ΑΣΚΗΣΕΙΣ

Α΄ ΟΜΑΔΑΣ

1. Να βρείτε τους τριγωνομετρικούς αριθμούς της γωνίας:

i) 1200° **ii)** −2850°.

$$\begin{array}{r|l} 1200 & 360 \\ \hline -1080 & 3 \\ 120 & \end{array} \qquad \begin{array}{r|l} 2850 & 360 \\ \hline -2520 & 7 \\ 330 & \end{array}$$

i) $1200^\circ = 3\cdot 360^\circ + 120^\circ$ $\qquad 120^\circ = 180^\circ - 60^\circ$

$ημ1200^\circ = ημ120^\circ = ημ(180^\circ-60^\circ) = ημ60^\circ = \frac{\sqrt{3}}{2}$

$συν1200^\circ = συν120^\circ = συν(180^\circ-60^\circ) = -συν60^\circ = -\frac{1}{2}$

$εφ1200^\circ = εφ120^\circ = εφ(180^\circ-60^\circ) = -εφ60^\circ = -\sqrt{3}$

$σφ1200^\circ = σφ120^\circ = σφ(180^\circ-60^\circ) = -σφ60^\circ = -\frac{\sqrt{3}}{3}$

ii) $2850^\circ = 7\cdot 360^\circ + 330^\circ$

$ημ(-2850^\circ) = ημ(-330^\circ) = -ημ330^\circ = -ημ(360^\circ-30^\circ) = -\overbrace{ημ(-30^\circ)}^{-ημ30^\circ} = ημ30^\circ = \frac{1}{2}$

$συν(-2850^\circ) = συν(-330^\circ) = συν330^\circ = συν(360^\circ-30^\circ) = συν(-30^\circ) = συν30^\circ = \frac{\sqrt{3}}{2}$

$εφ(-2850^\circ) = εφ(-330^\circ) = -εφ330^\circ = -εφ(360^\circ-30^\circ) = -εφ(-30^\circ) = εφ30^\circ = \frac{\sqrt{3}}{3}$

$σφ(-2850^\circ) = σφ(-330^\circ) = -σφ330^\circ = -σφ(360^\circ-30^\circ) = -σφ(-30^\circ) = σφ30^\circ = \sqrt{3}$

2. Να βρείτε τους τριγωνομετρικούς αριθμούς της γωνίας

i) $\frac{187\pi}{6}$ rad **ii)** $\frac{21\pi}{4}$ rad.

i) $\frac{187\pi}{6} = \frac{180\pi}{6} + \frac{7\pi}{6} = 30\pi + \frac{7\pi}{6} = 15\cdot 2\pi + \overset{210^\circ}{\frac{7\pi}{6}}$

$ημ\frac{187\pi}{6} = ημ\frac{7\pi}{6} = ημ\left(\pi+\frac{\pi}{6}\right) = -ημ\frac{\pi}{6} = -\frac{1}{2}$

$συν\frac{187\pi}{6} = συν\frac{7\pi}{6} = συν\left(\pi+\frac{\pi}{6}\right) = -συν\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$εφ\frac{187\pi}{6} = εφ\frac{7\pi}{6} = εφ\left(\pi+\frac{\pi}{6}\right) = εφ\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$σφ\frac{187\pi}{6} = σφ\frac{7\pi}{6} = σφ\left(\pi+\frac{\pi}{6}\right) = σφ\frac{\pi}{6} = \sqrt{3}$

ii) $\frac{21\pi}{4} = \frac{21}{8}\cdot 2\pi = \left(2+\frac{5}{8}\right)\cdot 2\pi = 2\cdot 2\pi + \overset{225^\circ}{\frac{5\pi}{4}}$

$ημ\frac{21\pi}{4} = ημ\frac{5\pi}{4} = ημ\left(\pi+\frac{\pi}{4}\right) = -ημ\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$συν\frac{21\pi}{4} = συν\frac{5\pi}{4} = συν\left(\pi+\frac{\pi}{4}\right) = -συν\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$

$εφ\frac{21\pi}{4} = εφ\frac{5\pi}{4} = εφ\left(\pi+\frac{\pi}{4}\right) = εφ\frac{\pi}{4} = 1$

$σφ\frac{21\pi}{4} = σφ\frac{5\pi}{4} = σφ\left(\pi+\frac{\pi}{4}\right) = σφ\frac{\pi}{4} = 1$

3. Σε κάθε τρίγωνο ΑΒΓ να αποδείξετε ότι:

i) $\eta\mu A = \eta\mu(B+\Gamma)$ ii) $\sigma\upsilon\nu A + \sigma\upsilon\nu(B+\Gamma) = 0$

iii) $\eta\mu\frac{A}{2} = \sigma\upsilon\nu\frac{B+\Gamma}{2}$ iv) $\sigma\upsilon\nu\frac{A}{2} = \eta\mu\frac{B+\Gamma}{2}$.

Σε κάθε τρίγωνο ΑΒΓ

$\hat{A}+\hat{B}+\hat{\Gamma}=180^\circ$

$\Rightarrow \hat{A} = 180^\circ - (\hat{B}+\hat{\Gamma})$ (1)

άρα $\hat{A}$, $\hat{B}+\hat{\Gamma}$ παραπληρ.

άρα και $\frac{\hat{A}}{2} = 90^\circ - \frac{\hat{B}+\hat{\Gamma}}{2}$ (2)

συμπληρωματικές

i) $\eta\mu\hat{A} \overset{(1)}{=} \eta\mu[180^\circ - (\hat{B}+\hat{\Gamma})] = \eta\mu(\hat{B}+\hat{\Gamma})$

ii) Αρκεί ν.δ.ο. $\sigma\upsilon\nu\hat{A} = -\sigma\upsilon\nu(\hat{B}+\hat{\Gamma})$

$\sigma\upsilon\nu\hat{A} \overset{(1)}{=} \sigma\upsilon\nu[180^\circ - (\hat{B}+\hat{\Gamma})] = -\sigma\upsilon\nu(\hat{B}+\hat{\Gamma})$

iii) $\eta\mu\frac{\hat{A}}{2} \overset{(2)}{=} \eta\mu\left(90^\circ - \frac{\hat{B}+\hat{\Gamma}}{2}\right) = \sigma\upsilon\nu\frac{\hat{B}+\hat{\Gamma}}{2}$

iv) $\sigma\upsilon\nu\frac{\hat{A}}{2} \overset{(2)}{=} \sigma\upsilon\nu\left(90^\circ - \frac{\hat{B}+\hat{\Gamma}}{2}\right) = \eta\mu\frac{\hat{B}+\hat{\Gamma}}{2}$

4. Να απλοποιήσετε την παράσταση $\frac{\sigma\upsilon\nu(-\alpha)\cdot\sigma\upsilon\nu(180^\circ+\alpha)}{\eta\mu(-\alpha)\cdot\eta\mu(90^\circ+\alpha)}$.

$$\left.\begin{aligned}
\sigma\upsilon\nu(-\alpha) &= \sigma\upsilon\nu\alpha \\
\sigma\upsilon\nu(180^\circ+\alpha) &= -\sigma\upsilon\nu\alpha \\
\eta\mu(-\alpha) &= -\eta\mu\alpha \\
\eta\mu(90^\circ+\alpha) &= \sigma\upsilon\nu\alpha
\end{aligned}\right\} (1)$$

$$\frac{\sigma\upsilon\nu(-\alpha)\cdot\sigma\upsilon\nu(180^\circ+\alpha)}{\eta\mu(-\alpha)\cdot\eta\mu(90^\circ+\alpha)} \overset{(1)}{=} \frac{\sigma\upsilon\nu\alpha\cdot(-\sigma\upsilon\nu\alpha)}{-\eta\mu\alpha\cdot\sigma\upsilon\nu\alpha} = \frac{-\sigma\upsilon\nu^2\alpha}{-\eta\mu\alpha\cdot\cancel{\sigma\upsilon\nu\alpha}} = \frac{\sigma\upsilon\nu\alpha}{\eta\mu\alpha} = \sigma\varphi\alpha$$

5. Να αποδείξετε ότι: $\dfrac{\varepsilon\varphi(\pi-x)\cdot\sigma\upsilon\nu(2\pi+x)\cdot\sigma\upsilon\nu\left(\frac{9\pi}{2}+x\right)}{\eta\mu(13\pi+x)\cdot\sigma\upsilon\nu(-x)\cdot\sigma\varphi\left(\frac{21\pi}{2}-x\right)}=-1.$

$\varepsilon\varphi(\pi-x) = -\varepsilon\varphi x$

$\sigma\upsilon\nu(2\pi+x) = \sigma\upsilon\nu x$

$\sigma\upsilon\nu\left(\frac{9\pi}{2}+x\right) = \sigma\upsilon\nu\left(\frac{8\pi}{2}+\frac{\pi}{2}+x\right) = \sigma\upsilon\nu\left(2\cdot 2\pi+\frac{\pi}{2}+x\right) = \sigma\upsilon\nu\left(\frac{\pi}{2}+x\right) = -\eta\mu x$

$\eta\mu(13\pi+x) = \eta\mu(12\pi+\pi+x) = \eta\mu(6\cdot 2\pi+\pi+x) = \eta\mu(\pi+x) = -\eta\mu x$

$\sigma\upsilon\nu(-x) = \sigma\upsilon\nu x$

$\sigma\varphi\left(\frac{21\pi}{2}-x\right) = \sigma\varphi\left(\frac{21\pi}{4}\cdot 2\pi - x\right) = \sigma\varphi\left[\left(5+\frac{\pi}{4}\right)2\pi - x\right] = \sigma\varphi\left(5\cdot 2\pi+\frac{\pi}{2}-x\right) = \sigma\varphi\left(\frac{\pi}{2}-x\right) = \varepsilon\varphi x$

Άρα

$$\frac{\varepsilon\varphi(\pi-x)\cdot\sigma\upsilon\nu(2\pi+x)\cdot\sigma\upsilon\nu\left(\frac{9\pi}{2}+x\right)}{\eta\mu(13\pi+x)\,\sigma\upsilon\nu(-x)\,\sigma\varphi\left(\frac{21\pi}{2}-x\right)} = \frac{-\cancel{\varepsilon\varphi x}\cdot\cancel{\sigma\upsilon\nu x}\cdot(-\cancel{\eta\mu x})}{-\cancel{\eta\mu x}\cdot\cancel{\sigma\upsilon\nu x}\cdot\cancel{\varepsilon\varphi x}} = -1$$

6. Να δείξετε ότι έχει σταθερή τιμή η παράσταση:

$$\eta\mu^2(\pi-x)+\sigma\upsilon\nu(\pi-x)\sigma\upsilon\nu(2\pi-x)+2\eta\mu^2\left(\frac{\pi}{2}-x\right).$$

$$\left.\begin{aligned}
\eta\mu(\pi-x) &= \eta\mu x\\
\sigma\upsilon\nu(\pi-x) &= -\sigma\upsilon\nu x\\
\sigma\upsilon\nu(2\pi-x) &= \sigma\upsilon\nu(-x) = \sigma\upsilon\nu x\\
\eta\mu\left(\frac{\pi}{2}-x\right) &= \sigma\upsilon\nu x
\end{aligned}\right\}\ (1)$$

Άρα $\eta\mu^2(\pi-x)+\sigma\upsilon\nu(\pi-x)\,\sigma\upsilon\nu(2\pi-x)\cdot 2\eta\mu^2\left(\frac{\pi}{2}-x\right)$

$\overset{(1)}{=} \eta\mu^2 x + (-\sigma\upsilon\nu x)\,\sigma\upsilon\nu x + 2\,\sigma\upsilon\nu^2 x$

$= \eta\mu^2 x - \sigma\upsilon\nu^2 x + 2\sigma\upsilon\nu^2 x$

$= \eta\mu^2 x + \sigma\upsilon\nu^2 x$

$= 1$

H/W σελ. 71-72

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