Άσκηση 12 / Σελ. 64

(i) $$\frac{\varepsilon\varphi\alpha+\varepsilon\varphi\beta}{\varepsilon\varphi\beta+\varepsilon\varphi\alpha}=\frac{\frac{\eta\mu\alpha}{\sigma\upsilon\nu\alpha}+\frac{\sigma\upsilon\nu\beta}{\eta\mu\beta}}{\frac{\eta\mu\alpha}{\eta\mu\beta}+\frac{\sigma\upsilon\nu\beta}{\sigma\upsilon\nu\alpha}}=\frac{\frac{\eta\mu\alpha\cdot\eta\mu\beta}{\eta\mu\beta\cdot\sigma\upsilon\nu\alpha}+\frac{\sigma\upsilon\nu\alpha\cdot\sigma\upsilon\nu\beta}{\eta\mu\beta\cdot\sigma\upsilon\nu\alpha}}{\frac{\eta\mu\alpha\cdot\eta\mu\beta}{\eta\mu\alpha\cdot\sigma\upsilon\nu\beta}+\frac{\sigma\upsilon\nu\alpha\cdot\sigma\upsilon\nu\beta}{\sigma\upsilon\nu\beta\cdot\eta\mu\alpha}}=$$

$$=\frac{\frac{\eta\mu\alpha\cdot\eta\mu\beta+\sigma\upsilon\nu\alpha\cdot\sigma\upsilon\nu\beta}{\eta\mu\beta\cdot\sigma\upsilon\nu\alpha}}{\frac{\eta\mu\alpha\cdot\eta\mu\beta+\sigma\upsilon\nu\alpha\cdot\sigma\upsilon\nu\beta}{\eta\mu\alpha\cdot\sigma\upsilon\nu\beta}}=\frac{(\eta\mu\alpha\cdot\eta\mu\beta+\sigma\upsilon\nu\alpha\cdot\sigma\upsilon\nu\beta)(\eta\mu\alpha\cdot\sigma\upsilon\nu\beta)}{(\eta\mu\beta\cdot\sigma\upsilon\nu\alpha)(\eta\mu\alpha\cdot\eta\mu\beta+\sigma\upsilon\nu\alpha\cdot\sigma\upsilon\nu\beta)}=\frac{\eta\mu\alpha\cdot\sigma\upsilon\nu\beta}{\eta\mu\beta\cdot\sigma\upsilon\nu\alpha}=$$

$$=\varepsilon\varphi\alpha\cdot\sigma\varphi\beta=\boxed{\frac{\varepsilon\varphi\alpha}{\varepsilon\varphi\beta}}$$

$$\varepsilon\varphi\alpha\cdot\sigma\varphi\beta=\varepsilon\varphi\alpha\cdot\frac{1}{\varepsilon\varphi\beta}=\frac{\varepsilon\varphi\alpha}{\varepsilon\varphi\beta}$$

άσκ 2 - Β' ΟΜ / Σελ. 64

Ν.δ.ο. $\eta\mu^4x+\sigma\upsilon\nu^4x = 1-2\eta\mu^2x\cdot\sigma\upsilon\nu^2x$

$$\eta\mu^4x+\sigma\upsilon\nu^4x = \underbrace{(\eta\mu^2x)}_{a}{}^2 + \underbrace{(\sigma\upsilon\nu^2x)}_{b}{}^2 = (\eta\mu^2x+\sigma\upsilon\nu^2x)^2 - 2\eta\mu^2x\cdot\sigma\upsilon\nu^2x$$

$$= 1 - 2\eta\mu^2x\cdot\sigma\upsilon\nu^2x \quad \text{ο.ε.δ.}$$

β) $\eta\mu^6x+\sigma\upsilon\nu^6x = 1-3\eta\mu^2x\cdot\sigma\upsilon\nu^2x$

1ος τρόπος: $\eta\mu^6x+\sigma\upsilon\nu^6x = \underbrace{(\eta\mu^2x)}_{a}{}^3 + \underbrace{(\sigma\upsilon\nu^2x)}_{b}{}^3 =$

$$= (\eta\mu^2x+\sigma\upsilon\nu^2x)^3 - 3\eta\mu^2x\cdot\sigma\upsilon\nu^2x\cdot(\eta\mu^2x+\sigma\upsilon\nu^2x)$$

$$= 1^3 - 3\eta\mu^2x\cdot\sigma\upsilon\nu^2x\cdot 1 \quad \text{ο.ε.δ.}$$

$$a^2+b^2 = (a+b)^2 - 2ab$$

$$a^3+b^3 = (a+b)\cdot(a^2-ab+b^2)$$

$$a^3+b^3 = (a+b)^3 - 3ab\cdot(a+b)$$

$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$

$$(a+b)^3 = a^3+b^3+3ab\cdot(a+b) \Leftrightarrow$$

$$(a+b)^3 - 3ab\cdot(a+b) = a^3+b^3$$

Ασκ. 1 / Β' ΟΜ / Σελ 64

Αν $\eta\mu x + \sigma\upsilon\nu x = \alpha$

i) $\eta\mu x \cdot \sigma\upsilon\nu x$

Έχω $\eta\mu^2 x + \sigma\upsilon\nu^2 x = 1 \Leftrightarrow$

$$(\eta\mu x + \sigma\upsilon\nu x)^2 - 2\eta\mu x \cdot \sigma\upsilon\nu x = 1$$

$$\alpha^2 - 2\eta\mu x \cdot \sigma\upsilon\nu x = 1 \Leftrightarrow$$

$$-2\eta\mu x \cdot \sigma\upsilon\nu x = -\alpha^2 + 1$$

$$\frac{2\eta\mu x \cdot \sigma\upsilon\nu x}{2} = \frac{\alpha^2 - 1}{2}$$

ii) $\dfrac{\sigma\upsilon\nu x}{1} \quad \dfrac{\eta\mu x}{1}$

$$\frac{1}{\eta\mu x} + \frac{1}{\sigma\upsilon\nu x} =$$

$$\frac{\sigma\upsilon\nu x}{\eta\mu x \cdot \sigma\upsilon\nu x} + \frac{\eta\mu x}{\eta\mu x \cdot \sigma\upsilon\nu x} =$$

$$\frac{\sigma\upsilon\nu x + \eta\mu x}{\eta\mu x \cdot \sigma\upsilon\nu x} =$$

$$\frac{\alpha}{\frac{\alpha^2 - 1}{2}} = \frac{2\alpha}{\alpha^2 - 1}$$

(iii) $\mathcal{E}_{qx} + \mathcal{G}_{qx} = \frac{\overbrace{n\mu x}^{\sim t x}}{6\omega x} + \frac{\overbrace{6\omega x}^{\sim t x}}{n\mu x} = \frac{n\mu^2 x + 6\omega^2 x}{n\mu x + 6\omega x}$

(iv) $n\mu^3 x + 6\omega^3 x = \underbrace{(n\mu x + 6\omega x)}^{3} - 3\underbrace{n\mu x}\cdot 6\omega x \cdot \underbrace{(n\mu x + 6\omega x)}$

$\hookrightarrow (n\mu x + 6\omega x)\cdot\left(\underline{n\mu^2 x} - n\mu x \cdot 6\omega x + \underline{6\omega^2 x}\right)$

(v) $n\mu^6 x + 6\omega^6 x$

(vi) $n\mu^4 x + 6\omega^4 x$

vii) $\frac{1}{n\mu^2 x} + \frac{1}{6\omega^2 x}$

viii) $\frac{n\mu x}{6\omega^2 x} + \frac{6\omega x}{n\mu^2 x}$