5. Να αποδείξετε ότι:

i) $(\sqrt{8}-\sqrt{18})\cdot(\sqrt{50}+\sqrt{72}-\sqrt{32})=-14$

$(\sqrt{8}-\sqrt{18})\cdot(\sqrt{50}+\sqrt{72}-\sqrt{32})=$ $(\sqrt{2}\cdot\sqrt{4}-\sqrt{2}\cdot\sqrt{9})\cdot(\sqrt{2}\cdot\sqrt{25}+\sqrt{2}\cdot\sqrt{36}-\sqrt{2}\cdot\sqrt{16})$

$=(\sqrt{2}\cdot 2-\sqrt{2}\cdot 3)\cdot(\sqrt{2}\cdot 5+\sqrt{2}\cdot 6-\sqrt{2}\cdot 4)=(-\sqrt{2})\cdot 7\sqrt{2}=-7\cdot(\sqrt{2})^2=-7\cdot 2=-14$

ii) $(\sqrt{28}+\sqrt{7}+\sqrt{32})\cdot(\sqrt{63}-\sqrt{32})=31$

$(\sqrt{28}+\sqrt{7}+\sqrt{32})\cdot(\sqrt{63}-\sqrt{32})=$ $(\sqrt{4}\cdot\sqrt{7}+\sqrt{7}+\sqrt{2}\cdot\sqrt{16})\cdot(\sqrt{7}\cdot\sqrt{9}-\sqrt{2}\cdot\sqrt{16})$

$(2\sqrt{7}+\sqrt{7}+4\sqrt{2})\cdot(3\sqrt{7}-4\sqrt{2})=(3\sqrt{7}+4\sqrt{2})\cdot(3\sqrt{7}-4\sqrt{2})=$

$(3\sqrt{7})^2-(4\sqrt{2})^2=9\cdot 7-16\cdot 2=63-32=31$

8. Να αποδείξετε ότι:

i) $\sqrt[4]{3^3} \cdot \sqrt[3]{3} = 3 \cdot \sqrt[12]{3}$

$$\sqrt[4]{3^3} \cdot \sqrt[3]{3} = 3^{\frac{3}{4}} \cdot 3^{\frac{1}{3}} = 3^{\frac{3}{4}+\frac{1}{3}} = 3^{\frac{9}{12}+\frac{4}{12}} = 3^{\frac{13}{12}} = 3^{\frac{12}{12}+\frac{1}{12}} = 3 \cdot 3^{\frac{1}{12}} = 3\sqrt[12]{3}$$

ii) $\sqrt[9]{2^8} \cdot \sqrt[6]{2^5} = 2 \cdot \sqrt[18]{2^{13}}$

$$\sqrt[9]{2^8} \cdot \sqrt[6]{2^5} = 2^{\frac{8}{9}} \cdot 2^{\frac{5}{6}} = 2^{\frac{8}{9}+\frac{5}{6}} = 2^{\frac{16}{18}+\frac{15}{18}} = 2^{\frac{31}{18}} = 2^{\frac{18}{18}+\frac{13}{18}} =$$

$$2 \cdot 2^{\frac{13}{18}} = 2\sqrt[18]{2^{13}}$$

iii) $\sqrt{5^3} \cdot \sqrt[3]{5} \cdot \sqrt[6]{5^4} = 25 \cdot \sqrt{5}.$

$$\sqrt{5^3} \cdot \sqrt[3]{5} \cdot \sqrt[6]{5^4} = 5^{\frac{3}{2}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{4}{6}} = 5^{\frac{3}{2}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{2}{3}} =$$

$$5 \cdot 5^{\frac{1}{2}} \cdot 5^{\frac{1}{3}+\frac{2}{3}} = 5 \cdot 5 \cdot 5^{\frac{1}{2}} = 25\sqrt{5}$$