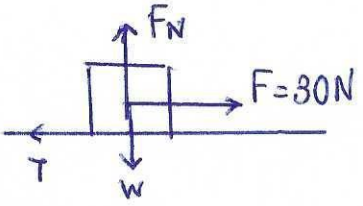
Λύσεις ασκήσεων φυλλαδίου Φ6

1) Στο σώμα ασκούνται



- βάρος $w = mg \Leftrightarrow w = 2\,kg \cdot 10\frac{m}{s^2} \Leftrightarrow w = 20N$
- η κάθετη αντίδραση από το έδαφος F_N
- η τριβή T

1ο βήμα

Συνθήκη ισορροπίας στον y-άξονα

$\Sigma F_y = 0 \Leftrightarrow F_N - w = 0 \Leftrightarrow F_N = w \Leftrightarrow F_N = 20\,N$

2ο βήμα

Εύρεση τριβής: $T = \mu F_N \Leftrightarrow T = \frac{1}{2} \cdot 20\,N \Leftrightarrow T = 10N$

3ο βήμα

Εφαρμόζω Β' Νόμο Νεύτωνα: $\Sigma F_x = ma \Leftrightarrow F - T = ma \Leftrightarrow$

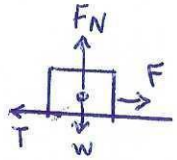
$30 - 10 = 2a \Leftrightarrow 20 = 2a \Leftrightarrow \underline{a = 10\,m/s^2}$

$\Delta x = v_0 \cdot \Delta t + \frac{1}{2} a \Delta t^2 \Leftrightarrow \Delta x = \frac{1}{2} \cdot 10 \cdot 3^2 = 45m$

2) Από τα κινηματικά χαρακτηριστικά βρίσκω την επιτάχυνση του σώματος.

$\Delta x = v_0 \Delta t + \frac{1}{2} a \Delta t^2 \Leftrightarrow 14 = 4 \cdot 2 + \frac{1}{2} a \cdot 2^2 \Leftrightarrow 14 = 8 + \frac{1}{2} a \cdot 4$

$14 - 8 = 2a \Leftrightarrow \underline{a = 3\,m/s^2}$

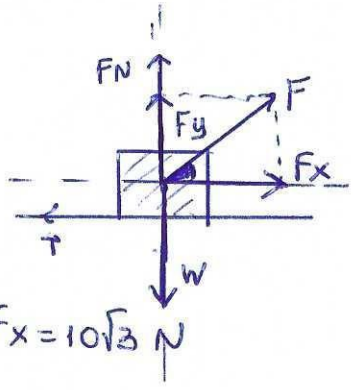


$\Sigma F_x = ma \Leftrightarrow F - T = ma \Leftrightarrow 25 - T = 5 \cdot 3 \Leftrightarrow T = 25 - 15 \Leftrightarrow \underline{T = 10\,N}$

$\Sigma F_y = 0 \Leftrightarrow w - F_N = 0 \Leftrightarrow F_N = mg \Leftrightarrow \underline{F_N = 50\,N}$

$T = \mu \cdot F_N \Leftrightarrow \mu = \frac{T}{F_N} \Leftrightarrow \mu = \frac{10}{50} \Leftrightarrow \mu = 1/5$

3) α) Αναλύω την F σε συνιστώσες



- $\eta\mu\phi = \frac{F_y}{F} \Leftrightarrow F_y = F\,\eta\mu\phi \Leftrightarrow F_y = 20 \cdot \frac{1}{2}$

$\Leftrightarrow F_y = 10\,N$

- $\sigma\upsilon\nu\phi = \frac{F_x}{F} \Leftrightarrow F_x = F\,\sigma\upsilon\nu\phi \Leftrightarrow F_x = 20 \cdot \frac{\sqrt{3}}{2} \Leftrightarrow F_x = 10\sqrt{3}\,N$

Συνθήκη ισορροπίας στον y-άξονα $\Rightarrow F_N$

$\Sigma F_y = 0 \Leftrightarrow F_N + F_y = w \Leftrightarrow F_N = mg - F_y \Leftrightarrow F_N = 30 - 10 \Leftrightarrow F_N = 20N$

Τριβή

$T = \mu \cdot F_N \Leftrightarrow T = \frac{\sqrt{3}}{5} \cdot 20 \Leftrightarrow T = 4\sqrt{3}\,N.$

Β' Νόμος Νεύτωνα (στον x-άξονα)

$\Sigma F_x = ma \Leftrightarrow F_x - T = ma \Leftrightarrow 10\sqrt{3} - 4\sqrt{3} = 3a \Leftrightarrow 3a = 6\sqrt{3} \Leftrightarrow$

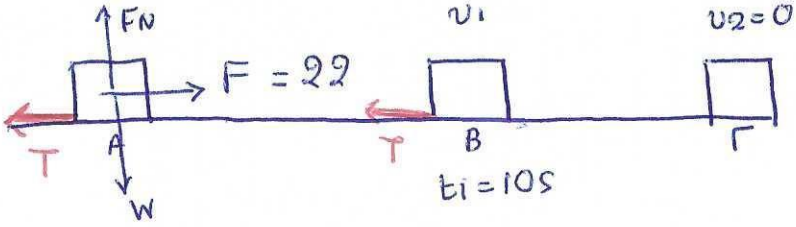
$\Leftrightarrow a = 2\sqrt{3}\ m/s^2$

β) $v = v_0 + a\Delta t \Leftrightarrow 6 = 0 + 2\sqrt{3}\,t \Leftrightarrow 2\sqrt{3}\,t = 6 \Leftrightarrow t = \frac{6}{2\sqrt{3}}\,s \Leftrightarrow$

$t = \frac{6\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}} \Leftrightarrow t = \frac{6\sqrt{3}}{6}\,s \Leftrightarrow t = \sqrt{3}\,s.$

$\Delta x = \frac{1}{2} a \Delta t^2 \Leftrightarrow \Delta x = \frac{1}{2} \cdot 2\sqrt{3} \cdot (\sqrt{3})^2 \Leftrightarrow \Delta x = 3\sqrt{3}\ m$

4)



$W = mg \Leftrightarrow$

$W = 20\,N$

- $\Sigma F_y = 0 \Leftrightarrow F_N - w = 0 \Leftrightarrow F_N = W = 20\,N$
- $T = \mu \cdot F_N \Leftrightarrow T = 0{,}5 \cdot 20 \Leftrightarrow T = 10\,N$

Από Α → Β $\Sigma F_x = m \cdot a_1 \Leftrightarrow F - T = m a_1 \Leftrightarrow 22 - 10 = 5a_1 \Leftrightarrow$

$12 = 5a_1 \Leftrightarrow a_1 = \frac{12}{5}\ m/s^2$ (επιταχυνόμενη)

- $\Delta x_1 = v_0 \cdot \Delta t_1 + \frac{1}{2} a_1 \Delta t_1^2 \Leftrightarrow \Delta x_1 = \frac{1}{2} \cdot \frac{12}{5} \cdot 100 \Leftrightarrow$

$\Delta x_1 = 120\,m$

- Η ταχύτητα v_1 στο σημείο Β: $v_1 = v_0 + a_1 \cdot \Delta t_1 \Leftrightarrow$

$v_1 = 0 + \frac{12}{5} \cdot 10 \Leftrightarrow$

$v_1 = 24\,m/s.$

Από Β → Γ $\Sigma F_x = m a_2 \Leftrightarrow$

$-T = m a_2 \Leftrightarrow -10 = 5a_2 \Leftrightarrow a_2 = -2\,m/s^2$

(επιβραδυνόμενη)

Αρχική ταχύτητα για 'αυτό το διάστημα κίνησης:

$v_1 = 24\ m/s.$

Βρίσκω σε πόσο χρόνο θα σταματήσει

$v_2 = v_1 + a_2 \cdot \Delta t_2 \Leftrightarrow 0 = 24 - 2\,\Delta t_2 \Leftrightarrow \Delta t_2 = 12\,sec$

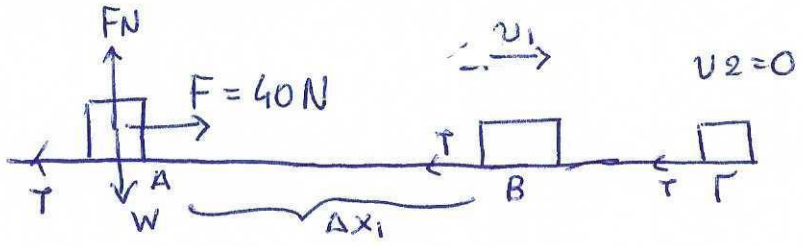
Βρίσκω πόσο διάστημα θα διανύσει (Β → Γ)

$\Delta x_2 = v_1 \cdot \Delta t_2 + \frac{1}{2} a_2 \Delta t_2^2 = 24 \cdot 12 - \frac{1}{2} \cdot 2 \cdot 12^2 = 144\,m$

Άρα $\Delta t_{ολ} = \Delta t_1 + \Delta t_2 = 10\,s + 12\,s = 22\,s$

$\Delta x_{ολ} = \Delta x_1 + \Delta x_2 = 120\,m + 144\,m = 264\,m$

5)



• $\Sigma F_y = 0 \Leftrightarrow F_N = w = mg \Leftrightarrow F_N = 50N$

$T = \mu \cdot F_N = \frac{1}{5} \cdot 50 = 10N$

A→B $\Sigma F_x = ma_1 \Leftrightarrow F - T = ma_1 \Leftrightarrow 40 - 10 = 5a_1 \Leftrightarrow a_1 = 6 m/s^2$

• $\Delta x_1 = \frac{1}{2} a \Delta t_1^2 \Leftrightarrow 12 = \frac{1}{2} \cdot 6 \Delta t_1^2 \Leftrightarrow \Delta t_1^2 = 4 \Leftrightarrow \Delta t_1 = 2s$

• $\upsilon_1 = \upsilon_0 + a_1 \Delta t_1 \Leftrightarrow \upsilon_1 = 6 \cdot 2 = 12 m/s$

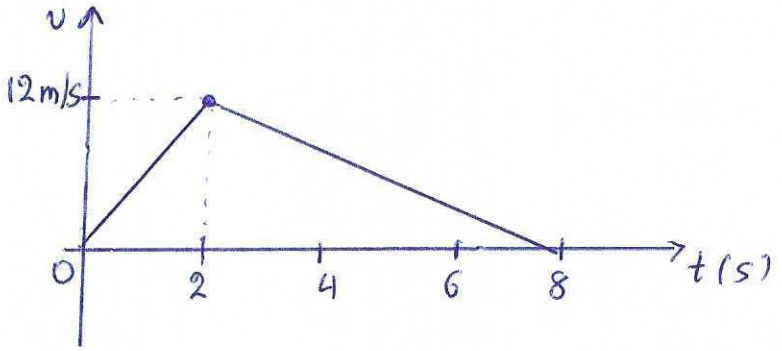
B→Γ $\Sigma F_x = ma_2 \Leftrightarrow -T = ma_2 \Leftrightarrow -10 = 5a_2 \Leftrightarrow a_2 = -2 m/s^2$

• $\upsilon_2 = \upsilon_1 + a \Delta t_2 \Leftrightarrow 0 = 12 - 2\Delta t_2 \Leftrightarrow \Delta t_2 = 6s.$

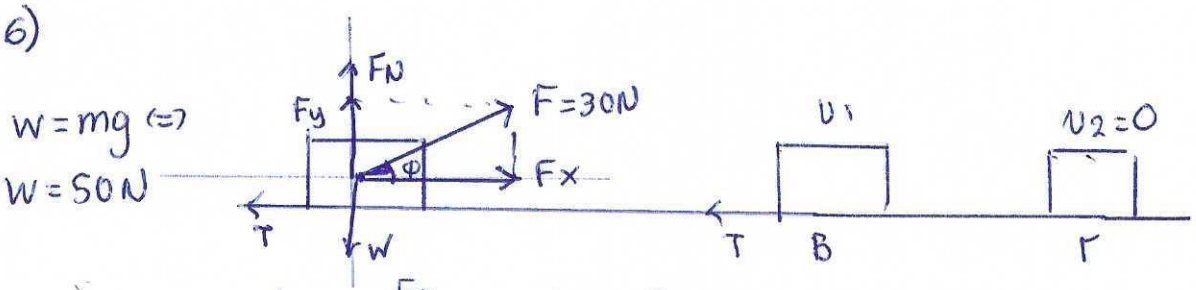
• $\Delta x_2 = \upsilon_1 \cdot \Delta t_2 + \frac{1}{2} a_2 \Delta t_2^2 \Leftrightarrow \Delta x_2 = 12 \cdot 6 - \frac{1}{2} \cdot 2 \cdot 36 = 36m$

αρα (α) $\Delta t_{ολ} = \Delta t_1 + \Delta t_2 = 2s + 6s = 8s$

$\Delta x_{ολ} = \Delta x_1 + \Delta x_2 = 12m + 36m = 48m$



6)



$\eta\mu\varphi = \frac{Fy}{F} \Leftrightarrow Fy = F\eta\mu\varphi \Leftrightarrow Fy = 30 \cdot \frac{\sqrt{3}}{2} \Leftrightarrow Fy = 15\sqrt{3} N$

$\sigma\upsilon\nu\varphi = \frac{Fx}{F} \Leftrightarrow Fx = F \cdot \sigma\upsilon\nu\varphi \Leftrightarrow Fx = 30 \cdot \frac{1}{2} \Leftrightarrow Fx = 15N$

▸ $\Sigma F_y = 0 \Leftrightarrow F_N + Fy = w \Leftrightarrow F_N = w - Fy \Leftrightarrow$

$F_N = 50 - 15\sqrt{3} \Leftrightarrow F_N = 5(10 - 3\sqrt{3}) N.$

▸ Τριβή $T = \mu \cdot F_N \Leftrightarrow T = \frac{1}{10 - 3\sqrt{3}} \cdot 5(10 - 3\sqrt{3}) = 5N$

(συνέχεια άσκησης 6)

(A → B) • $\Sigma F_x = ma_1 \Leftrightarrow F_x - T = ma_1 \Leftrightarrow 15 - 5 = 5a_1 \Leftrightarrow a_1 = 2\ m/s^2$

• $v_1 = v_0 + a_1 \Delta t_1 \Leftrightarrow v_1 = 0 + 2 \cdot 2 \Leftrightarrow v_1 = 4\ m/s$.

• $\Delta x_1 = v_0 \Delta t_1 + \frac{1}{2} a_1 \Delta t_1^2 \Leftrightarrow \Delta x_1 = \frac{1}{2} \cdot 2 \cdot 2^2 \Leftrightarrow \Delta x_1 = 4m$.

(B → Γ) • $\Sigma F_x = ma_2 \Leftrightarrow -T = ma_2 \Leftrightarrow -5 = 5a_2 \Leftrightarrow a_2 = -1\ m/s^2$

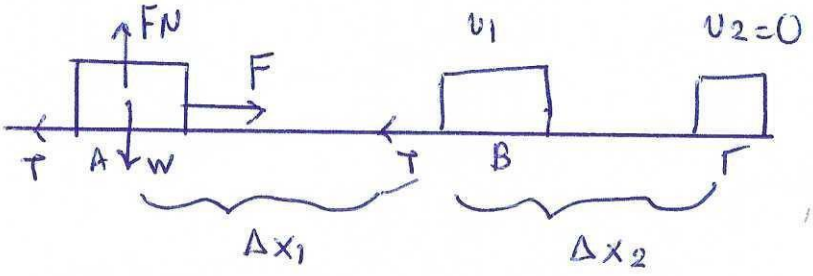
• $v_2 = v_1 + a_2 \Delta t_2 \Leftrightarrow 0 = 4 - 1 \cdot \Delta t_2 \Leftrightarrow \Delta t_2 = 4s$.

• $\Delta x_2 = v_1 \Delta t_2 + \frac{1}{2} a_2 \Delta t_2^2 \Leftrightarrow \Delta x_2 = 4 \cdot 4 - \frac{1}{2} \cdot 1\ 4^2$

$\Delta x_2 = 16 - 8 \Leftrightarrow \Delta x_2 = 8m$

$\Delta x_{ολ} = \Delta x_1 + \Delta x_2 = 4m + 8m = 12m$.

γ) **



• $\Sigma F_y = 0 \Leftrightarrow w = F_N \Leftrightarrow F_N = 50\ N$.

(A → B) $F - T = ma_1 \Leftrightarrow 50 - T = 5a_1 \Leftrightarrow a_1 = \frac{50 - T}{5}$ (1)

$\Delta x_1 = \frac{1}{2} a_1 \Delta t_1^2$ (2)

$v_1 = a_1 \cdot \Delta t_1$ (3)

(B → Γ) $\Sigma F_x = ma_2 \Leftrightarrow a_2 = -\frac{T}{m} \Leftrightarrow a_2 = -\frac{T}{5}$ (4)

$v_2 = v_1 + a_2 \cdot \Delta t_2 \Leftrightarrow 0 = a_1 \cdot \Delta t_1 + a_2 \cdot \Delta t_2 \Leftrightarrow$

$\Delta t_2 = \Delta t_1 \frac{a_1}{-a_2} \Leftrightarrow \Delta t_2 = \Delta t_1 \cdot \frac{\frac{50 - T}{5}}{+\frac{T}{5}} \Leftrightarrow \Delta t_2 = \Delta t_1 \cdot \frac{(50 - T)}{T}$ (5)

$\Delta x_2 = v_1 \cdot \Delta t_2 + \frac{1}{2} a_2 \Delta t_2^2 \Leftrightarrow$

$\Delta x_2 = a_1 \cdot \Delta t_1 \cdot \Delta t_2 + \frac{1}{2} a_2 \cdot \Delta t_2^2 \Leftrightarrow$

$\Delta x_2 = a_1 \cdot \Delta t_1 \cdot \Delta t_1 \frac{(50 - T)}{T} + \frac{1}{2} \cdot \left(-\frac{T}{5}\right) \cdot \Delta t_1^2 \frac{(50 - T)^2}{T^2}$

$\Delta x_2 = \frac{(50 - T)^2}{5 \cdot T} \cdot \Delta t_1^2 - \frac{1}{2} \cdot \frac{(50 - T)^2}{5 \cdot T} \Delta t_1^2 \Leftrightarrow$

$$\Delta x_2 = \frac{(50-T)^2}{10 \cdot T} \Delta t_1^2 \quad (6)$$

$$\frac{\Delta x_2}{\Delta x_1} = 4 \overset{(2),(6)}{\Leftrightarrow} \frac{\frac{(50-T)^2}{10T} \cancel{\Delta t_1^2}}{\frac{1}{2} \cdot \frac{(50-T)}{5} \cancel{\Delta t_1^2}} = 4 \Leftrightarrow$$

$$\frac{\frac{50-T}{\cancel{10}T}}{\frac{1}{\cancel{10}}} = 4 \Leftrightarrow \frac{50-T}{T} = 4 \Leftrightarrow 50-T = 4T \Leftrightarrow 5T = 50$$

$$\Leftrightarrow T = 10\,N$$

$$\mu = \frac{T}{F_N} \Leftrightarrow \mu = \frac{10}{50} \Leftrightarrow \mu = 0,2$$