Παραγοντική ολοκλήρωση:

$$\int_\alpha^\beta f'(x)\cdot g(x)\,dx = \left[f(x)\cdot g(x)\right]_\alpha^\beta - \int_\alpha^\beta f(x)\cdot g'(x)\,dx$$

Απόδειξη:

$$\left(f(x)\cdot g(x)\right)' = f'(x)\cdot g(x) + f(x)\cdot g'(x) \Leftrightarrow$$

$$\int_\alpha^\beta f'(x)\cdot g(x) = \int_\alpha^\beta \left(f(x)\cdot g(x)\right)' - \int_\alpha^\beta f(x)\cdot g'(x) \Leftrightarrow$$

$$\int_\alpha^\beta f'(x)\cdot g(x)\,dx = \left[f(x)\cdot g(x)\right]_\alpha^\beta - \int_\alpha^\beta f(x)\,g'(x)\,dx.$$

Υπολογισμοί ολοκληρωμάτων.

1η κατηγορία $\int_\alpha^\beta P(x)\cdot e^x\,dx$, $P(x)$ πολυώνυμο

1) $\int_0^1 x\cdot e^x\,dx = \int_0^1 x\cdot(e^x)'\,dx = \left[x\cdot e^x\right]_0^1 - \int_0^1 x'\cdot e^x\,dx =$

$1\cdot e^1 - 0\cdot e^0 - \int_0^1 e^x\,dx = e - \left[e^x\right]_0^1 = e-(e^1-e^0)$

$= e - e + 1 = 1.$

$$\int_\alpha^\beta \underbrace{f'(x)}\cdot \underbrace{g(x)}\,dx = \left[f(x)\cdot g(x)\right]_\alpha^\beta - \int_\alpha^\beta f(x)\cdot g'(x)\,dx$$

2) $\int_0^1 (x^2-3x)\cdot e^{2x}\,dx = \int_0^1 (x^2-3x)\left(\frac{1}{2}e^{2x}\right)'dx =$

$$= \underbrace{\left[\frac{1}{2}e^{2x}\cdot(x^2-3x)\right]_0^1}_{A} - \frac{1}{2}\int_0^1 \underbrace{(2x-3)}\cdot \underbrace{e^{2x}}\,dx = A - \frac{1}{2}\underbrace{\int_0^1 (2x-3)\cdot\left(\frac{1}{2}e^{2x}\right)'dx}_{I_2}$$

$$I_2 = \int_0^1 (2x-3)\left(\frac{1}{2}e^{2x}\right)' dx = \underbrace{\left[(2x-3)\cdot\frac{1}{2}\cdot e^{2x}\right]_0^1}_{B} - \int_0^1 \cancel{2}\cdot\frac{1}{\cancel{2}}e^{2x}\,dx =$$

$$= B - \left[\frac{1}{2}e^{2x}\right]_0^1 = \dots$$

__2η κατηγορία:__ $\int_a^b P(x)\cdot \text{ημ}x\,dx$ ή συνx.

$$\int_0^{\pi/2} x^2\cdot \text{ημ}2x\,dx = -\int_0^{\pi/2} x^2\left(\frac{1}{2}\,\text{συν}2x\right)' dx = -\left\{\underbrace{\left[x^2\cdot\frac{1}{2}\,\text{συν}2x\right]_0^{\pi/2}}_{A} - \underbrace{\int_0^{\pi/2}\cancel{2}x\cdot\frac{1}{\cancel{2}}\,\text{συν}2x\,dx}_{I_2}\right\}$$

$$I_2 = \int_0^{\pi/2} x\cdot\left(\frac{1}{2}\,\text{ημ}2x\right)' dx = \left[x\cdot\frac{1}{2}\,\text{ημ}2x\right]_0^{\pi/2} - \int_0^{\pi/2}\frac{1}{2}\,\text{ημ}2x\,dx = \dots$$

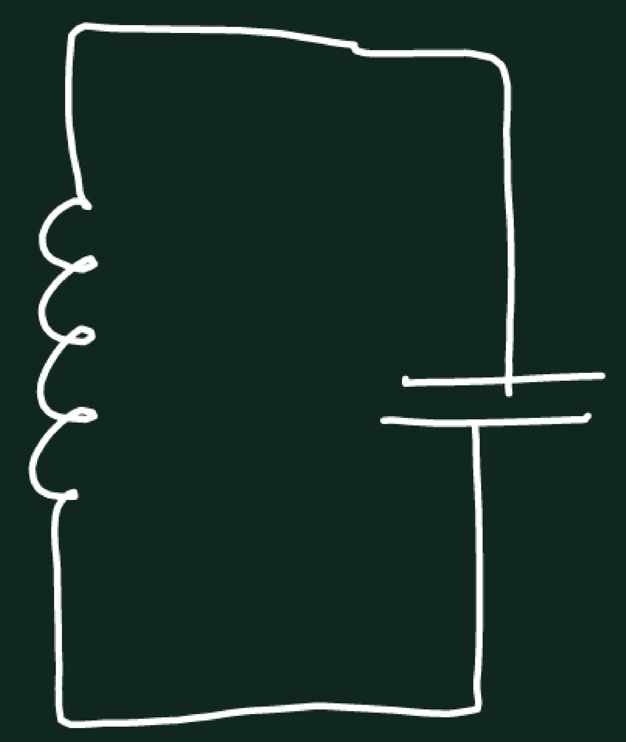
$$\parallel$$

$$-\frac{1}{2}\cdot\frac{1}{2}\left[\text{συν}2x\right]_0^{\pi/2}$$

3η κατηγορία : $\int_{\alpha}^{\beta} P(x)\cdot \ln x \, dx$ ⟶ "προετοιμάζω" το $P(x)$

$$\int_1^e (x^2+3x)\cdot \ln x \, dx = \int_1^e \left(\frac{x^3}{3}+\frac{3x^2}{2}\right)' \ln x \, dx =$$

$$= \underbrace{\left[\left(\frac{x^3}{3}+\frac{3x^2}{2}\right)\ln x\right]_1^e}_{A} - \underbrace{\int_1^e \left(\frac{x^3}{3}+\frac{3x^2}{2}\right)\cdot \frac{1}{x}\, dx}_{I_2}$$



$$I_2 = \int_1^e \left(\frac{x^2}{3}+\frac{3x}{2}\right) dx = \left[\frac{1}{3}\cdot\frac{1}{3}x^3 + \frac{3}{2}\cdot\frac{x^2}{2}\right]_1^e = \ldots$$