

Α' ΟΜΑΔΑΣ

1. Να λύσετε τις εξισώσεις

i) $\eta\mu x = 0$ ii) $\eta\mu x = \frac{\sqrt{2}}{2}$ iii) $\sigma\upsilon\nu x = 0$ iv) $\sigma\upsilon\nu x = \frac{\sqrt{2}}{2}$

i) $\eta\mu x = 0 \Leftrightarrow \eta\mu x = \eta\mu 0 \Leftrightarrow$

$$x = \begin{cases} 2k\pi + 0 = 2k\pi \\ 2k\pi + \pi - 0 = 2k\pi + \pi = (2k+1)\pi \end{cases} \quad k \in \mathbb{Z}$$

$$x = k\pi \quad (\text{Ε.Σ. Περίοδου})$$

$2k$: άρτιος
 $\begin{matrix} 2k+1 \\ \cup \\ 2k-1 \end{matrix}$: περιττός

ii) $\eta\mu x = \frac{\sqrt{2}}{2} \Leftrightarrow \eta\mu x = \eta\mu \frac{\pi}{4} \Leftrightarrow x = \begin{cases} 2k\pi + \frac{\pi}{4} \\ 2k\pi + \pi - \frac{\pi}{4} = 2k\pi + \frac{3\pi}{4} \end{cases} \quad k \in \mathbb{Z}$

iii) $\sigma\upsilon\nu x = 0 \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{2} \Leftrightarrow x = 2k\pi \pm \frac{\pi}{2} \Leftrightarrow x = k\pi + \frac{\pi}{2}$

$$x = \begin{cases} \underline{2k\pi} + \frac{\pi}{2} \\ 2k\pi - \frac{\pi}{2} = (2k-1)\pi + \pi - \frac{\pi}{2} = \underline{(2k-1)\pi} + \frac{\pi}{2} \end{cases} \quad k \in \mathbb{Z}$$

(Ε.Σ. Περίοδου.)

iv) $\sigma\upsilon\nu x = \frac{\sqrt{2}}{2} \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{\pi}{4} \Leftrightarrow x = 2k\pi \pm \frac{\pi}{4} \quad k \in \mathbb{Z}$

2. Να λύσετε τις εξισώσεις

$$\text{i) } \eta\mu x = -\frac{1}{2} \quad \text{ii) } \eta\mu x = -1 \quad \text{iii) } \sigma\upsilon\nu x = -\frac{\sqrt{2}}{2} \quad \text{iv) } \sigma\upsilon\nu x = -1$$

$$\text{i) } \eta\mu x = -\frac{1}{2} \Leftrightarrow \eta\mu x = -\eta\mu \frac{\pi}{6} \Leftrightarrow \eta\mu x = \eta\mu \left(-\frac{\pi}{6}\right)$$

$$x = \begin{cases} 2k\pi + \left(-\frac{\pi}{6}\right) = 2k\pi - \frac{\pi}{6} \\ 2k\pi + \pi - \left(-\frac{\pi}{6}\right) = 2k\pi + \pi + \frac{\pi}{6} = 2k\pi + \frac{7\pi}{6} \end{cases} \quad k \in \mathbb{Z}$$

$$\text{ii) } \eta\mu x = -1 \Leftrightarrow \eta\mu x = -\eta\mu \frac{\pi}{2} \Leftrightarrow \eta\mu x = \eta\mu \left(-\frac{\pi}{2}\right)$$

$$x = \begin{cases} 2k\pi + \left(-\frac{\pi}{2}\right) = 2k\pi - \frac{\pi}{2} \\ 2k\pi + \pi - \left(-\frac{\pi}{2}\right) = 2k\pi + \pi + \frac{\pi}{2} = 2k\pi + \frac{3\pi}{2} \end{cases} \quad k \in \mathbb{Z}$$

$x = 2k\pi - \frac{\pi}{2}$
(εξιστ. κ. 1
π < π, π/2.)

$$\text{iii) } \sigma\upsilon\nu x = -\frac{\sqrt{2}}{2} \Leftrightarrow \sigma\upsilon\nu x = -\sigma\upsilon\nu \frac{\pi}{4} \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \left(\pi - \frac{\pi}{4}\right)$$

$$\Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{3\pi}{4} \Leftrightarrow x = 2k\pi \pm \frac{3\pi}{4} \quad k \in \mathbb{Z}$$

$$\text{iv) } \sigma\upsilon\nu x = -1 \Leftrightarrow \sigma\upsilon\nu x = -\sigma\upsilon\nu 0 \Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu (\pi - 0)$$

$$\Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \pi \Leftrightarrow x = 2k\pi \pm \pi \Leftrightarrow x = 2k\pi + \pi$$

$k \in \mathbb{Z}$ (εξιστ. κ. 1
π < π, π/2.)

5. Να λύσετε τις εξισώσεις

i) $(1 - \eta \mu x)(2\eta \mu x - \sqrt{3}) = 0$ ii) $(2\eta \mu x + \sqrt{2})(1 - \sigma \upsilon \nu x) = 0$

i) $(1 - \eta \mu x) \cdot (2\eta \mu x - \sqrt{3}) = 0 \quad \Leftrightarrow$

$1 - \eta \mu x = 0 \quad \vee$

$\Leftrightarrow \eta \mu x = 1$

$\Leftrightarrow \eta \mu x = \eta \mu \frac{\pi}{2}$

$\Leftrightarrow x = \begin{cases} 2k\pi + \frac{\pi}{2} & k \in \mathbb{Z} \\ 2k\pi + \pi - \frac{\pi}{2} = 2k\pi + \frac{3\pi}{2} \end{cases}$

$\Leftrightarrow x = 2k\pi + \frac{\pi}{2}$

$2\eta \mu x - \sqrt{3} = 0$

$\Leftrightarrow \eta \mu x = \frac{\sqrt{3}}{2}$

$\Leftrightarrow \eta \mu x = \eta \mu \frac{\pi}{3}$

$\Leftrightarrow x = \begin{cases} 2k\pi + \frac{\pi}{3} & k \in \mathbb{Z} \\ 2k\pi + \pi - \frac{\pi}{3} = 2k\pi + \frac{2\pi}{3} \end{cases}$

ii) $(2\eta \mu x + \sqrt{2}) \cdot (1 - \sigma \upsilon \nu x) = 0 \quad \Leftrightarrow$

$2\eta \mu x + \sqrt{2} = 0 \quad \vee$

$\Leftrightarrow 2\eta \mu x = -\sqrt{2}$

$\Leftrightarrow \eta \mu x = -\frac{\sqrt{2}}{2}$

$\Leftrightarrow \eta \mu x = -\eta \mu \frac{\pi}{4}$

$\Leftrightarrow \eta \mu x = \eta \mu \left(-\frac{\pi}{4}\right)$

$\Leftrightarrow x = \begin{cases} 2k\pi + \left(-\frac{\pi}{4}\right) = 2k\pi - \frac{\pi}{4} & k \in \mathbb{Z} \\ 2k\pi + \pi - \left(-\frac{\pi}{4}\right) = 2k\pi + \pi + \frac{\pi}{4} = 2k\pi + \frac{5\pi}{4} \end{cases}$

$1 - \sigma \upsilon \nu x = 0$

$\Leftrightarrow \sigma \upsilon \nu x = 1$

$\Leftrightarrow \sigma \upsilon \nu x = \sigma \upsilon \nu 0$

$\Leftrightarrow x = 2k\pi \pm 0 \quad k \in \mathbb{Z}$

$\Leftrightarrow x = 2k\pi$

6. Να λύσετε τις εξισώσεις

i) $(\sqrt{3} + \varepsilon\varphi x)(1 - \varepsilon\varphi x) = 0$

ii) $(2\sigma\upsilon\nu x + 1)(\varepsilon\varphi^2 x - 3)\sigma\varphi x = 0$

ii) $(2\sigma\upsilon\nu x + 1) \cdot (\varepsilon\varphi^2 x - 3) \sigma\varphi x = 0 \quad \Leftrightarrow$

$$2\sigma\upsilon\nu x + 1 = 0$$

$$\vee \quad \varepsilon\varphi^2 x - 3 = 0$$

$$\vee \quad \sigma\varphi x = 0$$

$$\Leftrightarrow 2\sigma\upsilon\nu x = -1$$

$$\Leftrightarrow \varepsilon\varphi^2 x = 3$$

$$\Leftrightarrow \sigma\varphi x = \sigma\varphi \frac{\pi}{2}$$

$$\Leftrightarrow \sigma\upsilon\nu x = -\frac{1}{2}$$

$$\Leftrightarrow \varepsilon\varphi x = \sqrt{3} \quad \vee \quad \varepsilon\varphi x = -\sqrt{3}$$

$$\Leftrightarrow x = \kappa\pi + \frac{\pi}{2} \quad \kappa \in \mathbb{Z}$$

$$\Leftrightarrow \sigma\upsilon\nu x = -\sigma\upsilon\nu \frac{\pi}{3}$$

$$\Leftrightarrow \varepsilon\varphi x = \varepsilon\varphi \frac{\pi}{3}$$

$$\varepsilon\varphi x = -\varepsilon\varphi \frac{\pi}{3}$$

$$\Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \left(\pi - \frac{\pi}{3}\right)$$

$$\Leftrightarrow x = \kappa\pi + \frac{\pi}{3} \quad \kappa \in \mathbb{Z}$$

$$\varepsilon\varphi x = \varepsilon\varphi \left(-\frac{\pi}{3}\right)$$

$$x = \kappa\pi - \frac{\pi}{3}$$

$$\kappa \in \mathbb{Z}$$

$$\Leftrightarrow \sigma\upsilon\nu x = \sigma\upsilon\nu \frac{2\pi}{3}$$

$$\Leftrightarrow x = 2\kappa\pi \pm \frac{2\pi}{3} \quad \kappa \in \mathbb{Z}$$

8. Να λύσετε τις εξισώσεις

i) $2 \cdot \eta\mu 3x = \sqrt{3}$

ii) $\sigma\upsilon\nu \frac{x}{5} + 1 = 0$

iii) $3\varepsilon\varphi \frac{2x}{7} - \sqrt{3} = 0$

i) $2 \cdot \eta\mu 3x = \sqrt{3} \Leftrightarrow \eta\mu 3x = \frac{\sqrt{3}}{2} \Leftrightarrow \eta\mu 3x = \eta\mu \frac{\pi}{3}$

$\Leftrightarrow \text{Αρ.} \quad 3x = 2k\pi + \frac{\pi}{3} \Leftrightarrow x = \frac{2k\pi}{3} + \frac{\pi}{9} \quad k \in \mathbb{Z}$

$\quad \quad \quad 3x = 2k\pi + \pi - \frac{\pi}{3} \Leftrightarrow 3x = 2k\pi + \frac{2\pi}{3} \Leftrightarrow x = \frac{2k\pi}{3} + \frac{2\pi}{9}$

ii) $\sigma\upsilon\nu \frac{x}{5} + 1 = 0 \Leftrightarrow \sigma\upsilon\nu \frac{x}{5} = -1 \Leftrightarrow \sigma\upsilon\nu \frac{x}{5} = -\sigma\upsilon\nu 0$
ειδική περίπτωση.

$\Leftrightarrow \sigma\upsilon\nu \frac{x}{5} = \sigma\upsilon\nu \pi \Leftrightarrow \frac{x}{5} = 2k\pi + \pi \Leftrightarrow x = 10k\pi + 5\pi$
 $\quad \quad \quad \text{γ' } k \in \mathbb{Z}$

iii) $3\varepsilon\varphi \frac{2x}{7} - \sqrt{3} = 0 \Leftrightarrow \varepsilon\varphi \frac{2x}{7} = \frac{\sqrt{3}}{3} \Leftrightarrow \varepsilon\varphi \frac{2x}{7} = \varepsilon\varphi \frac{\pi}{6}$

$\Leftrightarrow \frac{2x}{7} = k\pi + \frac{\pi}{6} \stackrel{\cdot 7}{\Leftrightarrow} 2x = 7k\pi + \frac{7\pi}{6} \stackrel{\cdot 2}{\Leftrightarrow} x = \frac{7k\pi}{2} + \frac{7\pi}{12}$
 $\quad \quad \quad \text{γ' } k \in \mathbb{Z}$

9. Να λύσετε τις εξισώσεις

i) $\eta\mu\left(x + \frac{\pi}{3}\right) = -1$ ii) $2\sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = 1$ iii) $\varepsilon\varphi\left(\frac{\pi}{4} - 5x\right) = \sqrt{3}$

i) $\eta\mu\left(x + \frac{\pi}{3}\right) = -1 \Leftrightarrow \eta\mu\left(x + \frac{\pi}{3}\right) = -\eta\mu\frac{\pi}{2} \Leftrightarrow \eta\mu\left(x + \frac{\pi}{3}\right) = \eta\mu\left(-\frac{\pi}{2}\right)$

$\Leftrightarrow x + \frac{\pi}{3} = 2k\pi - \frac{\pi}{2} \Leftrightarrow x = 2k\pi - \frac{\pi}{2} - \frac{\pi}{3} \Leftrightarrow x = 2k\pi - \frac{5\pi}{6} \quad k \in \mathbb{Z}$

ii) $2\sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = 1 \Leftrightarrow \sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = \frac{1}{2} \Leftrightarrow \sigma\upsilon\nu\left(3x - \frac{\pi}{4}\right) = \sigma\upsilon\nu\frac{\pi}{3}$

$\Leftrightarrow \lambda_0 \quad 3x - \frac{\pi}{4} = 2k\pi + \frac{\pi}{3} \Leftrightarrow 3x = 2k\pi + \frac{\pi}{3} + \frac{\pi}{4} \Leftrightarrow$

$\Leftrightarrow 3x = 2k\pi + \frac{7\pi}{12} \Leftrightarrow x = \frac{2k\pi}{3} + \frac{7\pi}{36} \quad \mu, \quad k \in \mathbb{Z}$

$\therefore 3x - \frac{\pi}{4} = 2k\pi - \frac{\pi}{3} \Leftrightarrow 3x = 2k\pi + \frac{\pi}{3} - \frac{\pi}{4} \Leftrightarrow$

$\Leftrightarrow 3x = 2k\pi - \frac{\pi}{12} \Leftrightarrow x = \frac{2k\pi}{3} - \frac{\pi}{36} \quad \mu, \quad k \in \mathbb{Z}$

iii) $\varepsilon\varphi\left(\frac{\pi}{4} - 5x\right) = \sqrt{3} \Leftrightarrow \varepsilon\varphi\left(\frac{\pi}{4} - 5x\right) = \varepsilon\varphi\frac{\pi}{3}$

$\Leftrightarrow \frac{\pi}{4} - 5x = k\pi + \frac{\pi}{3} \Leftrightarrow -5x = k\pi + \frac{\pi}{3} - \frac{\pi}{4}$

$\Leftrightarrow -5x = k\pi + \frac{\pi}{12} \Leftrightarrow x = -\frac{k\pi}{5} - \frac{\pi}{60} \quad \mu, \quad k \in \mathbb{Z}$

10. Να λύσετε τις εξισώσεις

i) $2\eta\mu^2\omega + \eta\mu\omega - 1 = 0$

ii) $2\sigma\upsilon\nu^2x + 3\sigma\upsilon\nu x - 2 = 0$

i) $2\eta\mu^2\omega + \eta\mu\omega - 1 = 0$

Θέτω $\eta\mu\omega = x$ άρα $2x^2 + x - 1 = 0$

$a = 2$
 $b = 1$
 $\gamma = -1$

$\Delta = b^2 - 4a\gamma = 1^2 - 4 \cdot 2 \cdot (-1) = 1 + 8 = 9$

$x_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm \sqrt{9}}{2 \cdot 2} = \frac{-1 \pm 3}{4} = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$

Άρα $x = \frac{1}{2}$

ή $x = -1$

$\eta\mu\omega = \frac{1}{2}$

ή $\eta\mu\omega = -1$

$\Leftrightarrow \eta\mu\omega = \eta\mu \frac{\pi}{6}$

$\Leftrightarrow \eta\mu\omega = -\eta\mu \frac{\pi}{9}$

$\Leftrightarrow \omega = \begin{cases} 2k\pi + \frac{\pi}{6} & k \in \mathbb{Z} \\ 2k\pi + \pi - \frac{\pi}{6} = 2k\pi + \frac{5\pi}{6} \end{cases}$

$\Leftrightarrow \eta\mu\omega = \eta\mu\left(-\frac{\pi}{9}\right)$

$\Leftrightarrow \omega = 2k\pi - \frac{\pi}{9} \quad k \in \mathbb{Z}$