

Παράδειγμα 3

$$i) \lim_{x \rightarrow -\infty} (\sqrt{9x^2 + x + 1} - 2x) = \lim_{x \rightarrow -\infty} \left(x \left(\sqrt{9 + \frac{1}{x} + \frac{1}{x^2}} - 2 \right) \right)$$

$$= (-\infty) \cdot (3 - 2) = (-\infty) \cdot 1 = -\infty$$

$$ii) \lim_{x \rightarrow +\infty} (\sqrt{4x^2 + x + 2} - 2x) = \lim_{x \rightarrow +\infty} \frac{\cancel{4x^2} + x + 2 - \cancel{4x^2}}{(\sqrt{4x^2 + x + 2} + 2x)} = \lim_{x \rightarrow +\infty} \frac{x + 2}{\sqrt{4x^2 + x + 2} + 2x}$$

$$= \lim_{x \rightarrow +\infty} \frac{x + 2}{x \left(\sqrt{4 + \frac{1}{x} + \frac{1}{x^2}} + 2 \right)} = \lim_{x \rightarrow +\infty} \frac{x + 2}{2x + 2x} = \lim_{x \rightarrow +\infty} \frac{\cancel{x}}{4\cancel{x}} = \frac{1}{4}.$$

Παράδειγμα 4

$$i) \lim_{x \rightarrow -\infty} (|3 - x + x^5| + |2x - x^3|) \quad \lim_{x \rightarrow +\infty} (|3 - x + x^5| - |2x - x^3|)$$

$$iii) \lim_{x \rightarrow +\infty} \frac{1 + |x - x^2|}{2x^2 + 42}$$