

Γ.

15. Να αποδείξετε ότι:

$$\text{i. } \frac{1 - \eta\mu\alpha}{\sigma\upsilon\upsilon\alpha} = \frac{\sigma\upsilon\upsilon\alpha}{1 + \eta\mu\alpha}$$

$$\text{ii. } \frac{\sqrt{2}\sigma\upsilon\upsilon\chi - 1}{\eta\mu\chi + \sigma\upsilon\upsilon\chi} = \frac{\sigma\upsilon\upsilon\chi - \eta\mu\chi}{\sqrt{2}\sigma\upsilon\upsilon\chi + 1}$$

16. Να δείξετε ότι:

$$\text{i. } (\eta\mu\chi + \sigma\upsilon\upsilon\chi)^2 = 1 + 2\eta\mu\chi \cdot \sigma\upsilon\upsilon\chi$$

$$\text{ii. } (\eta\mu\chi + \sigma\upsilon\upsilon\chi)(\eta\mu\chi - \sigma\upsilon\upsilon\chi) + 2\sigma\upsilon\upsilon^2\chi = 1$$

17. Να δείξετε ότι:

$$\text{i. } \frac{\sigma\upsilon\upsilon^3\chi - \sigma\upsilon\upsilon\chi}{\eta\mu^3\chi - \eta\mu\chi} = \epsilon\phi\chi$$

$$\text{ii. } \frac{\eta\mu^4\chi - \eta\mu^2\chi}{\sigma\upsilon\upsilon^4\chi - \sigma\upsilon\upsilon^2\chi} = 1$$

$$\text{iii. } \frac{\sigma\upsilon\upsilon^2\alpha - \sigma\upsilon\upsilon^2\beta}{\eta\mu^2\alpha - \eta\mu^2\beta} = -1$$

18. Να δείξετε ότι: $\eta\mu^4\alpha - \sigma\upsilon\upsilon^4\alpha = \eta\mu^2\alpha - \sigma\upsilon\upsilon^2\alpha = 1 - 2\sigma\upsilon\upsilon^2\alpha = 2\eta\mu^2\alpha - 1$

19. Να αποδείξετε ότι:

$$\text{i. } 1 - \frac{\eta\mu^2\chi}{1 + \sigma\upsilon\upsilon\chi} = \sigma\upsilon\upsilon\chi$$

$$\text{ii. } 1 - \frac{\sigma\upsilon\upsilon^2\chi}{1 + \eta\mu\chi} = \eta\mu\chi$$

20. Να δείξετε ότι:

$$\text{i. } \frac{\sigma\upsilon\upsilon\alpha}{1 - \eta\mu\alpha} - \frac{1 - \eta\mu\alpha}{\sigma\upsilon\upsilon\alpha} = 2\epsilon\phi\alpha$$

$$\text{ii. } \frac{\eta\mu\alpha}{1 + \sigma\upsilon\upsilon\alpha} + \frac{\eta\mu\alpha}{1 - \sigma\upsilon\upsilon\alpha} = \frac{2}{\eta\mu\alpha}$$

21. Να δείξετε ότι: $\frac{\sigma\upsilon\upsilon^3\chi - \sigma\upsilon\upsilon\chi + \eta\mu\chi}{\sigma\upsilon\upsilon\chi} = \epsilon\phi\chi - \eta\mu^2\chi$

22. Να δείξετε ότι:

$$\text{i. } \eta\mu^2\chi \cdot \epsilon\phi\chi - \sigma\upsilon\upsilon^2\chi \cdot \sigma\phi\chi = \epsilon\phi\chi - \sigma\phi\chi$$

$$\text{ii. } \eta\mu^2\chi \cdot \epsilon\phi\chi + \sigma\upsilon\upsilon^2\chi \cdot \sigma\phi\chi + 2\eta\mu\chi \cdot \sigma\upsilon\upsilon\chi = \epsilon\phi\chi + \sigma\phi\chi$$

23. Να αποδείξετε ότι:

$$\text{i. } \epsilon\phi\chi + \sigma\phi\chi = \frac{1}{\eta\mu\chi \cdot \sigma\upsilon\upsilon\chi}$$

$$\text{ii. } \epsilon\phi^2\chi + \sigma\phi^2\chi + 2 = \frac{1}{\eta\mu^2\chi \cdot \sigma\upsilon\upsilon^2\chi}$$

7. Να δείξετε ότι:

i. $\eta\mu^2 26^\circ + \eta\mu^2 34^\circ + \eta\mu^2 64^\circ + \eta\mu^2 56^\circ = 2$

ii. $\varepsilon\varphi \frac{\pi}{7} \cdot \varepsilon\varphi \frac{3\pi}{7} \cdot \varepsilon\varphi \frac{5\pi}{14} \cdot \varepsilon\varphi \frac{\pi}{14} = 1$

8. Να δείξετε ότι:

i. $3\sigma\upsilon\nu^2(\pi - x) + \varepsilon\varphi(\pi + x) \cdot \sigma\varphi(\pi - x) + 3\eta\mu^2(-x) = 2$

ii. $2\eta\mu^2(\pi + x) + \varepsilon\varphi\left(\frac{\pi}{2} - x\right) \cdot \varepsilon\varphi(2\pi - x) + 2\eta\mu^2\left(\frac{\pi}{2} - x\right) = 1$

9. Να απλοποιήσετε τις παραστάσεις:

i. $A = \frac{\eta\mu\left(\frac{3\pi}{2} - \theta\right) \cdot \sigma\upsilon\nu\left(\frac{\pi}{2} + \theta\right) \cdot \varepsilon\varphi(2\pi - \theta)}{\sigma\upsilon\nu\left(\frac{5\pi}{2} + \theta\right) \cdot \sigma\varphi(-\theta) \cdot \sigma\varphi\left(\frac{\pi}{2} + \theta\right)}$

ii. $B = \frac{\eta\mu(2\pi + \theta) \cdot \sigma\varphi\left(\frac{3\pi}{2} - \theta\right) \cdot \sigma\upsilon\nu(3\pi + \theta)}{\sigma\upsilon\nu\left(\frac{3\pi}{2} + \theta\right) \cdot \varepsilon\varphi(\pi + \theta) \cdot \sigma\upsilon\nu\left(\frac{\pi}{2} + \theta\right)}$

10. Να υπολογίσετε τις τιμές των παραστάσεων:

i. $A = \frac{\eta\mu \frac{3\pi}{4} + \sigma\upsilon\nu \frac{2\pi}{3} - \eta\mu \frac{5\pi}{6}}{\eta\mu \frac{5\pi}{4} - \sigma\upsilon\nu \frac{4\pi}{3} - \eta\mu \frac{7\pi}{6}}$

ii. $B = \frac{-\eta\mu 135^\circ - \varepsilon\varphi 225^\circ + \sigma\upsilon\nu 120^\circ}{\eta\mu 225^\circ + \sigma\varphi 135^\circ + \sigma\upsilon\nu 240^\circ}$

11. Αν $\eta\mu(\pi + \omega) = -\frac{12}{15}$ και $0 < \omega < \frac{\pi}{2}$, να βρείτε το $\eta\mu\left(\frac{3\pi}{2} - \omega\right)$.

12. Να δείξετε ότι:

i. $\eta\mu^2\left(\frac{\pi}{3} + x\right) + \eta\mu^2\left(\frac{\pi}{6} - x\right) = 1$

ii. $\varepsilon\varphi(35^\circ - x) \varepsilon\varphi(55^\circ + x) = 1$