$v_1' = \frac{m - M}{m + M} \cdot v_1 = \frac{-1{,}5}{4{,}5} \cdot 6 = -2 \text{ м/с}$$

$$v_2' = \frac{2m}{m + M} \cdot v_1 = \frac{3}{4{,}5} \cdot 6 = 4 \text{ м/с}$$

$$\eta\% = \frac{\frac{1}{2} M v_2'^2}{\frac{1}{2} m v_1^2} \cdot 100\% = \frac{3 \cdot 4^2}{1{,}5 \cdot 36} = \frac{32}{36} = 88{,}8$$

δ)



ΑΔΜΕ

$\frac{1}{2}MV_1'^2 + 0 = 0 + Mg(l-h)$

$8 = 10(1,6-h)$

$8 = 16 - 10h \Rightarrow 10h = 8 \Rightarrow$

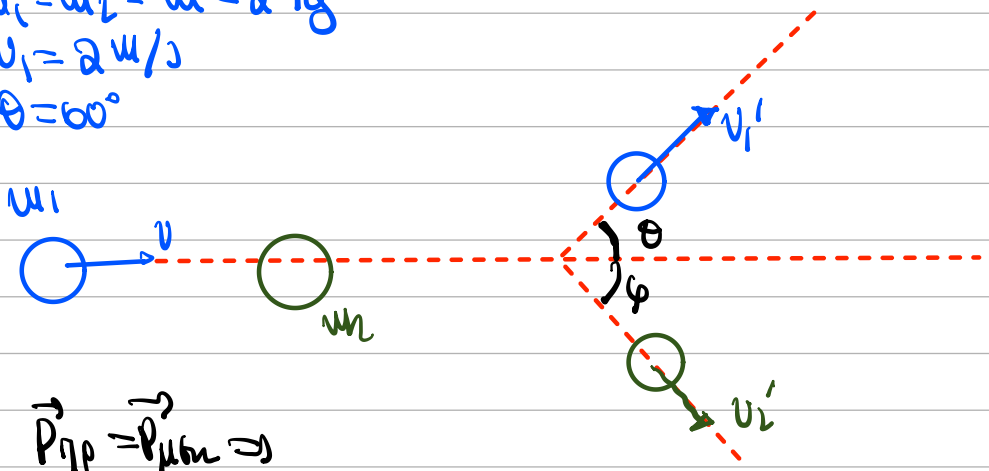
$h = 0.8.$

Άρα $\sigma\upsilon\nu\theta = \frac{h}{l} = \frac{0.8}{1,6} = \frac{1}{2}$

ε) $\Sigma F_{κ} = F_{κεν} \Rightarrow T_υ - Mg = M\frac{V_1'^2}{l} \Rightarrow$

$T_υ = 30 + 3\frac{16}{1,6} \Rightarrow$ $\boxed{T_υ = 60\ N}$

στ)



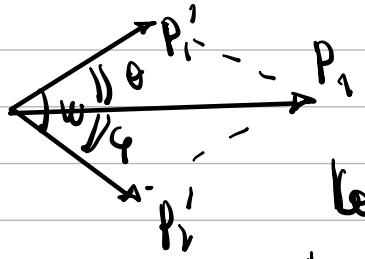
$m_1 = m_2 = m = 2\ kg$

$v_1 = 2\ m/s$

$\theta = 60°$

ΑΔΟ $\vec{P}_{πρ} = \vec{P}_{μετ} \Rightarrow$

$\vec{P}_1 = \vec{P}_1' + \vec{P}_2'$



$P_1^2 = P_1'^2 + P_2'^2 + 2P_1'P_2' \cdot \sigma\upsilon\nu\omega \Rightarrow$

$V_1^2 = V_1'^2 + V_2'^2 + 2V_1'V_2' \cdot \sigma\upsilon\nu\omega$ ①

$K_{πριν} = K_{μετ} \Rightarrow$

$\frac{1}{2}m_1V_1^2 = \frac{1}{2}m_1V_1'^2 + \frac{1}{2}m_2V_2'^2 \Rightarrow$

$V_1^2 = V_1'^2 + V_2'^2$ ②

① ② $\Rightarrow$ $2V_1'V_2' \cdot \sigma\upsilon\nu\omega = 0$ άρα $\sigma\upsilon\nu\omega = 0° \Rightarrow \omega = \frac{\pi}{2}$

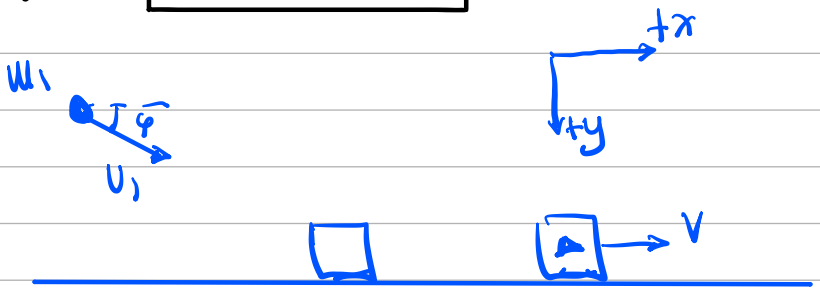
B. α) $\eta\mu\varphi = \frac{p_1'}{p_1} \Rightarrow \eta\mu 30^\circ = \frac{p_1'}{p_1} \Rightarrow \frac{1}{2} = \frac{m_1 v_1'}{m_1 v_1} \Rightarrow$

$2 v_1' = 2 \Rightarrow \boxed{v_1' = 1\ m/s}$

Από (2) $v_2'^2 = a^2 - 1^2 \Rightarrow \boxed{v_2' = \sqrt{3}\ m/s}$

β) $\Delta K_1 = K_{1\,\mu\varepsilon\tau} - K_{1\,\pi\rho\iota} \Rightarrow \Delta K_1 = \frac{1}{2} m_1 v_1'^2 - \frac{1}{2} m_1 v^2 =$

$= \frac{1}{2} 2 (1 - 4) \Rightarrow \boxed{\Delta K_1 = -3\,J}$



(B) $m_1 = 2\ kg$
$v_1 = 20\ m/s$
$\varphi = 60^\circ$
$m_2 = 3\ kg$

α) Χ.Α. $p_{x\,\pi\rho\iota} = p_{x\,\mu\varepsilon\tau} \Rightarrow m_1 v_1 \cdot \sigma\upsilon\nu 60^\circ = (m_1 + m_2) V \Rightarrow$
$2 \cdot 20 \frac{1}{2} = 5 V \Rightarrow V = 4\ m/s$

β) $Q = E_{\alpha\pi\omega\lambda} = K_{o\lambda\eta\,\pi\rho\iota} - K_{o\lambda\eta\,\mu\varepsilon\tau} = \frac{1}{2} m_1 v_1^2 - \frac{1}{2} (m_1 + m_2) V^2$
$Q = \frac{1}{2} \cdot 2 \cdot 400 - \frac{1}{2} 5 \cdot 16 \Rightarrow Q = 360\,J$

γ) $\alpha\% = \frac{\frac{1}{2} m_2 V^2}{\frac{1}{2} m_1 v_1^2} \cdot 100\% = \frac{3 \cdot 4^2}{2 \cdot 400} = \frac{48}{2 \cdot 400} = 0{,}06$
δηλ. 6%

δ) $\Delta p_{\sigma\upsilon\sigma\tau} = \Delta p_{\sigma\upsilon\sigma\tau,y} = \cancel{p_{y\,\mu\varepsilon\tau}} - p_{y\,\pi\rho\iota} \Rightarrow$

$\Delta p_{\sigma\upsilon\sigma\tau} = - m_1 \cdot v_1 \cdot \sigma\upsilon\nu 60^\circ = - 2 \cdot 20 \frac{\sqrt{3}}{2} \Rightarrow \Delta p_{\sigma\upsilon\sigma\tau} = -20\sqrt{3}\ kg\frac{m}{s}$

ε) $\Sigma F_y = \frac{\Delta p_{\sigma\upsilon\sigma,y}}{\Delta t} \Rightarrow m_2 g - N = \frac{\Delta p_{\sigma\upsilon\sigma}}{\Delta t} \Rightarrow$

$$N = 50 + \frac{20\sqrt{3}}{10^{-2}\sqrt{3}} \Rightarrow \boxed{N = 2050\ N}$$

(9) $m_1 = 3m$, $m_2 = m$, v_0

$$P_{\eta\rho} = P_{\mu\varepsilon\tau} \Rightarrow m_1 v_0 - m_2 v_0 = m_1 v_1' \cdot \sigma\upsilon\nu\theta$$
$$3mv_0 - mv_0 = 3m v_1' \cdot \sigma\upsilon\nu\theta$$
$$2mv_0 = 3m v_1' \cdot \sigma\upsilon\nu\theta \Rightarrow$$
$$\sigma\upsilon\nu\theta = \frac{2v_0}{3v_1'} \quad (1)$$

$$P_{y\pi\rho\iota\nu} = P_{y\mu\varepsilon\tau} \Rightarrow 0 = m_1 \cdot v_1' \eta\mu\theta - m_2 v_2' \Rightarrow$$
$$mv_2' = 3m v_1' \cdot \eta\mu\theta \Rightarrow$$
$$\eta\mu\theta = \frac{v_2'}{3v_1'} \quad (2)$$

$$K_{\pi\rho\iota\nu} = K_{\varepsilon\tau\alpha\iota} \Rightarrow \frac{1}{2} 3m v_0^2 + \frac{1}{2} m v_0^2 = \frac{1}{2} 3m v_1'^2 + \frac{1}{2} m v_2'^2$$
$$4v_0^2 = 3v_1'^2 + v_2'^2 \quad (3)$$

Από (1) και (2):
$$\sigma\upsilon\nu^2\theta = \frac{4v_0^2}{9v_1'^2}, \quad \eta\mu^2\theta = \frac{v_2'^2}{9v_1'^2} \Rightarrow 1 = \frac{4v_0^2}{9v_1'^2} + \frac{v_2'^2}{9v_1'^2} \Rightarrow$$
$$1 = \frac{4v_0^2 + v_2'^2}{9v_1'^2} \Rightarrow 9v_1'^2 = 4v_0^2 + v_2'^2 \Rightarrow v_2'^2 = 9v_1'^2 - 4v_0^2 \quad (4)$$
$$4v_0^2 = 3v_1'^2 + 9v_1'^2 - 4v_0^2 \Rightarrow 8v_0^2 = 12v_1'^2 \Rightarrow$$
$$\boxed{v_1' = v_0\sqrt{\frac{2}{3}}}$$

Από (3): $4v_0^2 = 3v_0^2 \frac{2}{3} + v_2'^2 \Rightarrow \boxed{v_2' = v_0\sqrt{2}}$

Από (2): $\eta\mu\theta = \frac{v_0\sqrt{2}\cdot\sqrt{3}}{3v_0\sqrt{2}} = \frac{\sqrt{3}}{3}$