Ασκ. 2 $\quad 2f'(x)+3x\cdot f(x) = 5x \quad (1)$ $\qquad f(0)=1$

$$f'(x)+\frac{3x}{2}f(x) = \frac{5x}{2} \iff$$

Μορφή $f'(x)+g(x)\cdot f(x)$

Θεωρώ $G(x)$ παράγουσα της $g(x)$

πολ/ζω με το $e^{G(x)}$.

$$f'(x)\cdot e^{\frac{3}{4}x^2} + \frac{3x}{2}\cdot e^{\frac{3}{4}x^2}\cdot f(x) = \frac{5x}{2}\cdot e^{\frac{3}{4}x^2}$$

$$\left(f(x)\cdot e^{\frac{3}{4}x^2}\right)' = \frac{5}{3}\frac{3x}{2}\cdot e^{\frac{3}{4}x^2} \iff$$

παράγουσα του $\frac{3x}{2}$: $\frac{3}{4}x^2$

πολ/ζω με $e^{\frac{3}{4}x^2}$

$$\left(f(x)\cdot e^{\frac{3}{4}x^2}\right)' = \left(\frac{5}{3}e^{\frac{3}{4}x^2}\right)'$$

έλεγχος: $\left(e^{\frac{3}{4}x^2}\right)' = e^{\frac{3}{4}x^2}\cdot\left(\frac{3}{4}x^2\right)'$

Δοκιμάζω: $\left(e^{\frac{3}{4}x^2}\right)' = \frac{3}{2}x\cdot e^{\frac{3}{4}x^2}$

Αφού $x\in\mathbb{R}$ (διάστημα)

$$f(x)\cdot e^{\frac{3}{4}x^2} = \frac{5}{3}e^{\frac{3}{4}x^2}+c$$

για $x=0$ $\quad 1\cdot e^0 = \frac{5}{3}e^0 + c \iff 1=\frac{5}{3}+c \iff c=-\frac{2}{3}$

$$f(x)\cdot e^{\frac{3}{4}x^2} = \frac{5}{3}e^{\frac{3}{4}x^2}-\frac{2}{3} \iff f(x) = \frac{5}{3}-\frac{2}{3}e^{-\frac{3}{4}x^2},\ x\in\mathbb{R}$$

Ασκ. 3 $f(0)=2,\quad f(x)>0\quad \forall x\in\mathbb{R}$

Βασική Εφαρμογή.

Αν $A'(x)=A(x)\quad \forall x\in\Delta$

τότε $A(x)=c\cdot e^{x},\quad x\in\Delta$.

$$f'(x)=f(x)\cdot\ln f(x)\xrightarrow{f(x)\neq 0}$$

$$\frac{f'(x)}{f(x)}=\ln f(x)\Leftrightarrow$$

$$\left(\ln f(x)\right)'=\ln f(x)\Leftrightarrow$$

$$\ln f(x)=c\cdot e^{x}\Leftrightarrow$$

$$\ln f(0)=c\cdot e^{0}\Leftrightarrow$$

$$\ln 2=c$$

$$\ln f(x)=\ln 2\cdot e^{x}\Leftrightarrow$$

$$e^{\ln f(x)}=e^{\ln 2\cdot e^{x}}\Leftrightarrow$$

$$f(x)=e^{\ln 2}\cdot e^{e^{x}}\Leftrightarrow$$

$$\boxed{f(x)=2\cdot e^{e^{x}}}\quad \boxed{x\in\mathbb{R}}$$

Ασκ. Εκτός $f(0)=1,\quad f'(0)=2$

$f(x)=;$

$$f''(x)=f(x)\Leftrightarrow$$

$$f''(x)+f'(x)=f'(x)+f(x)\Leftrightarrow$$

$$\left(f'(x)+f(x)\right)'=f'(x)+f(x)\Leftrightarrow$$

$$f'(x)+f(x)=c\cdot e^{x}\Rightarrow f'(0)+f(0)=c\cdot e^{0}\Leftrightarrow c=3$$

$$f'(x)+f(x)=3\cdot e^{x}$$ πολ/ζω με e^{x} [συντελεστής $f(x)$: 1, παράγουσα του 1: x]

$$f'(x)e^{x}+\left(e^{x}\right)'f(x)=3e^{2x}\Leftrightarrow\left(f(x)\cdot e^{x}\right)'=3\cdot e^{2x}\Leftrightarrow$$

$$\left(f(x)e^{x}\right)'=\left(\frac{3}{2}\cdot e^{2x}\right)'\Leftrightarrow f(x)\cdot e^{x}=\frac{3}{2}\cdot e^{2x}+c_1\Rightarrow$$

$$f(0)e^{0}=\frac{3}{2}\cdot e^{0}+c_1\Leftrightarrow 1=\frac{3}{2}+c_1\Leftrightarrow c_1=-\frac{1}{2}\Rightarrow f(x)e^{x}=\frac{3}{2}e^{2x}-\frac{1}{2}$$

$$f(x)=\frac{3}{2}e^{x}-\frac{1}{2}e^{-x}$$

4

$f'(x)+2f(x)=4e^{2x}$

$f'(x)\cdot e^{2x}+2\cdot e^{2x}\cdot f(x)=4\cdot e^{4x}$

$\left(f(x)\cdot e^{2x}\right)'=\left(e^{4x}\right)' \overset{\mathbb{R}}{\Leftrightarrow}$

$f(x)\cdot e^{2x}=e^{4x}+c_1 \Rightarrow$

$f(0)\cdot e^{0}=e^{0}+c_1 \Leftrightarrow$

$1=1+c_1 \Rightarrow c_1=0$

$f(x)\cdot e^{2x}=e^{4x} \Leftrightarrow$

$\boxed{f(x)=e^{2x},\ x\in\mathbb{R}.}$

$f''(x)=25f(x)$

$f''(x)+5f'(x)=5f'(x)+$
$+25f(x)$

Επίσης $=5\left(f'(x)+5f(x)\right)$

$f''(x)=4f(x) \quad f(0)=1,\ f'(0)=2$

$f''(x)+2f'(x)=2f'(x)+4f(x)$

$f''(x)+2f'(x)=2\left(\underbrace{f'(x)+2f(x)}_{A(x)}\right)$

$A'(x)=2A(x) \Leftrightarrow$

$A'(x)-2A(x)=0 \Leftrightarrow e^{-2x}\cdot A'(x)-2\cdot e^{-2x}\cdot A(x)=0$

$e^{-2x}\cdot A'(x)+\left(e^{-2x}\right)'\cdot A(x)=0 \Leftrightarrow$

$\left(e^{-2x}\cdot A(x)\right)'=0 \overset{\mathbb{R}}{\Leftrightarrow} e^{-2x}\cdot A(x)=c \Leftrightarrow$

$A(x)=c\cdot e^{2x} \Leftrightarrow f'(x)+2f(x)=c\cdot e^{2x} \overset{x=0}{\Rightarrow} f'(0)+2f(0)=c \Leftrightarrow$

$c=4 \Rightarrow f'(x)+2f(x)=4\cdot e^{2x}$

Ασκ. 4

$f''(x) = -f(x)$

Έστω $A(x) = (f(x))^2 + (f'(x))^2$

$A'(x) = 2f(x)\cdot f'(x) + 2f'(x)\cdot f''(x) \xrightarrow{(1)}$

$A'(x) = 2f(x)\cdot f'(x) + 2f'(x)\cdot(-f(x)) = 0$

$A'(x) = 0,\ x\in\mathbb{R} \to A(x) = c,\ x\in\mathbb{R}$

$f^2(x) + (f'(x))^2 = c \xrightarrow{x=0}$

$f^2(0) + (f'(0))^2 = c \Leftrightarrow c = 1$

$\boxed{f^2(x) + (f'(x))^2 = 1 \quad (2)}$

Θεωρώ τη συνάρτηση $B(x) = f(x) - \eta\mu x \Rightarrow$

$f(x) = B(x) + \eta\mu x$. $f'(x) = B'(x) + \sigma\upsilon\nu x \Rightarrow$

$f''(x) = B''(x) - \eta\mu x$

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$f''(x) + f(x) = 0$

$f''(x) = -f(x) \Leftrightarrow$

$B''(x) - \eta\mu x = -B(x) - \eta\mu x \Leftrightarrow$

$B''(x) = -B(x) \Leftrightarrow$

$B^2(x) + (B'(x))^2 = c_1 \quad (2)$

$B^2(0) + (B'(0))^2 = c_1 \quad (3)$

όμως $B(0) = f(0) - \eta\mu 0 = 0$

$B'(0) = f'(0) - \sigma\upsilon\nu 0 = 1 - 1 = 0$

$(3) \Rightarrow c_1 = 0$

$(2) \Rightarrow B^2(x) + (B'(x))^2 = 0 \quad \forall x\in\mathbb{R}$

$B(x) = 0$ και $B'(x) = 0$

$f(x) - \eta\mu x = 0$

$f(x) = \eta\mu x,\ x\in\mathbb{R}$