$$\lim_{x\to+\infty}\left[\ln(e^x+1)-x\right]=\lim_{x\to+\infty}\left\{x\cdot\left[\frac{\ln(e^x+1)}{x}-1\right]\right\}=0\cdot\infty \;\; \frac{0}{0}$$

μορφή	μέθοδος
$\infty-\infty$	κοινό παράγοντα $0\cdot\infty$
$0\cdot\infty$	$\frac{\infty}{\frac{1}{0}}=\frac{\infty}{\infty}$

$$\lim_{x\to+\infty}\frac{\ln(e^x+1)}{x}=\lim_{x\to+\infty}\frac{\frac{1}{e^x+1}\cdot e^x}{1}=\lim_{x\to+\infty}\frac{e^x}{e^x+1}=$$

$$=\lim_{y\to+\infty}\frac{y}{y+1}=1$$

$$\lim_{x\to+\infty}\frac{\frac{\ln(e^x+1)}{x}-1}{\frac{1}{x}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{\frac{\frac{1}{e^x+1}(e^x+1)'\cdot x-\ln(e^x+1)}{\cancel{x^2}}}{-\frac{1}{\cancel{x^2}}}=\lim_{x\to+\infty}-\left(\overset{+\infty}{\frac{x\cdot e^x}{e^x+1}}-\overset{-\infty}{\ln(e^x+1)}\right)$$

$$\lim_{x\to+\infty}\left[\ln(e^x+1)-x\right]=\lim_{x\to+\infty}\left[\ln(e^x+1)-\ln e^x\right]=\lim_{x\to+\infty}\left[\ln\left(\frac{e^x+1}{e^x}\right)\right]=0$$

$$\ln e^x=x$$

$$\lim_{x\to+\infty}\left[x\,\frac{\ln^2(x+1)-\ln^2 x}{\ln x}\right]=\lim_{x\to+\infty}\left[x\,\frac{(\ln(x+1)-\ln x)\cdot(\ln(x+1)+\ln x)}{\ln x}\right]=$$

$$=\lim_{x\to+\infty}\left[\underbrace{\overset{\infty}{x}\,\ln\overset{0}{\frac{x+1}{x}}}_{A(x)}\cdot\underbrace{\frac{\overset{\infty}{\ln(x^2+x)}}{\ln x\ \ \infty}}_{B(x)}\right]=$$

Σκέψη $\lim_{x\to+\infty}\frac{x+1}{x}=1$. $\ln 1=0$

$$0\cdot\infty=\frac{\infty}{\frac{1}{0}}=\frac{\infty}{\infty}$$

$$\frac{0}{\frac{1}{\infty}}=\frac{0}{0}$$

$$\underset{0}{\ln\frac{x+1}{x}}=\frac{1}{\frac{1}{\ln\frac{x+1}{x}}}\ \infty$$

Το $0\longrightarrow\frac{1}{\infty}$

$$\left[\ln(x^2+x)\right]'=\frac{1}{x^2+x}\cdot(2x+1)$$

$$(\ln x)'=\frac{1}{x}.\qquad \frac{\frac{2x+1}{x^2+x}}{\frac{1}{x}}=\frac{(2x+1)\cdot x}{x^2+x}$$

$$=\frac{2x^2+x}{x^2+x}\xrightarrow[x\to+\infty]{}2.\quad \lim_{x\to+\infty}B(x)=2$$

$$\lim_{x\to+\infty}\left[x\cdot \ln\frac{x+1}{x}\right]=\lim_{x\to+\infty}\frac{\ln\frac{x+1}{x}}{\frac{1}{x}}\overset{\frac{0}{0}}{\underset{DLH}{=}}\lim_{x\to+\infty}\frac{\frac{x}{x+1}\cdot\left(\frac{x+1}{x}\right)'}{-\frac{1}{x^2}}=$$

$$=\lim_{x\to+\infty}\frac{\frac{x}{x+1}\cdot\frac{1\cdot x-(x+1)\cdot 1}{x^2}}{-\frac{1}{x^2}}=\lim_{x\to+\infty}\frac{\frac{x}{x+1}\cdot\left(\frac{1}{x^2}\right)}{-\frac{1}{x^2}}=1.$$

Βρείτε τα $\alpha,\beta\in\mathbb{R}$ ώστε $\lim\limits_{x\to 2}\frac{x^2+\alpha x+\beta}{x-2}=5$

Λήμμα: Αν $\left.\begin{array}{l}\lim\limits_{x\to x_0}B(x)=0\\ \text{και } \lim\limits_{x\to x_0}\frac{A(x)}{B(x)}=\ell\in\mathbb{R}\end{array}\right\}\Longrightarrow \lim\limits_{x\to x_0}A(x)=0$

Θέτω $w(x)=\frac{A(x)}{B(x)}$, $\lim\limits_{x\to x_0}w(x)=\ell$

$A(x)=B(x)\cdot w(x)$.

$\lim\limits_{x\to x_0}\left[B(x)\cdot w(x)\right]=0\cdot\ell=0$

Έστω: $\lim\limits_{x\to 2} \dfrac{x^2+ax+b}{x-2} = 5$

Από λήμμα $\Rightarrow \lim\limits_{x\to 2}(x^2+ax+b)=0$

$\Rightarrow 4+2a+b=0 \Rightarrow$

$b=-2a-4$

$$\lim_{x\to 2} \underbrace{\frac{x^2+ax-2a-4}{x-2}}_{G(x)} = 5$$

όμως $\lim\limits_{x\to 2} G(x) \overset{\frac{0}{0}}{=} \lim\limits_{x\to 2} \dfrac{2x+a}{1} = 4+a$

$\Rightarrow 4+a=5 \Leftrightarrow a=1$

$\Rightarrow b=-2-4=-6.$

Επαλήθευση: $\lim\limits_{x\to 2} \dfrac{x^2+x-6}{x-2} = \dots$

Έστω f παρ/μη στο $a \in D_f$.

Τότε $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \overset{\frac{0}{0}}{\underset{DLH}{=}} \lim_{x \to a} \frac{f'(x)}{1} = \lim_{x \to a} f'(x)$

$\exists$?

Δηλ. η f' συνεχής στο a. Ουάου....