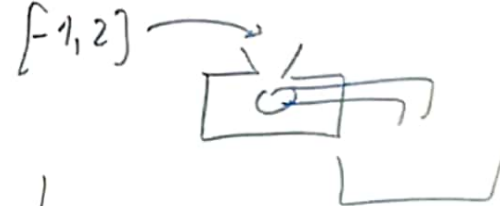


Def. 2 $\Delta = [-1, 2] = \text{H.O. m.s.f.}$



a) $g(x) = f(2x-1)$

$-1 \leq 2x-1 \leq 2 \Leftrightarrow$

$0 \leq 2x \leq 3 \Leftrightarrow$

$0 \leq x \leq 3/2$

$D_g = [0, 3/2]$

b) $h(x) = f(x^2-1)$

$-1 \leq x^2-1 \leq 2 \Leftrightarrow$

$0 \leq x^2 \leq 3 \Leftrightarrow$

$x^2 \leq 3 \Leftrightarrow$

$\sqrt{x^2} \leq \sqrt{3} \Leftrightarrow$

$|x| \leq \sqrt{3} \Leftrightarrow$

$-\sqrt{3} \leq x \leq \sqrt{3}$

$D_h = [-\sqrt{3}, \sqrt{3}]$

c) $\varphi(x) = f(\sqrt{x}) - 3f(\ln x)$

kar' approx

$x \geq 0$

kar $(x > 0)$

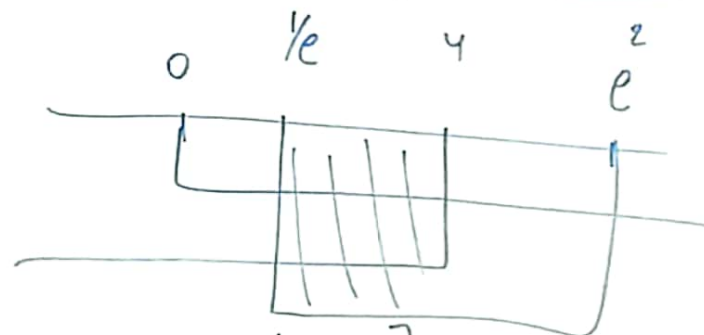
Elimination

$-1 \leq \sqrt{x} \leq 2 \quad -1 \leq \ln x \leq 2$

$\Leftrightarrow 0 \leq \sqrt{x} \leq 2 \quad \ln e^{-1} \leq \ln x \leq \ln e^2$

$x \leq 4$

$\frac{1}{e} \leq x \leq e^2$



$D_\varphi = [\frac{1}{e}, 4]$

ασκ. 3 / Σινυστική - άριστεύει

$$f, g: \mathbb{R} \rightarrow \mathbb{R}$$

$$(g \circ f)(x) = x^2 - x \quad (1) \quad \forall x \in \mathbb{R}$$

$$α) \quad g(x) = 3 - 5x \rightarrow f(x) = ?$$

$$(1) \Leftrightarrow g(f(x)) = x^2 - x \quad (\Rightarrow)$$

$$3 - 5f(x) = x^2 - x \quad (\Rightarrow)$$

$$+ 5f(x) = -x^2 + x + 3$$

$$f(x) = \frac{-x^2 + x + 3}{5}, \quad x \in \mathbb{R}$$

$$\hookrightarrow \text{όπου } x \rightarrow \frac{x+5}{2}$$

$$f(x) = 2x - 5, \quad g(x) = ?$$

$$(1) \Leftrightarrow g(f(x)) = x^2 - x \quad (\Rightarrow)$$

$$g(2x - 5) = x^2 - x$$

$$\textcircled{+} \text{ } \omega \quad 2x - 5 = \omega \quad \Leftrightarrow \quad 2x = \omega + 5$$

$$x = \frac{\omega + 5}{2}$$

$$g\left(2 \cdot \frac{\omega + 5}{2} - 5\right) = \left(\frac{\omega + 5}{2}\right)^2 - \frac{\omega + 5}{2} \quad (\Rightarrow)$$

$$g(\omega) = \left(\frac{\omega + 5}{2}\right)^2 - \frac{\omega + 5}{2}$$

$$g(x) = \left(\frac{x + 5}{2}\right)^2 - \frac{x + 5}{2}$$

αλγ. 5 | Συνθεση - αντισποση

$$(g \circ f)(x) = x - 1 \quad (1) \quad \forall x \in \mathbb{R}$$

$$g(x) = x^3 - 6x^2 + 12x - 1, \quad x \in \mathbb{R}$$

$$f = ?$$

$$(1) \Rightarrow g(f(x)) = x - 1 \Rightarrow$$

$$f^3(x) - 6f^2(x) + 12f(x) - 1 = x - 1$$

$$f^3(x) - 6f^2(x) + 12f(x) - 8 = x - 8$$

$$(f(x) - 2)^3 = x - 8 \quad (2)$$

$$\text{αν } x - 8 \geq 0 \Rightarrow x \geq 8$$

$$f(x) - 2 = \sqrt[3]{x - 8} \Rightarrow$$

$$f(x) = 2 + \sqrt[3]{x - 8}$$

$$\text{αν } x < 8 \Rightarrow x - 8 < 0$$

$$f(x) - 2 = -\sqrt[3]{|x - 8|} \Rightarrow$$

$$f(x) = 2 - \sqrt[3]{8 - x}$$

$$\text{Αρα } f(x) = \begin{cases} 2 + \sqrt[3]{x - 8}, & x \geq 8 \\ 2 - \sqrt[3]{8 - x}, & x < 8 \end{cases}$$

αλγ. 7 | Συναρτήσεις - άσκηση

d) $2f(x) - f(-x) = \sin 2x \quad (1)$

ΘΥΜΟΜΑΣΤΕ

$f(x) + f(x^2) = 15 - x$. { πάλι βρούμε 2 τιμές της f .
 $x = x^2 \Leftrightarrow x = 0, 1$

$x=0 \Rightarrow f(0) + f(0) = 15 \Leftrightarrow$

$2f(0) = 15 \Leftrightarrow f(0) = \frac{15}{2}$

$x=1 \Rightarrow f(1) + f(1) = 14 \Leftrightarrow$

$2f(1) = 14 \Leftrightarrow f(1) = 7$

Συναρτήσεις 6x6m

όπου $n \neq$ έχει 2 ορίσματα
και ίσως να βρούμε $n \neq$

Συναρτήσεις 4x5.

βρίσκω όπου $x = -x$

(1) $\Rightarrow 2f(-x) - f(x) = \sin(2(-x)) \Leftrightarrow$

$-f(x) + 2f(-x) = \sin 2x \quad (2)$

Αντί (1) $\Rightarrow 2f(x) - f(-x) = \sin 2x$
(2) $\Rightarrow -f(x) + 2f(-x) = \sin 2x$

$$\begin{cases} 2 & 4f(x) - 2f(-x) = 2\sin 2x \\ 1 & -f(x) + 2f(-x) = \sin 2x \end{cases}$$

$3f(x) = 3\sin 2x \Leftrightarrow f(x) = \sin 2x \quad \forall x \in \mathbb{R}$

αλγ. 7 | Συνθεση - αντιστοιχη

Συναρτησιακή εξίσωση
όπου $n \neq 1$ έχει 2 ορισματα,
και ίσως να βρεθεί $n \neq$

$$b) 3f(x) - 2f(1-x) = 5x^3 - 6x^2 + 6x \quad (1)$$

Βα)ω όπου x το $1-x$

$$3f(1-x) - 2f(1-(1-x)) = 5(1-x)^3 - 6(1-x)^2 + 6(1-x) \Leftrightarrow$$

$$3f(1-x) - 2f(x) = \underbrace{5(1-x)^3 - 6(1-x)^2 + 6(1-x)}_{A(x)} \quad (2)$$

Τωρα

$$(1) \Rightarrow \left. \begin{aligned} 3f(x) - 2f(1-x) &= 5x^3 - 6x^2 + 6x = B(x) \\ (2) \Rightarrow -2f(x) + 3f(1-x) &= A(x) \end{aligned} \right\} \begin{matrix} 3 \\ 2 \end{matrix}$$

$$\left. \begin{aligned} 9f(x) - 6f(1-x) &= B(x) \\ -4f(x) + 6f(1-x) &= A(x) \end{aligned} \right\} \xrightarrow{+} \frac{5f(x) = A(x) + B(x)}{5}$$

$$2 \cdot f(3x+2) - 7 \cdot f\left(\frac{2x-1}{4}\right) \Rightarrow 2f(y) - 7f(A(x))$$

Οιω $3x+2=y \Leftrightarrow x = \frac{y-2}{3} \quad y \rightarrow A(x)$

αλν. 8 / Σελ. 30 / Σχολ.

$$(B) \quad g(x) = x - 2\sqrt{x} + 1 = (\sqrt{x} - 1)^2$$

π.δ.ο.

$$(g \circ g)(x) = x, \quad \forall x \in [0, 1]$$

$$D_g = [0, +\infty)$$

$$D_{g \circ g} = \left\{ x \in D_g \mid g(x) \in D_g \right\} = [0, +\infty)$$

$$x \geq 0 \quad (\sqrt{x} - 1)^2 \geq 0$$

ισχύει πάντα.

$$g(g(x)) = g\left((\sqrt{x} - 1)^2\right) = \left(\sqrt{(\sqrt{x} - 1)^2} - 1\right)^2 = (|\sqrt{x} - 1| - 1)^2$$

$$1) \text{ Αν } \sqrt{x} - 1 > 0 \Leftrightarrow \sqrt{x} > 1 \Leftrightarrow x > 1 \Rightarrow (g \circ g)(x) = (\sqrt{x} - 2)^2$$

$$2) \text{ Αν } \sqrt{x} - 1 \leq 0 \Leftrightarrow \sqrt{x} \leq 1 \Leftrightarrow 0 \leq x \leq 1 \Rightarrow (g \circ g)(x) = (1 - \sqrt{x} - 1)^2 = (-\sqrt{x})^2 = x$$

$$(g \circ g)(x) = \begin{cases} (\sqrt{x} - 2)^2, & x > 1 \\ x, & 0 \leq x \leq 1 \end{cases}$$

$$\underline{a \in \mathbb{R} / \Sigma \varepsilon \lambda \cdot 30 / \Sigma x \in \mathbb{R} .}$$

$$f(x) = x + 1, \quad g(x) = ax + 2, \quad x \in \mathbb{R}$$

$$a = ? \text{ ώστε}$$

$$f \circ g = g \circ f$$

$$\left. \begin{aligned} \text{f21' de } x \in \mathbb{R}: \quad f(g(x)) &= (ax + 2) + 1 = ax + 3 \\ g(f(x)) &= a(x + 1) + 2 = ax + a + 2 \end{aligned} \right\}$$

$$a + 2 = 3 \quad (=) \quad a = 1$$

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