

11. Να λύσετε τις εξισώσεις

i) $x^2 - 7|x| + 12 = 0$

$$\begin{aligned}x^2 - 7|x| + 12 &= 0 \Leftrightarrow \\|x|^2 - 7|x| + 12 &= 0 \xleftrightarrow{y=|x| \geq 0} \\y^2 - 7y + 12 &= 0 \\ \Delta &= (-7)^2 - 4 \cdot 1 \cdot 12 = 49 - 48 = 1 \\ y_{1,2} &= \frac{7 \pm 1}{2} = \begin{matrix} 4 \\ 3 \end{matrix} \\ y=4 &\Leftrightarrow |x|=4 \Leftrightarrow x=\pm 4 \\ y=3 &\Leftrightarrow |x|=3 \Leftrightarrow x=\pm 3\end{aligned}$$

ii) $x^2 + 2|x| - 35 = 0$

$$\begin{aligned}x^2 + 2|x| - 35 &= 0 \Leftrightarrow \\|x|^2 + 2|x| - 35 &= 0 \xleftrightarrow{y=|x| \geq 0} \\y^2 + 2y - 35 &= 0 \\ \Delta &= 4 + 140 = 144 \\ y_{1,2} &= \frac{-2 \pm 12}{2} = \begin{matrix} 5 \\ -7 \end{matrix} \text{ απορ} \\ y=5 &\Leftrightarrow |x|=5 \Leftrightarrow \\ x &= \pm 5\end{aligned}$$

iii) $x^2 - 8|x| + 12 = 0$

$$\begin{aligned}x^2 - 8|x| + 12 &= 0 \\|x|^2 - 8|x| + 12 &= 0 \xleftrightarrow{y=|x| \geq 0} \\y^2 - 8y + 12 &= 0 \\ \Delta &= 64 - 48 = 16 \\ y_{1,2} &= \frac{8 \pm 4}{2} = \begin{matrix} 6 \\ 2 \end{matrix} \\ y=6 &\Leftrightarrow |x|=6 \Leftrightarrow x=\pm 6 \\ y=2 &\Leftrightarrow |x|=2 \Leftrightarrow x=\pm 2\end{aligned}$$

12. Να λύσετε την εξίσωση $(x - 1)^2 + 4|x - 1| - 5 = 0$.

$$\begin{aligned}(x-1)^2 + 4|x-1| - 5 &= 0 \Leftrightarrow \\ |x-1|^2 + 4|x-1| - 5 &= 0 \xleftrightarrow{y=|x-1| \geq 0} \\ y^2 + 4y - 5 &= 0\end{aligned}$$

$$\begin{aligned}\Delta &= 16 + 20 = 36 \\ y_{1,2} &= \frac{-4 \pm 6}{2} = < -5 \text{ απο.}\end{aligned}$$

$$|x-1|=1 \Leftrightarrow \begin{array}{l} x-1=1 \quad \vee \quad x-1=-1 \\ \boxed{x=2} \quad \vee \quad \boxed{x=0} \end{array}$$

14. Να λύσετε τις εξισώσεις

i) $\frac{x}{x+1} + \frac{x+1}{x} = \frac{13}{6}$

ΕΚΠ = $6x(x+1)$

$x \neq 0$

$x \neq -1$

$$6x \cancel{(x+1)} \frac{x}{\cancel{x+1}} + 6 \cancel{x(x+1)} \frac{x+1}{\cancel{x}} = \cancel{6x(x+1)} \frac{13}{\cancel{6}}$$

$$6x^2 + 6(x+1)^2 = 13x^2 + 13x$$

$$6x^2 + 6(x^2 + 2x + 1) = 13x^2 + 13x$$

$$6x^2 + 6x^2 + 12x + 6 = 13x^2 + 13x$$

$$12x^2 - 13x^2 + 12x - 13x + 6 = 0$$

$$-x^2 - x + 6 = 0 \quad \vee \quad x^2 + x - 6 = 0$$

$$x = 2 \quad \vee \quad x = -3$$

ii) $\frac{2}{x} + \frac{2x-3}{x-2} + \frac{2-x^2}{x^2-2x} = 0.$

ΕΚΠ = $x(x-2)$, $x \neq 0$, $x \neq 2$

$$\cancel{x(x-2)} \frac{2}{\cancel{x}} + x \cancel{(x-2)} \frac{2x-3}{\cancel{x-2}} + x \cancel{(x-2)} \frac{2-x^2}{\cancel{x(x-2)}} = 0$$

$$2(x-2) + x(2x-3) + 2 - x^2 = 0$$

$$2x - 4 + 2x^2 - 3x + 2 - x^2 = 0$$

$$x^2 - x - 2 = 0$$

$$\Delta = 1 + 8 = 9$$

$$x_{1,2} = \frac{1 \pm 3}{2} =$$

2 απορ
 -1

15. Να λύσετε τις εξισώσεις

i) $x^4 + 6x^2 - 40 = 0$

$$(x^2)^2 + 6x^2 - 40 = 0 \xleftrightarrow{x^2=y \geq 0}$$
$$y^2 + 6y - 40 = 0$$
$$\Delta = 36 + 160 = 196$$
$$y_{1,2} = \frac{-6 \pm 14}{2} = \begin{cases} 4 \\ -10 \end{cases}$$

απόρ.

$$x^2 = 4 \Leftrightarrow x = \pm 2$$

ii) $4x^4 + 11x^2 - 3 = 0$

$$4(x^2)^2 + 11x^2 - 3 = 0 \xleftrightarrow{x^2=y \geq 0}$$
$$4y^2 + 11y - 3 = 0$$
$$\Delta = 121 + 48 = 169$$
$$y_{1,2} = \frac{-11 \pm 13}{8} = \begin{cases} 1/4 \\ -3 \end{cases}$$

απόρ.

$$x^2 = \frac{1}{4} \Leftrightarrow x = \pm \frac{1}{2}$$

iii) $2x^4 + 7x^2 + 3 = 0$

$$2(x^2)^2 + 7x^2 + 3 = 0 \xleftrightarrow{y=x^2 \geq 0}$$
$$2y^2 + 7y + 3 = 0$$
$$\Delta = 49 - 24 = 25$$
$$y_{1,2} = \frac{-7 \pm 5}{4} = \begin{cases} -3 \\ -1/2 \end{cases}$$

αδύνατο