**5.** Να λύσετε τις εξισώσεις:

**i)** $x+\frac{1}{\alpha}=\alpha+\frac{1}{x},\quad \alpha\neq 0$ **ii)** $\frac{x}{\alpha}+\frac{\alpha}{x}=\frac{\alpha}{\beta}+\frac{\beta}{\alpha},\quad \alpha,\beta\neq 0.$

Λύση ii) Η εξίσωση ορίζεται όταν $x\neq 0$.

$\frac{x}{\alpha}+\frac{\alpha}{x}=\frac{\alpha}{\beta}+\frac{\beta}{\alpha} \iff$ | ΕΚΠ: $\alpha\cdot\beta\cdot x$

$\alpha\beta x\,\frac{x}{\alpha}+\alpha\beta x\,\frac{\alpha}{x}=\alpha\beta x\,\frac{\alpha}{\beta}+\alpha\beta x\,\frac{\beta}{\alpha} \iff$

$\beta\cdot x^2+\alpha^2\beta=\alpha^2 x+\beta^2 x \iff$

$\beta x^2-\alpha^2 x-\beta^2 x+\alpha^2\beta=0 \iff$ $\quad \alpha x^2+\beta x+\gamma=0$

$\beta x^2-(\alpha^2+\beta^2)x+\alpha^2\beta=0$

- $A=\beta$, $B=-(\alpha^2+\beta^2)$, $\Gamma=\alpha^2\beta$

$\Delta=B^2-4A\Gamma$

$=\left[-(\alpha^2+\beta^2)\right]^2-4\beta\cdot\alpha^2\beta$

$=(\alpha^2+\beta^2)^2-4\alpha^2\beta^2$

$=(\alpha^2)^2+2\alpha^2\beta^2+(\beta^2)^2-4\alpha^2\beta^2$

$=\alpha^4+2\alpha^2\beta^2+\beta^4-4\alpha^2\beta^2$

$=\alpha^4-2\alpha^2\beta^2+\beta^4$

$=(\alpha^2-\beta^2)^2\geq 0$ $\qquad \sqrt{(\alpha^2-\beta^2)^{\cancel{2}}}=|\alpha^2-\beta^2|$

$x=\frac{-B\pm\sqrt{\Delta}}{2A}=\frac{\alpha^2+\beta^2\pm(\alpha^2-\beta^2)}{2\beta}=$

$=\begin{cases}\frac{\alpha^2+\beta^2+(\alpha^2-\beta^2)}{2\beta}=\frac{\alpha^2+\cancel{\beta^2}+\alpha^2-\cancel{\beta^2}}{2\beta}=\frac{2\alpha^2}{2\beta}=\frac{\alpha^2}{\beta}\\[2ex] \frac{\alpha^2+\beta^2-(\alpha^2-\beta^2)}{2\beta}=\frac{\cancel{\alpha^2}+\beta^2-\cancel{\alpha^2}+\beta^2}{2\beta}=\frac{2\beta^2}{2\beta}=\beta\end{cases}$