

12. Να λύσετε τις εξισώσεις

ii) $\frac{3}{x+2} - \frac{2}{x} = \frac{x-4}{x^2+2x}$

$$\begin{aligned}x^2 + 2x &= x(x+2) \\ \text{ΕΚΓ} &= x(x+2) \\ x &\neq 0 \\ x &\neq -2\end{aligned}$$

$$\begin{aligned}x(\cancel{x+2}) \frac{3}{\cancel{x+2}} - \cancel{x(x+2)} \frac{2}{\cancel{x}} &= x(\cancel{x+2}) \frac{x-4}{\cancel{x(x+2)}} \\ 3x - 2(x+2) &= x-4 \\ 3x - 2x - 4 &= x-4 \\ x-4 &= x-4 \\ x-x &= 4-4 \\ 0x &= 0\end{aligned}$$

Ισχύει για κάθε $x \in \mathbb{R} - \{0, -2\}$

$$\text{iii) } \frac{1}{x+2} = \frac{x}{x^2-4}$$

$$x^2-4=(x-2)(x+2)$$

$$\in \mathcal{M} = (x-2)(x+2)$$

$$x \neq 2, x \neq -2$$

$$\cancel{(x-2)}\cancel{(x+2)} \frac{1}{\cancel{x+2}} = \cancel{(x-2)}\cancel{(x+2)} \frac{x}{\cancel{(x-2)}\cancel{(x+2)}}$$

$$x-2=x$$

$$x-x=2$$

$$0x=2 \quad \text{ΑΔΥΝΑΤΗ}$$

$$\text{iv) } \frac{x^2-x}{x^2-1} = \frac{x}{x+1}$$

$$x^2-1=(x-1)(x+1)$$

$$\in \mathcal{M} = (x-1)(x+1)$$

$$x \neq 1, x \neq -1$$

$$\cancel{(x-1)}\cancel{(x+1)} \frac{x^2-x}{\cancel{(x-1)}\cancel{(x+1)}} = \cancel{(x-1)}\cancel{(x+1)} \frac{x}{\cancel{x+1}}$$

$$x^2-x = x^2-x$$

$$x^2-x^2-x+x=0$$

$$0x=0$$

$$\text{Ισχύει για κάθε } x \in \mathbb{R} - \{-1, 1\}$$

15. Να λύσετε τις εξισώσεις

$$\text{i)} \frac{|x|+4}{3} - \frac{|x|+4}{5} = \frac{2}{3}$$

$$15 \frac{|x|+4}{3} - 15 \frac{|x|+4}{5} = 15 \cdot \frac{2}{3}$$

$$5(|x|+4) - 3(|x|+4) = 10$$

$$5|x|+20 - 3|x|-12 = 10$$

$$2|x| = 10 + 12 - 20$$

$$2|x| = 2$$

$$|x| = 1$$

$$x = 1 \text{ ή } x = -1$$

$$\text{ii)} \frac{2|x|+1}{3} - \frac{|x|-1}{2} = \frac{1}{2}$$

$$6 \frac{2|x|+1}{3} - 6 \frac{|x|-1}{2} = 6 \cdot \frac{1}{2}$$

$$2(2|x|+1) - 3(|x|-1) = 3$$

$$4|x|+2 - 3|x|+3 = 3$$

$$|x| = 3 - 3 - 2$$

$$|x| = -2$$

ΑΔΥΝΑΤΗ