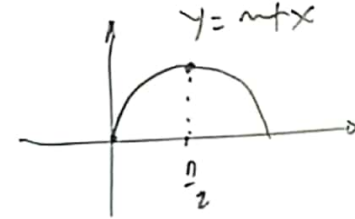


Agm.9 $0 < a < b < \frac{n}{2}$

$$\frac{b-a}{n\mu^2 b} < G_{\phi a} - G_{\phi b} < \frac{b-a}{n\mu^2 a} \quad (1) \quad \begin{matrix} b-a > 0 \\ \Longleftrightarrow \end{matrix}$$



$$\begin{aligned} \frac{1}{n\mu^2 b} &< \frac{G\varphi a - G\varphi b}{b-a} < \frac{1}{n\mu^2 a} \quad \Leftrightarrow \\ -\frac{1}{n\mu^2 b} &> \frac{G\varphi b - G\varphi a}{b-a} > -\frac{1}{n\mu^2 a} \quad \Leftrightarrow \end{aligned} \quad \boxed{-\frac{1}{n\mu^2 a} < \frac{G\varphi b - G\varphi a}{b-a} < -\frac{1}{n\mu^2 b} \quad (2)}$$

$f(x) = G\psi x$. $\heartsuit$ f maps G to $[a, b] \subset (0, \frac{a}{2})$ G onto π_1 $G\psi x$
 $\heartsuit$ f maps G to $(a, 0)$. $f'(x) = -\frac{1}{\eta\mu}x$

Ans O.M.T. $\Rightarrow \exists \xi \in (a, b) : \frac{G(b) - G(a)}{b - a} = f'(\xi) = -\frac{1}{\pi \mu^2 \xi} \quad (3)$

$$a < \xi < b \quad \left(\begin{smallmatrix} \text{up to } n \\ \text{on } [a, b] \end{smallmatrix} \right) \quad n\mu a < n\mu \xi < n\mu b \quad \left(\begin{smallmatrix} \text{up to } n \\ \text{on } [a, b] \end{smallmatrix} \right) \quad n\mu^2 a < n\mu^2 \xi < n\mu^2 b \quad \left(\begin{smallmatrix} \text{up to } n \\ \text{on } [a, b] \end{smallmatrix} \right) \quad \frac{1}{n\mu^2 a} > \frac{1}{n\mu^2 \xi} > \frac{1}{n\mu^2 b}$$

$$-\frac{1}{\eta \mu^2 a} < -\frac{1}{\eta \mu^2 \xi} < -\frac{1}{\eta \mu^2 b} \quad (=) -\frac{1}{\eta \mu^2 a} < \frac{G \mu b \cdot G \mu a}{b a} < -\frac{1}{\eta \mu^2 b} \quad \text{denn: } \{b \mu\} \geq 2$$

mv (2) (-1) (1) a.s.f.

Αθ. 3 - Α'ΟΜ / 2.13.1 / Σχολικό.

ήρπει $\frac{f(b) - f(a)}{b - a}$

ii) Αν $0 < a < b$, ν.δ.ο. $\frac{1}{b} < \frac{\ln b - \ln a}{b - a} < \frac{1}{a}$

Έστω $f(x) = \ln x$.

♡ f συνεχ στο $[a, b] \subseteq (0, +\infty)$ { Ανό θ.Μ.Τ. $\Rightarrow \exists \xi \in (a, b)$:

♡ $f'(x) = \frac{1}{x}$, $x \in (a, b)$ $\frac{\ln b - \ln a}{b - a} = f'(\xi) = \frac{1}{\xi}$ (1)

$a < \xi < b \Rightarrow \frac{1}{a} > \frac{1}{\xi} > \frac{1}{b} \Rightarrow \frac{1}{b} < \frac{1}{\xi} < \frac{1}{a} \stackrel{(1)}{\Rightarrow} \frac{1}{b} < \frac{\ln b - \ln a}{b - a} < \frac{1}{a}$

Άρα προβιζήναι: $f'(x) = \frac{1}{x}$, $f''(x) = -\frac{1}{x^2} < 0 \Rightarrow f'(x) \downarrow$ στο $(0, +\infty)$

$a < \xi < b \Rightarrow f'(a) > f'(\xi) > f'(b) \Rightarrow f'(b) < f'(\xi) < f'(a) \Rightarrow$

$\frac{1}{b} < \frac{\ln b - \ln a}{b - a} < \frac{1}{a}$

Αθ. 6: ν.δ.ο. $\ln x \leq x - 1$ (1) $\forall x > 0$

$\ln x - \ln 1 \leq x - 1$

(1) Αν $x > 1 \Rightarrow \frac{\ln x - \ln 1}{x - 1} < 1$ (2)

Έστω $f(t) = \ln t$

♡ f συνεχ στο $[1, x] \subseteq (0, +\infty)$

♡ $f'(t) = \frac{1}{t}$ στο $(1, x)$.

Ανό θ.Μ.Τ. $\Rightarrow \exists \xi \in (1, x)$: $f'(\xi) = \frac{\ln x - \ln 1}{x - 1}$ (3)

$\Rightarrow \frac{1}{\xi} = \frac{\ln x - \ln 1}{x - 1}$. $1 < \xi < x \Rightarrow 1 > \frac{1}{\xi} > \frac{1}{x} \Rightarrow 1 > \frac{\ln x - \ln 1}{x - 1}$ (2)

Agg. 10 $f(a) = a, f(b) = b.$

α) Θ.δ.ο. m εξίσωση

$$f(x) - a - b + x = 0 \quad (1) \text{ έχει λίσιν}$$

Έστω $A(x) = f(x) - a - b + x.$ $\forall A(x)$ συνεχ στο $[a, b]$ πράξουσ συνεχ

$$\left. \begin{aligned} A(a) &= f(a) - a - b + a = a - b < 0 \\ A(b) &= f(b) - a - b + b = b - a > 0 \end{aligned} \right\} \Rightarrow A(a) \cdot A(b) < 0.$$

Απο Θ. Bolzano $\Rightarrow$

$$\exists x_0 \in (a, b): A(x_0) = 0 \Leftrightarrow \text{έχει λίσιν m (1) ο.ε.δ.}$$

β) Σκίση: Θ.δ.ο. $f'(x_1) \cdot f'(x_2) = 1$

$$\text{Απο Θ.Μ.Τ.} \Rightarrow \exists x_1 \in (a, x_0): \frac{f(x_0) - f(a)}{x_0 - a} = f'(x_1) \Leftrightarrow \frac{a + b - x_0 - a}{x_0 - a} = f'(x_1) \quad (1)$$

$$\text{Απο Θ.Μ.Τ.} \Rightarrow \exists x_2 \in (x_0, b): \frac{f(b) - f(x_0)}{b - x_0} = f'(x_2) \Leftrightarrow \frac{b - a - b + x_0}{b - x_0} = f'(x_2) \quad (2)$$

$$(1), (2) \Rightarrow f'(x_1) \cdot f'(x_2) = \frac{b - x_0}{x_0 - a} \cdot \frac{x_0 - a}{b - x_0} = 1. \quad \text{ο.ε.δ.}$$