άσκ. 13) $f(x)=x^{\frac{1}{x}}$, $x>0$ $\qquad a^x=e^{x\ln a}$

$f(x)=e^{\frac{1}{x}\cdot\ln x}$. $\quad f'(x)=e^{\frac{1}{x}\ln x}\cdot\left(\frac{1}{x}\ln x\right)'=x^{\frac{1}{x}}\cdot\dfrac{\cancel{\frac{1}{x}\cdot x}-\ln x\cdot 1}{x^2}=$

$=\dfrac{x^{1/x}\cdot(1-\ln x)}{x^2}$. Λύνω την ανίσωση $f'(x)>0 \iff 1-\ln x>0 \iff$

$\ln x<1 \iff 0<x<e$. Αφού f συνεχής στα $(0,e]$, $[e,+\infty)$

$\Rightarrow f\uparrow$ στο $(0,e]$ και $f\downarrow$ στο $[e,+\infty)$

	0	e	$+\infty$
$f'(x)$	$+$	0	$-$
$f(x)$	$\nearrow$		$\searrow$

(ii) $a^b<b^a \iff \left(a^b\right)^{\frac{1}{ab}}<\left(b^a\right)^{\frac{1}{ab}} \iff$

$a^{\frac{\cancel{b}}{a\cancel{b}}}<b^{\frac{\cancel{a}}{\cancel{a}b}} \iff a^{\frac{1}{a}}<b^{\frac{1}{b}} \iff f(a)<f(b) \xLeftrightarrow[\text{στο }[e,+\infty)]{f(x)\downarrow} a>b$ τα οποία ισχύει

Συνήθης μέθοδος $\quad a^b<b^a \iff \ln a^b<\ln b^a \iff b\ln a<a\ln b \iff \boxed{\dfrac{\ln a}{a}<\dfrac{\ln b}{b}}$

$\frac{1}{a}\ln a<\frac{1}{b}\ln b \iff \cancel{\ln} a^{\frac{1}{a}}<\cancel{\ln} b^{\frac{1}{b}}$ $\quad \boxed{A(x)=\dfrac{\ln x}{x}}$

(γ) $e^n \lesseqgtr n^e \iff \ln e^n \lesseqgtr \ln n^e \iff n\cdot\ln e\le e\ln n \iff \boxed{\dfrac{\ln e}{e}\lesseqgtr\dfrac{\ln n}{n}}$.

0	e	$+\infty$
$\nearrow$		$\searrow$

$e^n>n^e \iff \left(e^n\right)^{\frac{1}{en}}>\left(n^e\right)^{\frac{1}{en}} \iff e^{\frac{1}{e}}>n^{\frac{1}{n}} \iff f(e)>f(n) \iff e<n$

$\nu^{\frac{1}{\nu}}>(\nu+1)^{\frac{1}{\nu+1}} \iff f(\nu)>f(\nu+1) \xLeftrightarrow{\nu>e} \nu<\nu+1$.

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άσκ 6 | Σ.174 / Σχόλιο:

$f(x) = \frac{\ln x}{x}$

x	0	e	$+\infty$
f	$\nearrow$		$\searrow$

(ii) Ν.δ.ο. $\boxed{a^{a+1} > (a+1)^a \quad (1)}$ $a>e$

$\ln a^{a+1} > \ln(a+1)^a \Leftrightarrow$

$(a+1)\ln a > a\ln(a+1) \Leftrightarrow$

$\frac{\ln a}{a} > \frac{\ln(a+1)}{a+1} \Leftrightarrow \boxed{f(a) > f(a+1). \ (2)}$

Όμως $e<a<a+1 \xRightarrow{f \searrow} f(a) > f(a+1)$. Άρα ισχύει $(2) \Leftrightarrow (1)$

(iii) $2^x = x^2$, $x>0$ $\qquad \ln x^2 = 2\ln|x|$, $x \in \mathbb{R}^*$

Για $x<e$

$\ln 2^x = \ln x^2 \Leftrightarrow$

$x \ln 2 = 2 \cdot \ln x \Leftrightarrow$

$\frac{\ln 2}{2} = \frac{\ln x}{x} \Leftrightarrow$

$f(x) = f(2) \xLeftrightarrow[f\nearrow]{(0,e)}$

$\boxed{x=2}$

$\frac{\ln 2}{2} = \frac{\ln x}{x}$

$f(x) = f(2)$

$x=2$

Λάθος σ.δ.

Για $x>e \Rightarrow 2^x = x^2 \Leftrightarrow$

$\ln 2^x = \ln x^2 \Leftrightarrow$

$x \ln 2 = 2\ln x \Leftrightarrow$

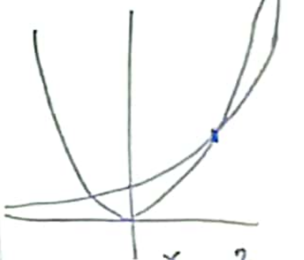
$\frac{\ln 2}{2} = \frac{\ln x}{x} \Leftrightarrow$

$\frac{\ln 4}{4} = \frac{\ln x}{x} \Leftrightarrow$

$f(x) = f(4) \xLeftrightarrow[f:1-1]{(e,+\infty)}$

$\boxed{x=4}$

$\frac{\ln 4}{4} = \frac{\ln 2^2}{4} = \frac{2\ln 2}{4} = \frac{\ln 2}{2}$



$A(x) = 2^x - x^2$

$A(-1) = 2^{-1} - (-1)^2 = \frac{1}{2} - 1$

$A(0) = 2^0 - 0^2 = 1 > 0$

ασκ. 19

$f(3) - f(e) > \ln(\ln 3)$ ΘΜΤ.

Από ΘΜΤ. $\Rightarrow \exists\ \xi \in (e, 3)$: $\dfrac{f(3)-f(e)}{3-e} = f'(\xi) > \dfrac{1}{\xi \ln \xi} > \dfrac{1}{3\ln 3}$

$$\left.\begin{array}{l} e < \xi < 3 \Rightarrow \ln e < \ln \xi < \ln 3 \\ e < \xi < 3 \end{array}\right\} \quad e < \xi \ln \xi < 3 \ln 3 \Leftrightarrow$$

$$\frac{1}{e} > \frac{1}{\xi \ln \xi} > \frac{1}{3\ln 3}$$

Έχω δείξει: $\dfrac{f(3)-f(e)}{3-e} > \dfrac{1}{3\ln 3} \Leftrightarrow f(3) - f(e) > \dfrac{3-e}{3\ln 3} > \ln(\ln 3)$

Έχω $f'(x) > \dfrac{1}{x \ln x} \Leftrightarrow f'(x) - \dfrac{1}{x\ln x} > 0 \Leftrightarrow f'(x) - \dfrac{(\ln x)'}{\ln x} > 0 \Leftrightarrow$

$f'(x) - (\ln(\ln x))' > 0 \Leftrightarrow (\underbrace{f(x) - \ln(\ln x)}_{A(x)})' > 0$

$e < 3 \Rightarrow A(e) < A(3) \Leftrightarrow f(e) - \ln(\ln e) < f(3) - \ln(\ln 3)$

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