Τριγωνομετρικές εξισώσεις.

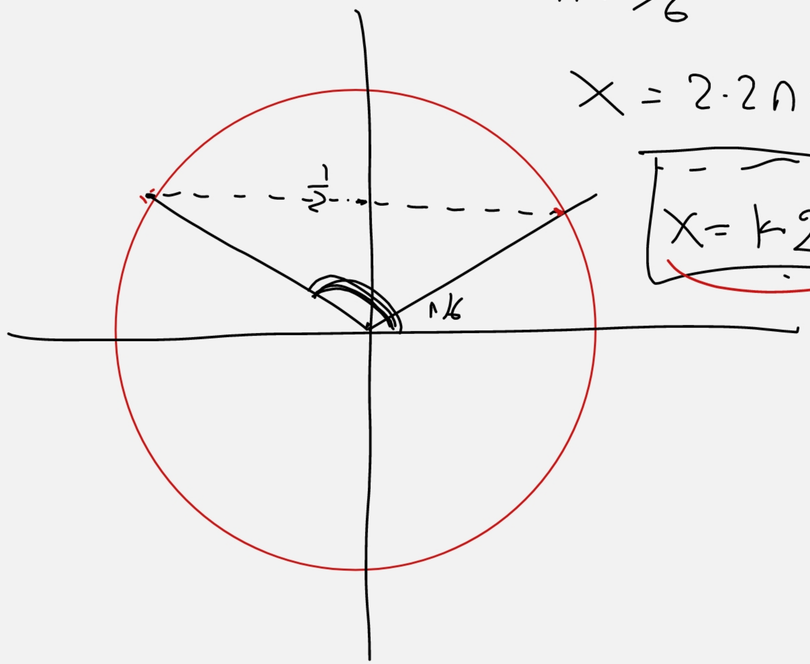
1) Να λυθεί η εξίσωση

$$\eta\mu x = \frac{1}{2} \longrightarrow \eta\mu x = \eta\mu \frac{\pi}{6}$$

$$x = \frac{\pi}{6} \qquad x = \pi - \frac{\pi}{6}$$

$$x = 2\pi + \frac{\pi}{6} \qquad x = 2\pi + \pi - \frac{\pi}{6}$$

$$x = 2\cdot 2\pi + \frac{\pi}{6} \quad , \quad x = 3\cdot 2\pi + \pi - \frac{\pi}{6}$$



$$\boxed{x = k\cdot 2\pi + \frac{\pi}{6}}, \quad \boxed{x = k\cdot 2\pi + \pi - \frac{\pi}{6}}$$

$k \in \mathbb{Z}$

$$\eta\mu x = \eta\mu \frac{\pi}{8} \Leftrightarrow$$

$$x = 2k\pi + \frac{\pi}{8} \quad \text{ή}$$

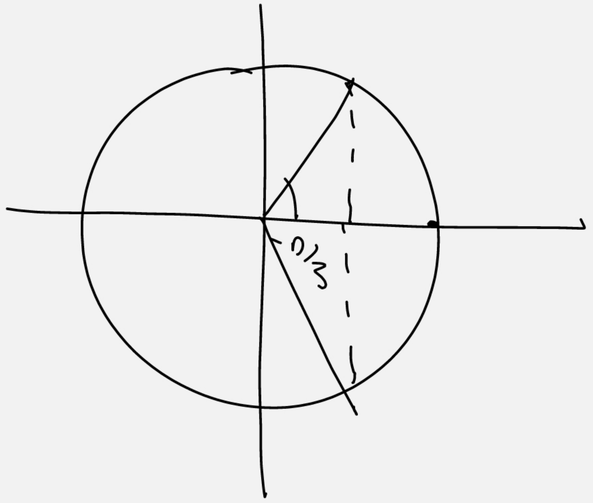
$$x = 2k\pi + \pi - \frac{\pi}{8}$$

2) Να λυθεί η εξίσωση

$\sigma\upsilon\nu x = \frac{1}{2} \Longleftrightarrow$

$x = \pi/3$ | $x = -\pi/3$

$x = k\cdot 2\pi + \pi/3$ | $x = k\cdot 2\pi - \pi/3$



3) Να λυθεί η εξίσωση

$\varepsilon\varphi x = \sqrt{3} \Longleftrightarrow$

$x = \pi/3$ $\qquad$ $x = \pi + \pi/3$

$x = 2k\pi + \pi/3$ $\qquad$ $x = 2k\pi + \pi + \pi/3$

χρησιμοποιούμε $+\frac{\pi}{3}$

$x = (2k+1)\pi + \pi/3$

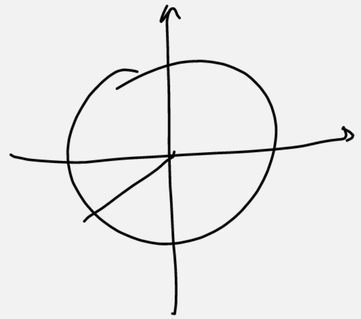
περιττά πολ. του π $+\frac{\pi}{3}$

Γενικά $x = k\pi + \frac{\pi}{3}$

$k \in \mathbb{Z}$

Διότι

$\varepsilon\varphi\left(\pi + \frac{\pi}{3}\right) = \varepsilon\varphi\frac{\pi}{3}$



Βασικές τριγωνομετρικές εξισώσεις (Απ' έξω)

1) $\eta\mu x = \eta\mu a \iff x = 2k\pi + a \ , \ x = 2k\pi + \pi - a \ , \ k \in \mathbb{Z}$

2) $\sigma\upsilon\nu x = \sigma\upsilon\nu a \iff x = 2k\pi \pm a \ , \ k \in \mathbb{Z}$

3) $\left.\begin{array}{l} \varepsilon\varphi x = \varepsilon\varphi a \\ \sigma\varphi x = \sigma\varphi a \end{array}\right\} \iff x = k\pi + a \ , \ k \in \mathbb{Z}$

Σχολικό βιβλίο – Λυμένα παραδείγματα

Σ.84 / παρ. 1

$\eta\mu x = -\frac{\sqrt{3}}{2} \iff$

$\eta\mu x = -\eta\mu\frac{\pi}{3} \iff$

$\eta\mu x = \eta\mu\left(-\frac{\pi}{3}\right) \iff$

$\iff x = 2k\pi - \frac{\pi}{3} \ , \ x = 2k\pi + \pi + \frac{\pi}{3}$

$k \in \mathbb{Z}$

Σελ. 84 Παρ. 2

$\eta\mu\left(2x + \frac{\pi}{4}\right) = \frac{1}{2} \iff$

$\eta\mu\left(2x + \frac{\pi}{4}\right) = \eta\mu\frac{\pi}{6} \iff$

$2x + \frac{\pi}{4} = 2k\pi + \frac{\pi}{6} \iff$

$2x = 2k\pi + \frac{\pi}{6} - \frac{\pi}{4} \iff$

$2x = 2k\pi + \frac{2\pi - 3\pi}{12} \iff$

$x = \frac{2k\pi}{2} - \frac{\pi}{24} \iff x = k\pi - \frac{\pi}{24}$

$2x + \frac{\pi}{4} = 2k\pi + \pi - \frac{\pi}{6}$

$2x = 2k\pi + \pi - \frac{\pi}{6} - \frac{\pi}{4} \iff$

$2x = 2k\pi + \frac{12\pi - 2\pi - 3\pi}{12} \iff$

$\frac{2x}{2} = \frac{2k\pi}{2} + \frac{7\pi}{12 \cdot 2} \iff x = k\pi + \frac{7\pi}{24}$

Παραδ. 2 / Σ. 86

$$\sigma\upsilon\nu 2x = -\frac{\sqrt{3}}{2} \Longleftrightarrow$$

$$\sigma\upsilon\nu 2x = -\sigma\upsilon\nu\frac{\pi}{6} \Longleftrightarrow$$

$$\sigma\upsilon\nu 2x = \sigma\upsilon\nu\left(\pi - \frac{\pi}{6}\right) \Longleftrightarrow$$

$$\sigma\upsilon\nu 2x = \sigma\upsilon\nu\frac{5\pi}{6} \Longleftrightarrow$$

$$\frac{2}{2}x = \frac{2k\pi}{2} \pm \frac{5\pi}{6\cdot 2} \Longleftrightarrow$$

$$x = k\pi \pm \frac{5\pi}{12}, \quad k \in \mathbb{Z}$$

Διάβασμα

$\sigma\upsilon\nu(-\alpha) = \sigma\upsilon\nu\alpha$

$\sigma\upsilon\nu(\pi - \alpha) = -\sigma\upsilon\nu\alpha$

όλα τα γνωστά παραδείγματα

των σελ. 84 - 87

μέχρι 1, 2, 3 / Σελ. 88.