Ερωτήσεις κατανόησης (Μαθ/τικά Θ.Π. – γ' Λυκείου)

II

1. $$\lim_{h\to 0}\frac{\varepsilon\varphi\left(\frac{\pi}{6}+h\right)-\varepsilon\varphi\frac{\pi}{6}}{h}=\varepsilon\varphi'\left(\frac{\pi}{6}\right)$$

αλλά $(\varepsilon\varphi x)'=\dfrac{1}{\sigma\upsilon\nu^2 x}$, άρα $\varepsilon\varphi'\left(\frac{\pi}{6}\right)=\dfrac{1}{\sigma\upsilon\nu^2\frac{\pi}{6}}=\dfrac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}=$

$$=\frac{1}{\frac{3}{4}}=\frac{4}{3}\quad (\text{B})$$

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2. $$\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}=\left(\frac{1}{x}\right)'=-\frac{1}{x^2}\quad (\Gamma)$$

2ος τρόπος $$\lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h}=\lim_{h\to 0}\frac{x-x-h}{h\,x(x+h)}=\lim_{h\to 0}\frac{-h}{h\,x(x+h)}=$$

$$=\lim_{h\to 0}\frac{-1}{x(x+h)}=-\frac{1}{x^2}$$

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3. $f(x)=5^{3x}$

. $f(x)=e^{3x\cdot\ln 5}$ άρα $f'(x)=e^{3x\cdot\ln 5}\cdot(3x\cdot\ln 5)'=$

$$=5^{3x}\cdot 3\ln 5=5^{3x}\cdot\ln 5^3=5^{3x}\cdot\ln 125\quad (\text{E})$$

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4. $f(x)=\sigma\upsilon\nu^3(x+1)$ $\quad f'(\pi)=;$

. $f'(x)=3\,\sigma\upsilon\nu^2(x+1)\cdot(\sigma\upsilon\nu(x+1))'=3\,\sigma\upsilon\nu^2(x+1)\cdot(-\eta\mu(x+1))(x+1)'$

$$=-3\,\sigma\upsilon\nu^2(x+1)\cdot\eta\mu(x+1)$$

οπότε $f'(\pi)=-3\,\sigma\upsilon\nu^2(\pi+1)\cdot\eta\mu(\pi+1)\quad (\Gamma)$

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5. $f(x) = (x^2-1)^3 \qquad f^{(7)}(0) = ;$

$\cdot\ f'(x) = 3(x^2-1)^2(x^2-1)' = 3(x^2-1)^2 \cdot 2x = 6x\,(x^2-1)^2$

$\cdot\ f''(x) = 12x\,(x^2-1)(x^2-1)' + 6\,(x^2-1) = 12x\,(x^2-1)\cdot 2x + 6(x^2-1) =$

$= 24x^2(x^2-1) + 6(x^2-1) = 6(x^2-1)(4x^2+1)$

$\cdot\ f^{(3)}(x) = 6\,(2x)(4x^2+1) + 6\,(x^2-1)\cdot 8x =$

$= 12x\,(4x^2+1) + 48x(x^2-1) =$

$= 48x^3 + 12x + 48x^3 - 48x = 96x^3 + 12x - 48x =$

$= 96x^3 - 36x$

$\cdot\ f^{(4)}(x) = 3\cdot 96x^2 - 36$

$\cdot\ f^{(5)}(x) = 3\cdot 96 \cdot 2x$

$\cdot\ f^{(6)}(x) = 6\cdot 96$

$\cdot\ f^{(7)}(x) = 0 \qquad f^{(7)}(0) = 0.$

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6. $f(x) = \ln x$ και $g(x) = 2x^2$

Για να είναι η (ε_1) εφαπτομένη της C_f, παράλληλη στην (ε_2) εφαπτομένη της C_g παράλληλη στο x_0.

Αρκεί $f'(x_0) = g'(x_0) \Rightarrow$

$\Rightarrow \frac{1}{x_0} = 4x_0 \Leftrightarrow x_0^2 = \frac{1}{4} \Leftrightarrow |x_0|^2 = \frac{1}{4} \Leftrightarrow |x_0| = \frac{1}{2}$

αλλά $x_0 > 0$, επομένως $|x_0| = x_0$. Δηλ. $x_0 = \frac{1}{2}$. (Γ)

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7. $f(x) = e^{\beta x}, \quad g(x) = e^{\alpha x}$

$f'(x) = \beta e^{\beta x}, \quad g'(x) = \alpha \cdot e^{\alpha x}$

και αν $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)}{g'(x)} \Rightarrow \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)} = \frac{f'(x)}{g'(x)} \Rightarrow$

$\Leftrightarrow \frac{\beta e^{\beta x}\cdot e^{\alpha x} - e^{\beta x}\cdot \alpha \cdot e^{\alpha x}}{e^{2\alpha x}} = \frac{\beta e^{\beta x}}{\alpha \cdot e^{\alpha x}} \Leftrightarrow \frac{e^{\beta x + \alpha x}(\beta - \alpha)}{e^{2\alpha x}} = \frac{\beta}{\alpha}\cdot e^{\beta x - \alpha x}$

$\Leftrightarrow e^{\beta x - \alpha x}(\beta - \alpha) = \frac{\beta}{\alpha}\cdot e^{\beta x - \alpha x} \Leftrightarrow \beta - \alpha = \frac{\beta}{\alpha} \Leftrightarrow \beta - \frac{\beta}{\alpha} = \alpha \Leftrightarrow$

$\Leftrightarrow \beta\left(1 - \frac{1}{\alpha}\right) = \alpha \Leftrightarrow \beta \cdot \frac{\alpha - 1}{\alpha} = \alpha \Leftrightarrow \beta = \frac{\alpha^2}{\alpha - 1} \quad \alpha \neq 1$ (Ε)

8. Αν $f'(x)>0 \ \forall x \in [-1,1]$ και $f(0)=0$ τότε από Θ.Μ.Τ. είναι $\xi \in (-1,0)$ ώστε

$$f'(\xi) = \frac{f(0)-f(-1)}{0+1} > 0 \Leftrightarrow \frac{-f(-1)}{+1} > 0 \Rightarrow f(-1) < 0$$

ομοίως Θ.Μ.Τ.

και $\xi_1 \in (0,1)$

$$f'(\xi_1) = \frac{f(1)-f(0)}{1-0} > 0 \Leftrightarrow \frac{f(1)}{1} > 0 \Leftrightarrow f(1) > 0 \quad (\Gamma)$$

2ος τρόπος

με $f'(x)>0 \ \forall x \in [-1,1]$ η $f \uparrow \,|[-1,1]$

• $-1 < 0 < 1$

και $-1 < x < 0 \overset{f\uparrow}{\Rightarrow} f(-1) < f(x) < f(0) \Leftrightarrow f(-1) < 0$

και $0 < x < 1 \overset{f\uparrow}{\Rightarrow} f(0) < f(x) < f(1) \Leftrightarrow f(1) > 0 \quad (\Gamma)$

Σχ. κατ. θγ. 18ο

III

1. (α) → (Ε) παρ 1. $f(x) = x^2+1 \Rightarrow f'(x) = 2x$

(β) → (Α) παρ 2. $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x & x<0 \end{cases}$

$f'(x) = \begin{cases} 1, & x>0 \\ -1, & x<0 \end{cases}$

(γ) → (Β) παρ 3. $f(x) = \alpha x^4 + \beta x^3 + \gamma x^2 + \delta x \quad , \alpha \neq 0$

$f'(x) = 4\alpha x^3 + 3\beta x^2 + 2\gamma x + \delta$

(δ) → (Δ) παρ. α) $f(x) = 2x, \quad f'(x) = 2$ σταθερή!

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2. Ασύμπτωτη της f στο +∞

1) $f(x) = x + \frac{1}{x^2} \quad , x \neq 0$

· $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \left(1 + \frac{1}{x^3}\right) = 1 = \lambda$, αφού $\lim_{x\to+\infty} \frac{1}{x^3} = 0$.

· $\lim_{x\to+\infty} [f(x) - \lambda x] = \lim_{x\to+\infty} \left[x + \frac{1}{x^2} - x\right] = \lim_{x\to+\infty} \frac{1}{x^2} = 0 = \beta$

Ασύμπτωτη $y = x$ (Δ.)

2) $f(x) = -x + 1 + \frac{1}{e^x}$

· $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \left(-1 + \frac{1}{x} + \frac{1}{x \cdot e^x}\right) = -1 = \lambda$

αφού $\lim_{x\to+\infty} \frac{1}{x} = 0$ και $\lim_{x\to+\infty} \frac{1}{x \cdot e^x} = 0$ επειδή $\lim_{x\to+\infty} x = +\infty$

και $\lim_{x\to+\infty} e^x = +\infty$

· $\lim_{x\to+\infty} [f(x) - \lambda x] = \lim_{x\to+\infty} \left(-x + 1 + \frac{1}{e^x} + x\right) =$

$= \lim_{x\to+\infty} \left(1 + \frac{1}{e^x}\right) = 1 = \beta$ αφού $\lim_{x\to+\infty} \frac{1}{e^x} = 0$

Ασύμπτωτη $y = -x + 1$ (Γ.)

3. $f(x) = 2 + \dfrac{3}{x-2}$

· $\lim\limits_{x\to+\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to+\infty} \left(\dfrac{2}{x} + \dfrac{3}{x(x-2)} \right) = 0$

αφού $\lim\limits_{x\to+\infty} \dfrac{2}{x} = 0$ και $\lim\limits_{x\to+\infty} \dfrac{3}{x^2-2x} = \lim\limits_{x\to+\infty} \dfrac{3}{x^2} = 0$

· $\lim\limits_{x\to+\infty} \left(2 + \dfrac{3}{x-2} \right) = 2$ αφού $\lim\limits_{x\to+\infty} \dfrac{3}{x-2} = \lim\limits_{x\to+\infty} \dfrac{3}{x} = 0$.

Ασύμπτωτη $y = 2$ A.