ENVIRONMENTAL SYSTEMS AND SOCIETIES

Maths Skills for ESS

Key points, key terms and worked examples

Course Companion (Oxford, 2024) · pages 762–773

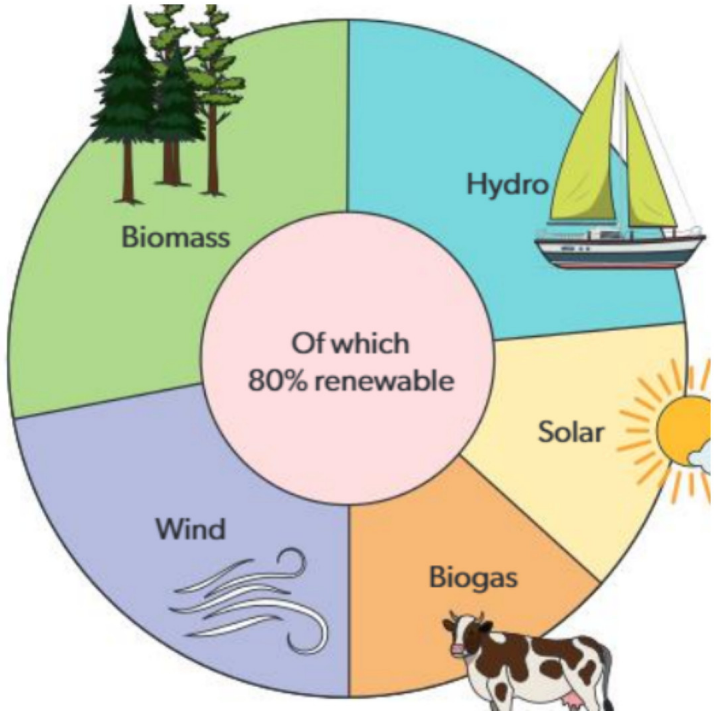


Figure 8 · A doughnut pie chart

OVERVIEW

The maths skills you may need

Basic maths

- Arithmetic and approximations
- Fractions, decimals, percentages and ratios
- Mean, median and mode

Units and notation

- SI units for time, length, area and volume
- Standard notation: 2×10^6 m = 2 million metres
- Direct and inverse proportion

Graphs and data

- Gradients, intercepts and areas
- Scatter plot, line of best fit, box-and-whisker plot
- Bar chart, histogram, pie chart, kite diagram

Statistics and indices

- SD, chi-squared, t-test, ANOVA, Spearman's rank
- Simpson's reciprocal index and Lincoln index
- Natural increase rate and doubling time

You do not have to memorize equations.

SOME BASIC MATHS

Four key terms

Approximation

Something similar to but not exactly equal to something else, e.g. rounding a number up or down.

Example: 95,637 → 95,640 (nearest 10) · 95,600 (nearest 100) · 96,000 (nearest 1,000)

Fractions, decimals, percentages

Ways of showing numbers that are parts of a whole; you can convert between them.

Example: ¼ = 0.25 = 25%

Ratio

Shows how much there is of one thing compared with another, usually written as a:b.

Example: 1 part oil to 4 parts water gives a ratio of 1:4

Frequency

The number of times a value occurs in a set of data.

Example: if 5 of your class gained a grade 7, the frequency of grade 7 is 5

WORKED EXAMPLE

Mean, median and mode

Data set **3, 4, 5, 5, 5, 8, 8, 11, 15, 16**

Mean

8

Add all the values, then divide by the number of values.

80 ÷ 10 = 8

Median

6.5

The middle value when the data are ordered from least to greatest.

Middle values 5 and 8: (5 + 8) ÷ 2 = 6.5

Mode

5

The value that occurs most often: the most common number.

5 appears three times

UNITS AND NOTATION

SI units and prefixes

Table 1 · Units used in ESS

Unit	Examples	Symbol(s)
time	second, minute, hour, year	s, min, h, yr
length	micrometre, millimetre, metre	μm, mm, m
area	metre squared, hectare	m^2, ha (= 10,000 m^2)
volume	cubic metre	m^3
mass	milligram, gram, kilogram, tonne	mg, g, kg, t

Table 2 · Unit prefixes

Prefix	Base 10	Symbol
peta	10^{15}	P
giga	10^{9}	G
kilo	10^{3}	k
milli	10^{-3}	m
micro	10^{-6}	μ

Productivity

gram per metre squared per year = $g/m^2/yr$ or $g\ m^{-2}\ yr^{-1}$

Carbon cycle

gigatonnes of carbon per year = GtC/yr or $GtC\ yr^{-1}$

UNITS AND NOTATION

Abbreviations and proportion

Particulate matter (PM)

Pollution by particles in the air.

PM_{10}: particles with diameters generally 10 μm and smaller

PM2.5: fine inhalable particles 2.5 μm in diameter and smaller

For scale: a human hair is about 70 μm in diameter.

1 Gt (gigatonne) = 1 billion metric tonnes = 10^9 tonnes

Direct and inverse proportion

Direct proportion

The ratio between two quantities remains the same: when one variable increases, so does the other.

Inverse (indirect) proportion

As one quantity increases, the other decreases.

Example: as vehicle speed increases, journey time decreases

GRAPHS

Five steps in graph plotting

1. Identify the variables
2. Determine the variable range and scale
3. Label each axis with units; give a title
4. Determine the data points and plot them
5. Draw a straight line, line of best fit or freehand curve

Independent variable

The one the investigator changes, e.g. temperature, light intensity, distance from the sea. Plot on the **x-axis**.

Dependent variable

Changes because the independent variable has changed. Plot on the **y-axis**.

Controlled variable

A variable that is kept constant during the investigation.

GRAPHS

Gradients and intercepts

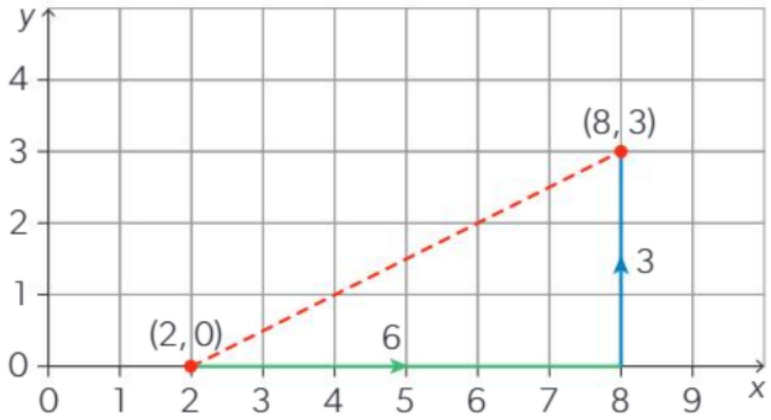


Figure 1 · A positive (uphill) gradient

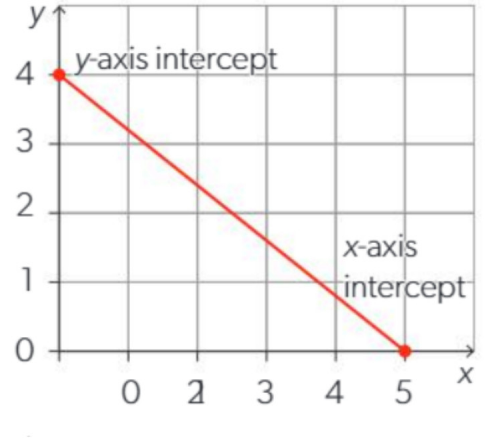


Figure 2 · Intercepts

Gradient: the steepness of a line; the rate of change of one measure against another. Steeper = higher rate; a negative value = decreasing. For a curve, draw a tangent.

Intercepts: the x-intercept is where a line crosses the x-axis; the y-intercept is where it crosses the y-axis. **Linear** = straight line; **non-linear** = curve.

GRAPHS

Scatter plots and point-to-point lines

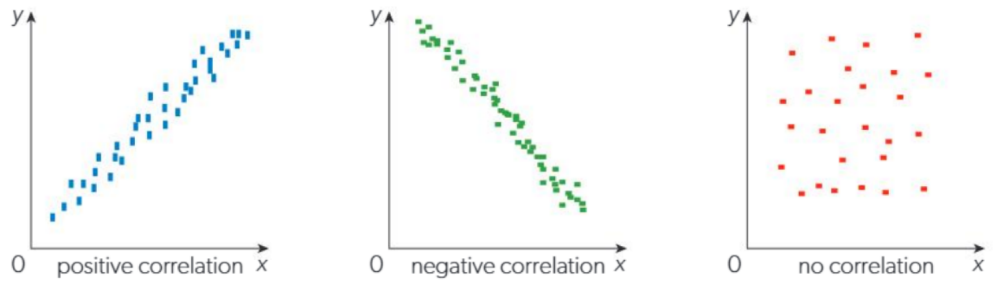


Figure 3 · Types of correlation in scatter plots

Scatter plot: plots each pair of values to show whether two variables are related, e.g. air temperature and ice cream sales.

- **Positive**: as one increases, the other increases
- **Negative**: as one increases, the other decreases
- **None**: no clear relationship

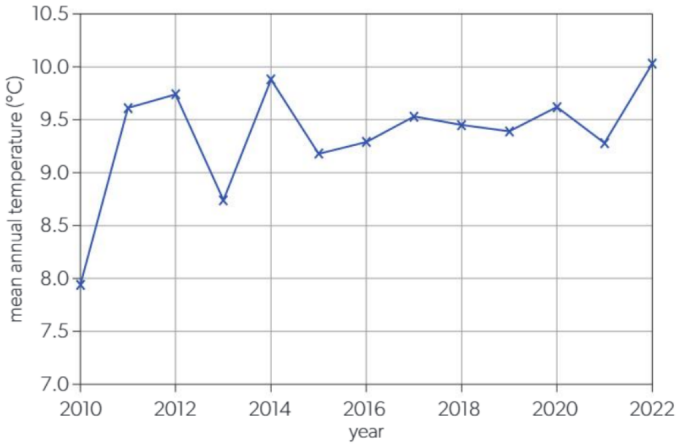


Figure 4 · UK mean annual temperature, 2010–2022

Point-to-point line: joins adjacent points with straight lines; used to show changes over time. In a scatter graph, draw a **line of best fit** instead.

GRAPHS AND DATA

Box-and-whisker plots and data types

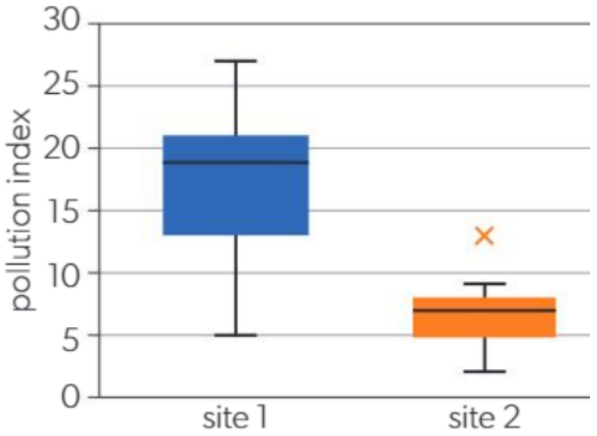


Figure 5 · Pollution levels at two sites

- Centre line = **median**; box edges = **upper and lower quartiles**
- Whiskers = range, to the maximum and minimum
- **Outliers**: small crosses or dots (one at site 2)

Useful when there are outliers in the data. The type of data also decides the graph and the statistical test.

Categorical data

Can be divided into groups, e.g. nationality, age groupings, educational level.

Quantitative data

Counted or measured in numbers, e.g. height, age in years, distance.

Discrete: particular values only (shoe size) · **Continuous**: any value (wingspan)

GRAPHS

Bar charts and histograms

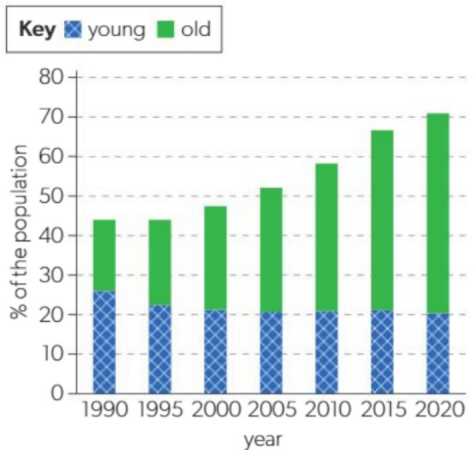


Figure 6 · Stacked bar chart: dependency ratio in Japan, 1990–2020

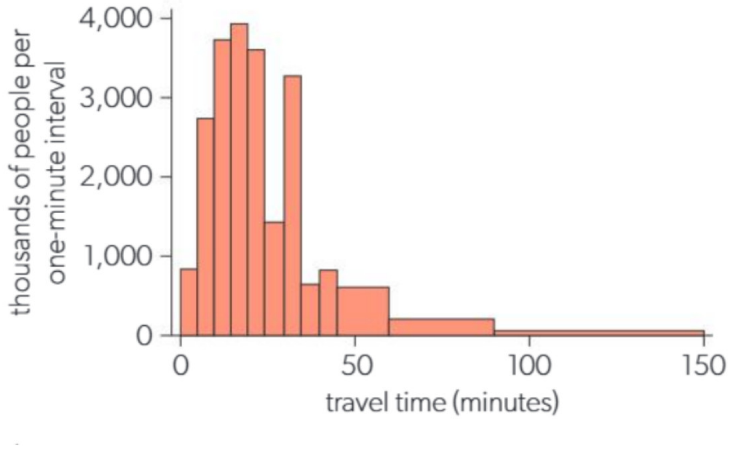


Figure 7 · Histogram: travel time to work in the USA

Bar chart: shows **categorical** data. Bars are the same width with spaces between; length is proportional to the quantity. A **stacked** bar has two or more colour-coded stacks per group.

Histogram: shows **quantitative** data. The bars can be different widths and there is no space between them.

Maths skills for ESS · pp. 767–768

GRAPHS

Pie charts and kite diagrams

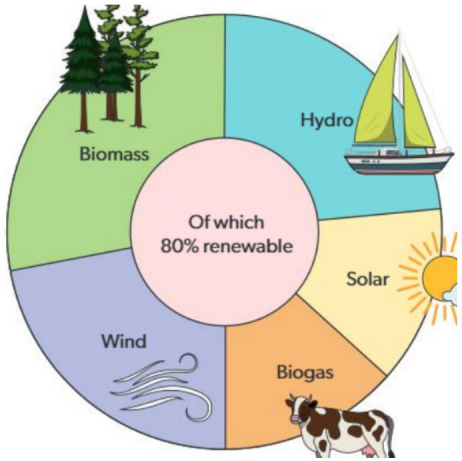


Figure 8 · A doughnut pie chart

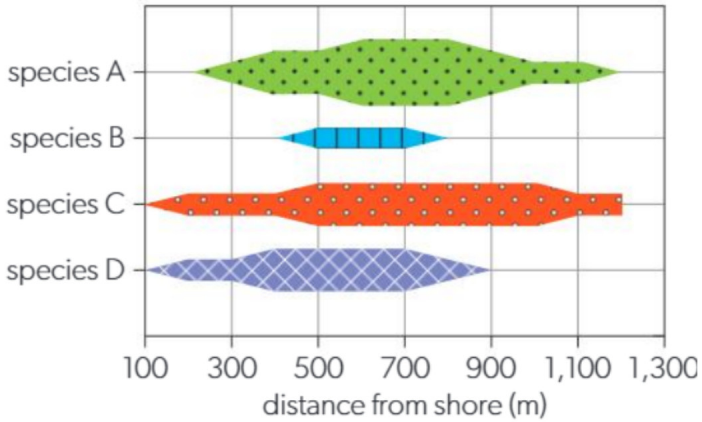


Figure 9 · Abundance of four species along a transect

Pie chart: a circular chart for **categorical** data; segments show the relative sizes of the categories. A hollow pie chart is a **“doughnut”** pie chart.

Kite diagram: observations at points along a transect: species presence or absence, abundance and distance. Used to hypothesize biotic–abiotic relationships; hard to draw by hand.

STATISTICAL TESTS

Decide which test to use before you start collecting data.

Ask at the stage of designing your research question and methodology.

Standard deviation | Chi-squared test | t-test | ANOVA | Correlation coefficient | Spearman's rank

STATISTICAL TESTS

Standard deviation

Standard deviation (SD): a measure of dispersement, i.e. how much your data is spread out around the mean.

Normal distribution: a bell-shaped curve with the mean in the centre; each segment is one SD away from the mean.

$$s = \frac{\sum(x - \bar{x})^2}{n - 1}$$

Figure 10 · You do not need to learn this

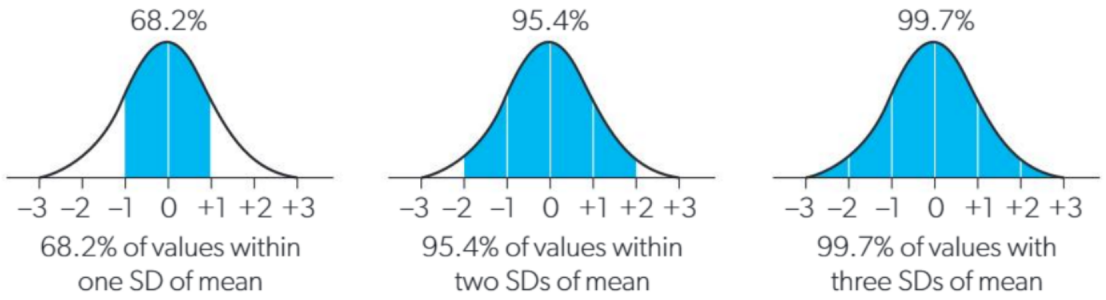


Figure 11 · 68.2% within ±1 SD · 95.4% within ±2 SD · 99.7% within ±3 SD

STATISTICAL TESTS

The chi-squared test, χ^2

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i}$$

Figure 12 · Chi-squared equation

Tests whether a difference between two **categorical** variables is due to chance or to a relationship, e.g. the distribution of two species.

O = observed values you obtain
E = expected values, based on the null hypothesis

1. **Hypotheses**: null = no difference in the distribution of the two species; alternative = there is a difference.
2. **Table**: make a frequency table of observed (O) and expected (E) results, then apply the formula.
3. **Probability table**: find a published table of all possible outcomes and their likelihoods.
4. **Degrees of freedom**: Df = N − 1, where N is the sample size.
5. **p-value**: $p < 0.05$ = statistically significant, so reject the null hypothesis; $p > 0.05$ = the null hypothesis is considered true.

STATISTICAL TESTS

The t-test and ANOVA

t-test

Compares the **means of two samples**.

Null hypothesis: the difference in group means is zero.

Use with a small, random sample compared with a fixed value or a second sample's mean.

$p < 0.05$ = significant. The greater the magnitude of t, the more evidence against the null.

ANOVA

Analysis of variance compares the **means of three or more groups**; a generalization of the t-test.

One-way: one independent variable.

Two-way: two independent variables.

Chi-squared or t-test?

Both can test for differences between two groups.

Chi-squared: two categorical variables.

t-test: a dependent quantitative variable and an independent categorical variable with two groups.

STATISTICAL TESTS

Correlation coefficients

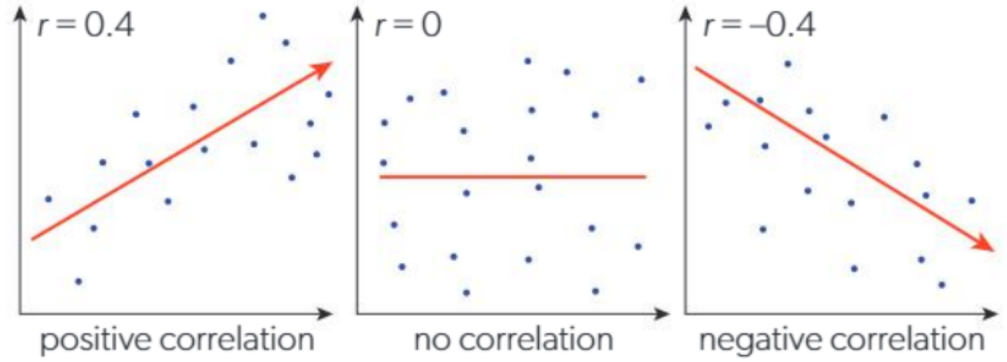


Figure 13 · Correlation

+1
Strong positive, e.g. shoe size and foot length

0
No relationship between the variables

−1
Strong negative relationship

Measure the **strength of the relationship** between two variables, as a value between −1 and +1.

Spearman's rank (ρ) gives a numerical value for the degree of correlation between two data sets: +1 = perfect association of rank, −1 = perfect negative association.

Correlation does not necessarily indicate causation

Ice cream sales and shark attacks are strongly correlated: in warmer weather more people buy ice cream and more go into the sea.

DIVERSITY

Simpson’s reciprocal index

$$D = \frac{N(N-1)}{\sum n(n-1)}$$

- **D** = Simpson’s reciprocal index
- **N** = total number of organisms of all species found
- **n** = number of individuals of a particular species

Species diversity: a function of the number of species and their relative abundance; a measure of **species richness** and **species evenness**.

1

Lowest value: just one species

max

Equal to the number of species in the sample

The higher the number, the greater the diversity, so ecosystems can be compared or tracked over time.

Be careful: “Simpson’s diversity index” can mean three related indices; the other two range from 0 to 1.

POPULATION SIZE

The Lincoln index

Estimates the population size of a **motile** population. Also called the **capture–mark–release–recapture** method.

1

Capture a sample in a defined, marked area: **n_1**

2

Mark them: not easily removable, no harm to survival or behaviour

3

Release them to rejoin the population

4

Capture a second sample: **n_2**

5

Count the marked individuals in the second sample: **m_2**

$$N = \frac{n_1 \times n_2}{m_2}$$

Apply the formula: multiply the two sample sizes and divide by the number of marked individuals recaptured.

POPULATION CALCULATIONS

Growth and doubling time

Natural increase rate (NIR)

The birth rate minus the death rate of a population over time.

$$\text{NIR} = \frac{\text{number of births} - \text{number of deaths}}{\text{population}}$$

Population growth rate

The change in the number of individuals over a specific period of time.

$$GR = N \div t$$

GR = growth rate (number of individuals) · N = change in population · t = time

Doubling time

The time it takes for a population to double in size.

$$\text{doubling time} = \frac{70}{\text{annual growth rate}} = \frac{70}{GR}$$

EXAMPLE Annual growth rate 3.5% → doubling time = 70 ÷ 3.5 = **20 years**

RECAP

Which tool for which question?

Tool	Use it to…
Standard deviation	show how much data is spread out around the mean
Chi-squared test	test whether a difference between two categorical variables is due to chance
t-test	compare the means of two samples
ANOVA	compare the means of three or more groups
Spearman’s rank	put a number on the strength of correlation between two data sets
Simpson’s reciprocal index	express species diversity as a number
Lincoln index	estimate the population size of motile organisms