

9. Να λύσετε τις εξισώσεις

i) $\eta\mu\left(x + \frac{\pi}{3}\right) = -1$

$$\eta\mu\left(x + \frac{\pi}{3}\right) = \eta\mu\left(-\frac{\pi}{2}\right)$$

$$\left. \begin{aligned} x + \frac{\pi}{3} &= 2k\pi - \frac{\pi}{2} \\ x + \frac{\pi}{3} &= 2k\pi + \pi + \frac{\pi}{2} \end{aligned} \right\} k \in \mathbb{Z}$$

$$\left. \begin{aligned} x + \frac{\pi}{3} &= 2k\pi - \frac{\pi}{2} \\ x + \frac{\pi}{3} &= 2k\pi + \frac{3\pi}{2} \end{aligned} \right\} k \in \mathbb{Z}$$

$$\begin{aligned} x + \frac{\pi}{3} &= 2k\pi - \frac{\pi}{2}, \quad k \in \mathbb{Z} \\ x &= 2k\pi - \frac{\pi}{2} - \frac{\pi}{3}, \quad k \in \mathbb{Z} \\ x &= 2k\pi - \frac{5\pi}{6}, \quad k \in \mathbb{Z} \end{aligned}$$

ii) $2\sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = 1$

$$\begin{aligned} 6\omega\left(3x - \frac{\pi}{4}\right) &= \frac{1}{2} \\ \sigma\omega\left(3x - \frac{\pi}{4}\right) &= 6\omega\frac{\pi}{3} \\ 3x - \frac{\pi}{4} &= 2k\pi \pm \frac{\pi}{3} \quad k \in \mathbb{Z} \end{aligned}$$

$$\left. \begin{aligned} 3x &= 2k\pi + \frac{\pi}{3} + \frac{\pi}{4} \\ 3x &= 2k\pi - \frac{\pi}{3} + \frac{\pi}{4} \end{aligned} \right\} k \in \mathbb{Z}$$

$$\left. \begin{aligned} 3x &= 2k\pi + \frac{7\pi}{12} \\ 3x &= 2k\pi - \frac{\pi}{12} \end{aligned} \right\} k \in \mathbb{Z}$$

$$\begin{aligned} x &= \frac{2k\pi}{3} + \frac{7\pi}{36} \\ x &= \frac{2k\pi}{3} - \frac{\pi}{36} \end{aligned}$$

iii) $\varepsilon\varphi\left(\frac{\pi}{4} - 5x\right) = \sqrt{3}$

$$\varepsilon\varphi\left(\frac{\pi}{4} - 5x\right) = \varepsilon\varphi\frac{\pi}{3}$$

$$\frac{\pi}{4} - 5x = k\pi + \frac{\pi}{3}, \quad k \in \mathbb{Z}$$

$$-5x = k\pi + \frac{\pi}{3} - \frac{\pi}{4}, \quad k \in \mathbb{Z}$$

$$-5x = k\pi + \frac{\pi}{12}, \quad k \in \mathbb{Z}$$

$$x = -\frac{k\pi}{5} - \frac{\pi}{60}$$

10. Να λύσετε τις εξισώσεις

i) $2\eta\mu^2\omega + \eta\mu\omega - 1 = 0$

Θέτω $\eta\mu\omega = x$

$$2x^2 + x - 1 = 0$$

$$\Delta = \beta^2 - 4\alpha\gamma = 1^2 - 4 \cdot 2 \cdot (-1)$$

$$= 1 + 8 = 9$$

$$x_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} = \frac{-1 \pm 3}{4}$$

$$x_1 = \frac{1}{2}$$

$$x_2 = -1$$

$$\bullet \eta\mu\omega = \frac{1}{2} \Leftrightarrow \eta\mu\omega = \eta\mu\frac{\pi}{6} \Leftrightarrow$$

$$\left\{ \omega = 2k\pi + \frac{\pi}{6} \text{ ή } \omega = 2k\pi + \pi - \frac{\pi}{6}, k \in \mathbb{Z} \right\} \Leftrightarrow$$

$$\left\{ \omega = 2k\pi + \frac{\pi}{6} \text{ ή } \omega = 2k\pi + \frac{5\pi}{6}, k \in \mathbb{Z} \right\}.$$

$$\bullet \eta\mu\omega = -1 \Leftrightarrow \eta\mu\omega = \eta\mu\left(-\frac{\pi}{2}\right) \Leftrightarrow$$

$$\left\{ \omega = 2k\pi - \frac{\pi}{2} \text{ ή } \omega = 2k\pi + \pi + \frac{\pi}{2}, k \in \mathbb{Z} \right\} \Leftrightarrow$$

$$\left\{ \omega = 2k\pi - \frac{\pi}{2} \text{ ή } \omega = 2k\pi + \frac{3\pi}{2}, k \in \mathbb{Z} \right\} \Leftrightarrow$$

$$\omega = 2k\pi - \frac{\pi}{2}, k \in \mathbb{Z}$$

$$\text{ii) } 2\sigma\upsilon\nu^2x + 3\sigma\upsilon\nu x - 2 = 0$$

$$\Theta \dot{\epsilon} \tau \omega \quad \sigma \omega x = y$$

$$2y^2 + 3y - 2 = 0$$

$$\Delta = b^2 - 4ac = 3^2 - 4 \cdot 2 \cdot (-2) = 9 + 16 = 25$$

$$y_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-3 \pm 5}{4}$$

$$\boxed{y_1 = \frac{1}{2}}, y_2 = -2 \text{ απορ.}$$

$$\sigma \omega x = \frac{1}{2} \leftrightarrow$$

$$\sigma \omega x = \sigma \omega \frac{7}{3} \leftrightarrow$$

$$x = 2k\pi \pm \frac{7}{3}, k \in \mathbb{Z}$$

$$\text{iii) } 3\varepsilon\varphi^2 t = 3 + 2\sqrt{3}\varepsilon\varphi t$$

$$3\varepsilon\varphi^2 t - 2\sqrt{3}\varepsilon\varphi t - 3 = 0$$

$$\partial \varepsilon \varphi t = x$$

$$3x^2 - 2\sqrt{3}x - 3 = 0$$

$$\Delta = (-2\sqrt{3})^2 - 4 \cdot 3 \cdot (-3) \\ = 12 + 36 = 48$$

$$x_{1,2} = \frac{2\sqrt{3} \pm 4\sqrt{3}}{6}$$

$$x_1 = \sqrt{3}$$

$$x_2 = -\frac{\sqrt{3}}{3}$$

$$\bullet \varepsilon\varphi t = \sqrt{3} \Leftrightarrow \varepsilon\varphi t = \varepsilon\varphi \frac{\pi}{3} \Leftrightarrow$$

$$t = k\pi + \frac{\pi}{3}, k \in \mathbb{Z}$$

$$\bullet \varepsilon\varphi t = -\frac{\sqrt{3}}{3} \Leftrightarrow \varepsilon\varphi t = -\varepsilon\varphi \frac{\pi}{6} \Leftrightarrow$$

$$\varepsilon\varphi t = \varepsilon\varphi \left(-\frac{\pi}{6}\right) \Leftrightarrow$$

$$t = k\pi - \frac{\pi}{6}, k \in \mathbb{Z}$$

11. Να λύσετε τις εξισώσεις

i) $\eta\mu^2 x + 5\sigma\upsilon\nu^2 x = 4$

$$\eta\mu^2 x + 5\sigma\omega^2 x = 4 \Leftrightarrow$$

$$\eta\mu^2 x + \sigma\omega^2 x + 4\sigma\omega^2 x = 4 \Leftrightarrow$$

$$1 + 4\sigma\omega^2 x = 4 \Leftrightarrow$$

$$4\sigma\omega^2 x = 3 \Leftrightarrow$$

$$\sigma\omega^2 x = \frac{3}{4} \Leftrightarrow$$

$$\sigma\omega x = \pm \frac{\sqrt{3}}{2}$$

$$\bullet \quad 6\upsilon\nu x = \frac{\sqrt{3}}{2} \Leftrightarrow 6\omega x = 6\omega \frac{\pi}{6}$$

$$x = 2k\pi \pm \frac{\pi}{6}, \quad x \in \mathbb{Z}$$

$$\bullet \quad 6\omega x = -\frac{\sqrt{3}}{2} \Leftrightarrow$$
$$\sigma\omega x = -\sigma\omega \frac{\pi}{6} \Leftrightarrow$$

$$\sigma\omega x = 6\omega \left(\pi - \frac{\pi}{6}\right) \Leftrightarrow$$

$$6\upsilon\nu x = \sigma\omega \frac{5\pi}{6} \Leftrightarrow$$

$$x = 2k\pi \pm \frac{5\pi}{6}, \quad x \in \mathbb{Z}$$