**9.** Αν είναι x = r ημθσυνφ , y = r ημθημφ και z = r συνθ , να δείξετε ότι $x^2 + y^2 + z^2 = r^2$.

$$x^2+y^2+z^2=(r\,\text{ημ}\theta\,\text{συν}\varphi)^2+(r\,\text{ημ}\theta\,\text{ημ}\varphi)^2+(r\,\text{συν}\theta)^2=$$

$$\underbrace{r^2\text{ημ}^2\theta\,\text{συν}^2\varphi+r^2\text{ημ}^2\theta\,\text{ημ}^2\varphi}+r^2\text{συν}^2\theta=$$

$$r^2\text{ημ}^2\theta\left(\cancelto{1}{\text{συν}^2\varphi+\text{ημ}^2\varphi}\right)+r^2\text{συν}^2\theta=$$

$$r^2\text{ημ}^2\theta+r^2\text{συν}^2\theta=r^2\left(\cancelto{1}{\text{ημ}^2\theta+\text{συν}^2\theta}\right)=r^2$$

**10.** Να αποδείξετε ότι:

**i)** $\dfrac{\eta\mu\alpha}{1+\sigma\upsilon\nu\alpha} = \dfrac{1-\sigma\upsilon\nu\alpha}{\eta\mu\alpha}$

$\Leftrightarrow \eta\mu^2\alpha = (1-\sigma\upsilon\nu\alpha)(1+\sigma\upsilon\nu\alpha)$

$\Leftrightarrow \eta\mu^2\alpha = 1-\sigma\upsilon\nu^2\alpha \Leftrightarrow$

$\Leftrightarrow \eta\mu^2\alpha + \sigma\upsilon\nu^2\alpha = 1$

ΙΣΧΥΕΙ

**ii)** $\sigma\upsilon\nu^4\alpha - \eta\mu^4\alpha = 2\sigma\upsilon\nu^2\alpha - 1$ .

$\Leftrightarrow (\sigma\upsilon\nu^2\alpha)^2 - (\eta\mu^2\alpha)^2 = 2\sigma\upsilon\nu^2\alpha - 1 \Leftrightarrow$

$(\sigma\upsilon\nu^2\alpha - \eta\mu^2\alpha)(\overbrace{\cancel{\sigma\upsilon\nu^2\alpha + \eta\mu^2\alpha}}^{1}) = 2\sigma\upsilon\nu^2\alpha - 1 \Leftrightarrow$

$\sigma\upsilon\nu^2\alpha - \eta\mu^2\alpha = 2\sigma\upsilon\nu^2\alpha - 1 \Leftrightarrow$

$0 = 2\sigma\upsilon\nu^2\alpha - \sigma\upsilon\nu^2\alpha + \eta\mu^2\alpha - 1 \Leftrightarrow$

$0 = \sigma\upsilon\nu^2\alpha + \eta\mu^2\alpha - 1 \Leftrightarrow$

$0 = 1 - 1$

ΙΣΧΥΕΙ

**11.** Να αποδείξετε ότι:

i) $\dfrac{\eta\mu\theta}{1+\sigma\upsilon\nu\theta}+\dfrac{1+\sigma\upsilon\nu\theta}{\eta\mu\theta}=\dfrac{2}{\eta\mu\theta}$

ii) $\dfrac{\sigma\upsilon\nu x}{1-\eta\mu x}+\dfrac{\sigma\upsilon\nu x}{1+\eta\mu x}=\dfrac{2}{\sigma\upsilon\nu x}.$

$$\frac{\eta\mu\theta}{1+\sigma\upsilon\nu\theta}+\frac{1+\sigma\upsilon\nu\theta}{\eta\mu\theta}=$$

$$\frac{\eta\mu^2\theta+(1+\sigma\upsilon\nu\theta)^2}{\eta\mu\theta(1+\sigma\upsilon\nu\theta)}=$$

$$\frac{\eta\mu^2\theta+\sigma\upsilon\nu^2\theta+1+2\sigma\upsilon\nu\theta}{\eta\mu\theta(1+\sigma\upsilon\nu\theta)}=\frac{1+1+2\sigma\upsilon\nu\theta}{\eta\mu\theta(1+\sigma\upsilon\nu\theta)}=$$

$$\frac{2+2\sigma\upsilon\nu\theta}{\eta\mu\theta(1+\sigma\upsilon\nu\theta)}=\frac{2(\cancel{1+\sigma\upsilon\nu\theta})}{\eta\mu\theta(\cancel{1+\sigma\upsilon\nu\theta})}=$$

$$\frac{2}{\eta\mu\theta}.$$

$$\frac{\sigma\upsilon\nu x}{1-\eta\mu x}+\frac{\sigma\upsilon\nu x}{1+\eta\mu x}=\frac{\sigma\upsilon\nu x(1+\eta\mu x)+\sigma\upsilon\nu x(1-\eta\mu x)}{(1-\eta\mu x)(1+\eta\mu x)}$$

$$=\frac{\sigma\upsilon\nu x(1+\cancel{\eta\mu x}+1-\cancel{\eta\mu x})}{1-\eta\mu^2x}=$$

$$\frac{\cancel{\sigma\upsilon\nu x}\cdot 2}{\sigma\upsilon\nu^{\cancel{2}}x}=\frac{2}{\sigma\upsilon\nu x}$$