

$$\gamma. \lim_{x \rightarrow 3} \frac{x^2 - 9}{(x-3)^3(x+3)} =$$

$$\lim_{x \rightarrow 3} \frac{\cancel{(x-3)} \cancel{(x+3)}}{(x-3)^{\cancel{3}^2} \cancel{(x+3)}} = \lim_{x \rightarrow 3} \frac{1}{(x-3)^2} = +\infty$$

$$\text{αφαί } (x-3)^2 \geq 0$$

$$\delta. \lim_{x \rightarrow 1^-} \frac{x^2 - 2x + 1}{|x| - 1} =$$

$$\lim_{x \rightarrow 1^-} \frac{(x-1)^2}{x-1} = \lim_{x \rightarrow 1^-} (x-1) = 0$$

Όριο συναρτήσεων στο άπειρο

Παράδειγμα 1

$$i) \lim_{x \rightarrow +\infty} (-5x^{11} + 3x^5 + 2002) = \lim_{x \rightarrow +\infty} -5x^{11} = -5 \cdot (+\infty) = -\infty$$

$$ii) \lim_{x \rightarrow -\infty} \frac{3x^5 + 5x^3 + 10}{6x^9 + 4x - 3} = \lim_{x \rightarrow -\infty} \frac{3x^5}{6x^9} = \lim_{x \rightarrow -\infty} \frac{1}{2x^4} = 0$$

Παράδειγμα 2

$$\lim_{x \rightarrow +\infty} \left(\sqrt{4x^2 + 42x + 12} - \sqrt{25x^2 - 17x + 1} \right) =$$

$$\lim_{x \rightarrow +\infty} \left(\sqrt{x^2 \left(4 + \frac{42}{x} + \frac{12}{x^2} \right)} - \sqrt{x^2 \left(25 - \frac{17}{x} + \frac{1}{x^2} \right)} \right) =$$

$$\lim_{x \rightarrow +\infty} \left(x \sqrt{4 + \frac{42}{x} + \frac{12}{x^2}} \right) - \lim_{x \rightarrow +\infty} \left(x \sqrt{25 - \frac{17}{x} + \frac{1}{x^2}} \right) =$$

$$\lim_{x \rightarrow +\infty} (2x) - \lim_{x \rightarrow +\infty} (5x) = \lim_{x \rightarrow +\infty} (2x - 5x) = \lim_{x \rightarrow +\infty} (-3x) = -\infty$$