Ασκήσεις στις Αλγεβρικές παραστάσεις

Θέμα 1ο

Να απλοποιήσετε τις παρακάτω παραστάσεις :

α) $\mathbf{A} = \left(\frac{1}{x+y}+\frac{1}{x-y}\right) : \frac{2y}{x^2-y^2}$ **β)** $\mathbf{B} = \left(\frac{\alpha}{\alpha+2\beta}+\frac{\alpha}{\alpha-2\beta}\right) : \frac{2\alpha^2}{\alpha^2-4\beta^2}$

Απ. Α = $\frac{x}{y}$ ***, Β = 1***

γ) $\mathbf{\Gamma} = \left(\frac{1+x}{1-x}-\frac{1-x}{1+x}\right)\left(\frac{3}{4x}+\frac{x}{4}-x\right)$

Απ. Γ = 3

δ) $\mathbf{\Delta} = \left[\left(\frac{\alpha^2}{\beta^2}-\frac{\beta^2}{\alpha^2}\right):\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)\right] : \left[\left(\frac{1}{\alpha^2}-\frac{1}{\beta^2}\right):\left(\frac{1}{\alpha}-\frac{1}{\beta}\right)\right]$

Απ. Δ = α – β

ε) $\mathbf{E} = \left(\frac{x+y}{x-y}-\frac{x-y}{x+y}-\frac{4x^2}{x^2-y^2}\right)\cdot\frac{x^2-2xy+y^2}{4xy}\cdot\frac{xy+y^2}{x-y}$

Απ. Ε = y – x

Θέμα 2ο

Να μετατρέψετε σε απλά τα παρακάτω σύνθετα κλάσματα

α) $\mathbf{A} = \dfrac{3\alpha-\beta}{\alpha+\beta+\dfrac{\alpha-\beta}{1+\dfrac{\alpha-\beta}{\alpha+\beta}}}$ **β)** $\mathbf{B} = \dfrac{\alpha^3-\beta^3}{\alpha^2-\beta^2+\dfrac{2\beta^2}{1+\dfrac{\alpha+\beta}{\alpha-\beta}}}$

Απ. Α = $\dfrac{2\alpha}{\alpha+\beta}$ ***β) Β = α***

Θέμα 3ο

Να αποδείξετε ότι

α) $\dfrac{\dfrac{2xy}{x+y}-y}{\dfrac{1}{x}+\dfrac{1}{y-2x}}+\dfrac{\dfrac{2xy}{x+y}-x}{\dfrac{1}{y}+\dfrac{1}{x-2y}} = xy$

β) $\dfrac{\dfrac{\alpha^2+\beta^2}{\beta}+2\alpha}{\dfrac{1}{\beta}+\dfrac{1}{\alpha}}+\dfrac{2\beta+\dfrac{\alpha^2+\beta^2}{\alpha}}{\dfrac{1}{\beta}+\dfrac{1}{\alpha}} = (\alpha+\beta)^2$

Θέμα 4ο

Να εκτελέσετε τις πράξεις και να βρείτε την τιμή των παρακάτω παραστάσεων

α) **Α =** $\dfrac{x^4-(x-1)^2}{(x^2+1)^2-x^2}+\dfrac{x^2-(x^2-1)^2}{x^2\cdot(x+1)^2-1}+\dfrac{x^2(x-1)^2-1}{x^4-(x+1)^2}$

β) **Β =** $\dfrac{(2\alpha-3\beta)^2-\alpha^2}{4\alpha^2-(3\beta+\alpha)^2}+\dfrac{4\alpha^2-(3\beta-\alpha)^2}{9(\alpha^2-\beta^2)}+\dfrac{9\beta^2-\alpha^2}{(2\alpha+3\beta)^2-\alpha^2}$

Απ. α) Α = 1 , β) Β= 1

Θέμα 5ο

Να απλοποιήσετε τις παραστάσεις

α) **Α =** $\dfrac{\dfrac{1}{\dfrac{1}{\beta}-\dfrac{1}{\alpha}}}{\dfrac{\alpha+\beta}{\alpha-\beta}-\dfrac{\alpha-\beta}{\alpha+\beta}}$

β) **Β =** $\dfrac{\dfrac{x}{1+\dfrac{1}{x}}+1-\dfrac{1}{x+1}}{\dfrac{x}{1-\dfrac{1}{x}}-x-\dfrac{1}{x-1}}$

Απ. α) Α = - $\dfrac{\alpha+\beta}{4}$ ***β) Β = x***

γ) **Γ =** $\dfrac{(\dfrac{1}{\alpha}+\dfrac{1}{\beta}-\dfrac{\gamma}{\alpha\beta})\cdot(\alpha+\beta+\gamma)}{\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}+\dfrac{2}{\alpha\beta}-\dfrac{\gamma^2}{\alpha^2\beta^2}}$

δ) **Δ =** $\dfrac{\dfrac{x^3-y^3}{x^2+y^2}\cdot\dfrac{x^2-y^2}{x^3+y^3}\,(\dfrac{1}{x^2}+\dfrac{1}{y^2})}{\dfrac{(x+y)^2-xy}{(x-y)^2+xy}\cdot(\dfrac{1}{y}-\dfrac{1}{x})}$

Απ. γ) Γ = αβ , δ) Δ= $\dfrac{x-y}{xy}$

Θέμα 6ο

Να γίνουν όλες οι δυνατές απλοποιήσεις

α) **Α =** $\dfrac{\dfrac{\beta^2+\alpha}{\beta^2}-\dfrac{\alpha^2+\beta}{\alpha^2}}{\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}-\dfrac{2}{\alpha\beta}}\cdot\dfrac{\alpha^2-2\alpha\beta+\beta^2}{(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)}$

β) **Β =** $\dfrac{\dfrac{\alpha-\beta}{\beta-\gamma}-\dfrac{\beta-\gamma}{\alpha-\beta}}{\dfrac{\alpha-\beta-1}{\alpha-\beta}-\dfrac{\beta-\gamma-1}{\beta-\gamma}}$

Απ. α) Α= 1 , β) Β = α – γ

γ) **Γ =** $\dfrac{\dfrac{1}{(\beta-1)(\beta+1)}}{1-\dfrac{1}{1-\dfrac{1}{\beta^2}}}$

δ) **Δ =** $\dfrac{x+1}{x+\dfrac{1}{2+\dfrac{2x}{1-x}}}$

Απ. α) Γ= - 1 , β) Δ = 2

Θέμα 7ο

Να γίνουν όλες οι δυνατές απλοποιήσεις

α) **Α =** $\dfrac{\dfrac{\chi}{\alpha}+\dfrac{\alpha}{\chi}+2-\dfrac{(\alpha-\chi)^2}{\alpha\chi}}{4\alpha\chi\cdot(\dfrac{1}{\chi}-\dfrac{1}{\alpha})(\dfrac{1}{\chi}+\dfrac{1}{\alpha})}\cdot\dfrac{\chi^2-\alpha^2}{\alpha\chi}$

β) **Β** $\dfrac{\dfrac{1-x}{1-x+x^2}+\dfrac{1+x}{1+x+x^2}}{\dfrac{1+x}{1+x+x^2}-\dfrac{1-x}{1-x+x^2}}$

γ) **Γ =** $\dfrac{\dfrac{x-\alpha}{1+\alpha x}-\dfrac{x-\beta}{1+\beta\chi}}{1+\dfrac{(\chi-\alpha)(\chi-\beta)}{(1+\alpha\chi)(1+\beta\chi)}}\cdot\dfrac{1+\alpha\beta}{\alpha-\beta}$

Απ. α) Α = - 1 , β) Β = $\dfrac{1}{x^3}$ ***, γ) Γ = -1***

Θέμα 8ο

Να αποδείξετε ότι

$$\left(\frac{x-y}{x+y}+\frac{x+y}{x-y}\right)\cdot\left(\frac{x^2+y^2}{2xy}+1\right)\cdot\frac{1-\dfrac{y}{x}}{\dfrac{1+\dfrac{y}{x}}{\dfrac{x}{y}+\dfrac{y}{x}}}=1$$

Θέμα 9ο

Να απλοποιήσετε τις παραστάσεις:

α) **Α =** $\alpha^2\beta^2\left[\dfrac{1}{(\alpha+\beta)^2}\left(\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}\right)+\dfrac{2}{(\alpha+\beta)^3}\left(\dfrac{1}{\alpha}+\dfrac{1}{\beta}\right)\right]$

β) **Β =** $\dfrac{1}{\alpha^3+\beta^3}\left(\dfrac{\alpha^2}{\beta^2}-2+\dfrac{\beta^2}{\alpha^2}\right)\dfrac{\alpha^4\beta^4}{\alpha\beta+\beta^2}\cdot\dfrac{\dfrac{\alpha}{\beta}-1+\dfrac{\beta}{\alpha}}{\alpha^3-2\alpha^2\beta+\alpha\beta^2}$

γ) $\Gamma = \frac{1}{(\alpha+\beta)^3}\left(\frac{1}{\alpha^3}+\frac{1}{\beta^3}\right)+\frac{3}{(\alpha+\beta)^4}\left(\frac{1}{\alpha^2}+\frac{1}{\beta^2}\right)+\frac{6}{(\alpha+\beta)^5}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$

Απ. A = 1 , B= 1 , Γ = $\frac{1}{\alpha^3\beta^3}$

Θέμα 10ο

Να απλοποιήσετε τις παρακάτω παραστάσεις:

1. $A = \frac{x}{\alpha x-2\alpha^2}-\frac{2}{x^2+x-2\alpha x-2\alpha}\left(1+\frac{3x+x^2}{3+x}\right)$ *Απ.A =* $\frac{1}{\alpha}$

2. $B = \left(\frac{x^2}{y^2}-1\right)\left(\frac{x}{x-y}-1\right)+\left(\frac{x^3}{y^3}-1\right)\left(\frac{x^2+xy}{x^2+xy+y^2}-1\right)$ *Απ. B = 2*

3. $\Gamma = \frac{1}{x+1}+\frac{1}{1-x}-\frac{2x}{1-x^2}$ *Απ. Γ =* $\frac{2}{x-1}$

4. $\Delta = \frac{\alpha^2+\alpha\beta+\beta^2}{\alpha+\beta}-\frac{\alpha^2-\alpha\beta+\beta^2}{\alpha-\beta}+\frac{2\beta^3-\beta^2+\alpha^2}{\alpha^2-\beta^2}$ *Απ. Δ = 1*

5. $E = \left[\left(\frac{1+x}{1-x}-\frac{1-x}{1+x}\right)x^2\right]:\left[\left(\frac{1+x}{1-x}-1\right)\left(1-\frac{1}{1+x}\right)\right]$

Απ. E = 2χ

Θέμα 11ο

Να απλοποιήσετε τις παρακάτω παραστάσεις:

α) $A = \frac{3\alpha x^3+3\alpha^3x-6\alpha^2x^2}{\alpha x^3-\alpha^3x}\cdot\frac{\chi+\alpha}{\chi-\alpha}$ *Απ. A = 3*

β) $A = \frac{(\alpha^2+\beta^2-\gamma^2)^2-(\alpha^2-\beta^2+\gamma^2)^2}{4\alpha\beta^2+4\alpha\beta\gamma}\cdot\frac{\beta}{\gamma-\beta}$ *Απ. A = - α*

γ) $B = \frac{\alpha^3+\beta^3}{\alpha-\frac{\beta}{1+\frac{\beta}{\alpha-\beta}}}-\frac{\alpha^3-\beta^3}{\alpha+\frac{\beta}{1-\frac{\beta}{\alpha+\beta}}}$

Απ. B = 2αβ