Να αποδείξετε ότι $\frac{\eta\mu\theta}{1+\sigma\upsilon\nu\theta} + \frac{1+\sigma\upsilon\nu\theta}{\eta\mu\theta} = \frac{2}{\eta\mu\theta}$

$$\frac{\eta\mu\theta}{1+\sigma\upsilon\nu\theta} + \frac{1+\sigma\upsilon\nu\theta}{\eta\mu\theta} =$$

$$\boxed{\eta\mu^2\theta = (\eta\mu\theta)^2}$$

$$= \frac{(\eta\mu\theta)^2 + (1+\sigma\upsilon\nu\theta)^2}{(1+\sigma\upsilon\nu\theta)(\eta\mu\theta)} = \frac{\eta\mu^2\theta + 1 + 2\sigma\upsilon\nu\theta + \sigma\upsilon\nu^2\theta}{(1+\sigma\upsilon\nu\theta)(\eta\mu\theta)} =$$

$$= \frac{1+1+2\sigma\upsilon\nu\theta}{(1+\sigma\upsilon\nu\theta)(\eta\mu\theta)} = \frac{2+2\sigma\upsilon\nu\theta}{(1+\sigma\upsilon\nu\theta)(\eta\mu\theta)} = \frac{2\cancel{(1+\sigma\upsilon\nu\theta)}}{\cancel{(1+\sigma\upsilon\nu\theta)}(\eta\mu\theta)} = \frac{2}{\eta\mu\theta}$$

Να λυθεί η εξίσωση:

$$\frac{\eta\mu x}{1+\sigma\upsilon\nu x} + \frac{1+\sigma\upsilon\nu x}{\eta\mu x} = 2\sqrt{2} \Leftrightarrow$$

$$\frac{2}{\eta\mu x} = 2\sqrt{2} \Leftrightarrow$$

$$2\sqrt{2}\cdot \eta\mu x = 2 \Leftrightarrow$$

$$\eta\mu x = \frac{2}{2\sqrt{2}} \Leftrightarrow$$

$$\eta\mu x = \frac{\sqrt{2}}{2} \Leftrightarrow$$

$$\eta\mu x = \eta\mu\frac{\pi}{4}$$

$$x = 2\kappa\pi + \frac{\pi}{4} \quad \text{ή} \quad x = 2\kappa\pi + \pi - \frac{\pi}{4},\ \kappa\in\mathbb{Z}$$

Να δείξετε ότι η συνάρτηση $f(x) = \frac{\eta\mu x}{1+\sigma\upsilon\nu x} + \frac{1+\sigma\upsilon\nu x}{\eta\mu x}$ είναι περιοδική.

Λύση: Από προηγούμενο ερώτημα έχω αποδείξει ότι $f(x) = \frac{2}{\eta\mu x}$.

$$f(x+2\pi) = \frac{2}{\eta\mu(x+2\pi)} = \frac{2}{\eta\mu x} = f(x)$$

$f(x) = 2\,\eta\mu\, 4x$

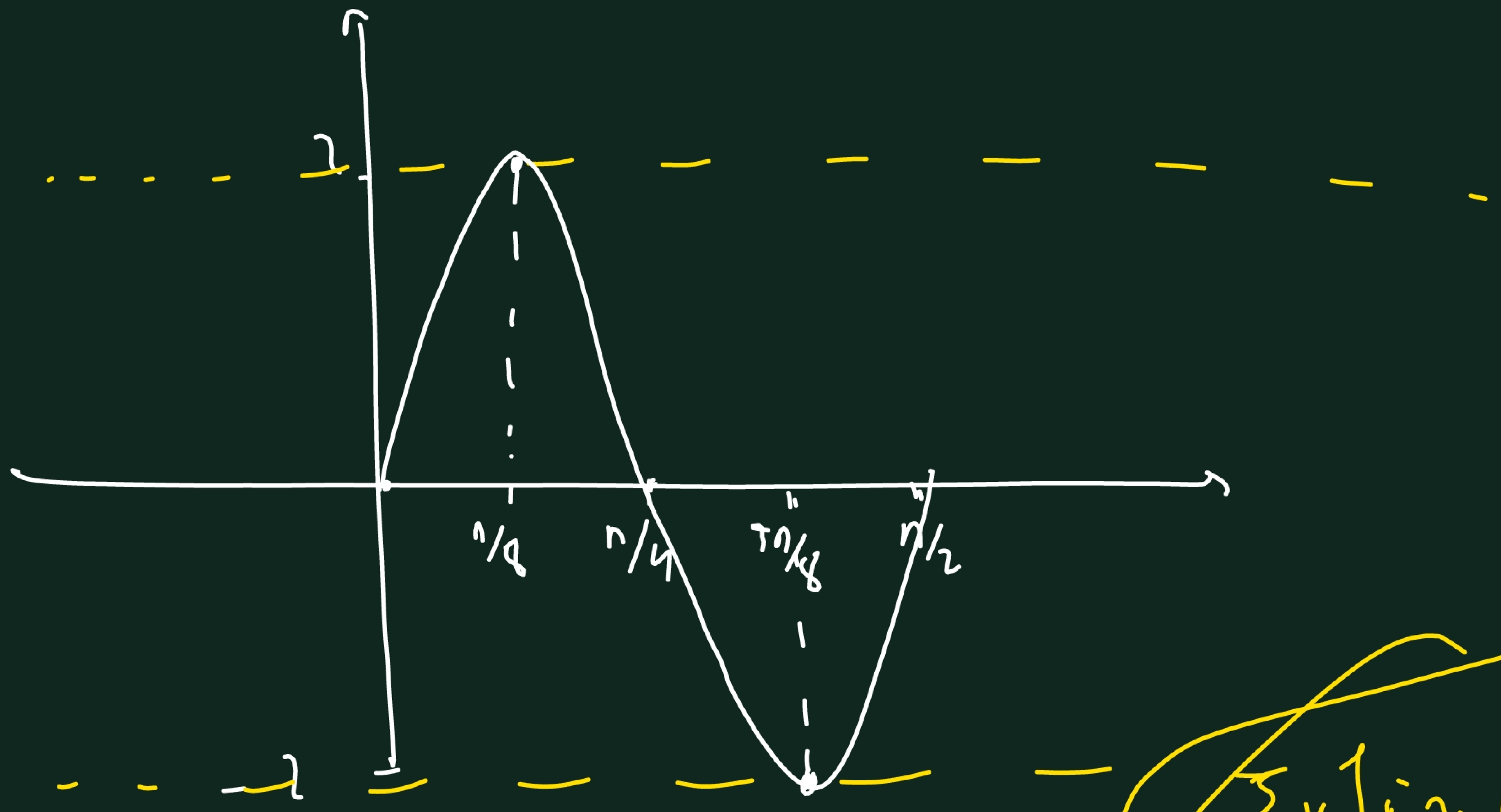
$T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

$f_{max} = 2, \quad f_{min} = -2$

$\dfrac{\pi}{2} \longrightarrow \dfrac{\pi}{2\cdot 4} = \dfrac{\pi}{8}$

$\dfrac{\pi}{8} \qquad \dfrac{2\pi}{8} \qquad \dfrac{3\pi}{8} \qquad \dfrac{4\pi}{8}$

$\dfrac{2\pi}{8} \downarrow \dfrac{\pi}{4} \qquad \dfrac{4\pi}{8} \downarrow \dfrac{\pi}{2}$



$\eta\mu x$

Συζήτηση

$f\left(x + \frac{\pi}{2}\right) =$

$= 2\cdot\eta\mu\left[4\left(x + \frac{\pi}{2}\right)\right] =$

$= 2\,\eta\mu\,(4x + \cancel{2\pi}) =$

$= 2\,\eta\mu\,4x = f(x)$

$f(x) = \rho\,\eta\mu\,\omega x.$ $\qquad T = \dfrac{2\pi}{\omega}$

$$f\left(x+\frac{2\pi}{\omega}\right) = \rho\cdot\eta\mu\left[\omega\cdot\left(x+\frac{2\pi}{\omega}\right)\right] = \rho\cdot\eta\mu\left(\omega x+\cancel{2\pi}\right) = \rho\cdot\eta\mu\,\omega x = f(x)$$

Να δείξετε ότι η συνάρτηση $f(x) = 2\eta\mu x + 3\sigma\upsilon\nu x$ είναι περιοδική

$$f(x+2\pi) = 2\eta\mu(x+2\pi) + 3\sigma\upsilon\nu(x+2\pi) \Longleftrightarrow$$

$$2\eta\mu x + 3\sigma\upsilon\nu x = f(x)$$