

Να λύσει η εξίσωση:

$$\log x^4 = (\log x^2)^2 \quad (1)$$

Πρόσεται:

$$x^4 > 0 \Leftrightarrow \text{κεν} \quad x^2 > 0 \Leftrightarrow x \neq 0$$

$$(1) \Leftrightarrow 4 \log |x| = (2 \log |x|)^2 \Leftrightarrow$$

$$\text{Θέτω } \log |x| = y$$

$$4y = (2y)^2 \Leftrightarrow 4y = 4y^2 \Leftrightarrow$$

$$0 = 4y^2 - 4y \Leftrightarrow 4y(y-1) = 0 \Leftrightarrow$$

$$\textcircled{y=0} \text{ ή } y-1=0 \Leftrightarrow \textcircled{y=1}$$

$$y=0 \Leftrightarrow$$

$$\log |x| = 0 \Leftrightarrow$$

$$\log |x| = \log 1 \Leftrightarrow$$

$$|x| = 1 \Leftrightarrow$$

$$x = \pm 1$$

Δευτὶς

$$y=1 \Leftrightarrow$$

$$\log |x| = 1 \Leftrightarrow$$

$$\log |x| = \log 10 \Leftrightarrow$$

$$|x| = 10 \Leftrightarrow$$

$$x = \pm 10$$

Δευτὶς

i) $\log \sqrt{x} = \sqrt{\log x}$

Προϋπόθεση:

$x > 0$



Από $x \geq 1$

$\frac{1}{2} \log x = \sqrt{\log x}$ Ορίζω $\log x = y$

$\frac{1}{2} y = \sqrt{y}, y \geq 0 \iff \frac{1}{4} y^2 = y \iff$

$\log x \geq 0 \iff$
 $\log x \geq \log 1 \iff$
 $x \geq 1$

$\log_{\frac{1}{2}} x \geq \log_{\frac{1}{2}} 1 \iff$
 $x \leq 1$

$\frac{1}{4} y^2 - y = 0 \iff$

$y \left(\frac{1}{4} y - 1 \right) = 0 \iff$

$y = 0$ ή $\frac{1}{4} y = 1 \iff y = 4$

$\log x = 0 \iff$

$\log x = \log 1 \iff$

$x = 1$

8 σημ. in

$\log x = 4 \iff$

$\log x = 4 \cdot 1 \iff$

$\log x = 4 \cdot \log 10 \iff$

$\log x = \log 10^4 \iff$

$x = 10^4$ 8 σημ. in.

Αου. 5 - Β'ΟΜ / Σελ. 185

(ii) $\ln^4 x - 5 \ln^2 x + 4 = 0 \Leftrightarrow$

$$(\ln x)^4 - 5(\ln x)^2 + 4 = 0, \quad x > 0$$

Θέτουμε $\ln x = y$

$$y^4 - 5y^2 + 4 = 0$$

Θέτουμε $y^2 = u \Rightarrow y^4 = u^2$

$$u^2 - 5u + 4 = 0 \Leftrightarrow u_{1,2} = \frac{5 \pm 1}{2}$$

$$y^2 = 4$$

$$y = \pm 2$$

$$y^2 = 1$$

$$y = \pm 1$$

$$\ln x^4 - 5 \ln x^2 + 4 = 0 \Leftrightarrow$$

$$\ln x^4 - \ln (x^2)^5 = -4 \Leftrightarrow$$

$$\ln \frac{x^4}{x^{10}} = -4 \Leftrightarrow$$

$$\ln \frac{1}{x^6} = -4 \cdot \ln e \Leftrightarrow$$

$$\ln \frac{1}{x^6} = \ln e^{-4} \Leftrightarrow$$

$$\frac{1}{x^6} = e^{-4} \Leftrightarrow x^6 = e^4 \Leftrightarrow$$

$$x = \pm \sqrt[6]{e^4}$$

10 επιδ.

$$\ln x = 2.1, \quad \ln x = -2.1, \quad \ln x = 1, \quad \ln x = -1$$

$$\ln x = 2.1 \Leftrightarrow \ln x = -2.1 \Leftrightarrow$$

$$\ln x = \ln e^2 \Leftrightarrow \ln x = \ln e^{-2}$$

$$x = e^2$$

$$x = e^{-2}$$

$$\ln x = \ln e$$

$$x = e$$

$$\ln x = -\ln e \Leftrightarrow$$

$$\ln x = \ln e^{-1}$$

$$x = e^{-1}$$

Ασκήσεις Να λυθούν οι εξισώσεις:

$$1) \frac{\log x + 3}{\log x} + \frac{\log x - 2}{\log x - 3} = \frac{9}{2}$$

$$2) \log(\log(3x-5)) = 0$$

$$3) \log(\log(2x^2 + x - 11)) = 0$$

$$4) \log(\log x) = 4$$

$$\log(\log(3x-5)) = 0$$

Definition: $3x-5 > 0$

kon: $\log(3x-5) > 0 \Leftrightarrow$

$$\log(3x-5) > \log 1 \quad \left(\begin{array}{l} \log x \\ \leftarrow \end{array} \right)$$

$$3x-5 > 1 \quad \Leftrightarrow \quad 3x > 6 \\ x > 2$$

$$A > 0$$

$$A > 1$$

$$\log(\log(3x-5)) = \log 1 \Leftrightarrow$$

$$\log(3x-5) = 1 \Leftrightarrow$$

$$\log(3x-5) = \log 10 \Leftrightarrow 3x-5 = 10$$

$$3x = 15 \Leftrightarrow x = 5 \quad \text{Domain}$$