

ΑΛΓΕΒΡΑ Β ΛΥΚΕΙΟΥ
ΤΡΙΓΩΝΟΜΕΤΡΙΑ
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Τριγωνομετρικοί αριθμοί γωνίας

1. Ποιες από τις παρακάτω τιμές δε μπορεί να είναι ημίτονο γωνίας;

α. $\frac{1}{2}$ β. $-\frac{3}{2}$ γ. $\frac{\sqrt{2}}{2}$ δ. $-\frac{1}{2}$ ε. $\frac{\sqrt{3}}{2}$ στ. $-\frac{\sqrt{8}}{4}$ ζ. $\frac{3}{\sqrt{3}}$

2. Αν $|\eta\mu x| + |\sigma\upsilon\nu x| = 2$ τότε η γωνία x ισούται με:

α. 0° β. 90° γ. 180° δ. 270° ε. 45° στ. Τίποτα από αυτά

3. Να βρείτε το πρόσημο των παρακάτω γινομένων:

α. $\eta\mu 80^\circ \cdot \sigma\upsilon\nu 260^\circ$ β. $\sigma\upsilon\nu 120^\circ \cdot \epsilon\phi 310^\circ$
 γ. $\eta\mu 100^\circ \cdot \sigma\upsilon\nu 100^\circ \cdot \epsilon\phi 100^\circ$ δ. $\epsilon\phi 240^\circ \cdot \sigma\upsilon\nu 320^\circ \cdot \sigma\phi 510^\circ$

4. Η τιμή του γινομένου $\sigma\upsilon\nu 0^\circ \cdot \sigma\upsilon\nu 90^\circ \cdot \sigma\upsilon\nu 180^\circ \cdot \sigma\upsilon\nu 270^\circ \cdot \sigma\upsilon\nu 360^\circ$ είναι:

α. -1 β. 1 γ. 0 δ. 2 ε. $\frac{1}{2}$

5. Να υπολογίσετε την τιμή της παράστασης:

$$\sigma\upsilon\nu^2 0 + \sigma\upsilon\nu^2 \frac{\pi}{6} + \eta\mu^2 \frac{\pi}{4} - \eta\mu^2 \frac{\pi}{3} - \epsilon\phi^2 \frac{\pi}{4}$$

6. Να υπολογίσετε τους παρακάτω τριγωνομετρικούς αριθμούς:

α. $\epsilon\phi 1845^\circ$ β. $\eta\mu 2580^\circ$ γ. $\sigma\phi \frac{97\pi}{6}$ δ. $\sigma\upsilon\nu \frac{11\pi}{2}$

7. Εάν $\eta\mu x = \frac{\sqrt{2}}{3}$ και $90^\circ < x < 180^\circ$, να υπολογίσετε τους υπόλοιπους τριγωνομετρικούς αριθμούς της γωνίας x.

8. Εάν $\epsilon\phi \theta = -\frac{3}{4}$ και $270^\circ < \theta < 360^\circ$, να υπολογίσετε τους υπόλοιπους τριγωνομετρικούς αριθμούς της γωνίας θ.

2. Αποδεικτικές ασκήσεις

Να αποδείξετε ότι για οποιεσδήποτε γωνίες x, y ισχύουν οι σχέσεις:

- $(\eta\mu x - \sigma\upsilon\nu x)^2 = 1 - 2 \cdot \eta\mu x \cdot \sigma\upsilon\nu x$
- $\eta\mu^4 x - \sigma\upsilon\nu^4 x = \eta\mu^2 x - \sigma\upsilon\nu^2 x = 1 - 2 \cdot \sigma\upsilon\nu^2 x = 2 \cdot \eta\mu^2 x - 1$
- $\sigma\upsilon\nu^2 x \cdot \sigma\upsilon\nu^2 y - \eta\mu^2 x \cdot \eta\mu^2 y + 1 = \sigma\upsilon\nu^2 x + \sigma\upsilon\nu^2 y$
- $(1 + \eta\mu x + \sigma\upsilon\nu x)^2 = 2 \cdot (1 + \sigma\upsilon\nu x) \cdot (1 + \eta\mu x)$

$$5. \quad \frac{1 - \varepsilon\varphi^2 x}{1 + \varepsilon\varphi^2 x} = 1 - 2 \cdot \eta\mu^2 x$$

$$6. \quad 1 - \frac{\sigma\upsilon\nu^2 x}{1 + \eta\mu x} = \eta\mu x$$

$$7. \quad \varepsilon\varphi^2 x \cdot \sigma\upsilon\nu^2 x + \sigma\varphi^2 x \cdot \eta\mu^2 x = 1$$

$$8. \quad \eta\mu^3 x + \eta\mu x \cdot \sigma\upsilon\nu^2 x = \eta\mu x$$

$$9. \quad \frac{\eta\mu x - \sigma\upsilon\nu x + 1}{\eta\mu x + \sigma\upsilon\nu x - 1} = \frac{\sigma\upsilon\nu x}{1 - \eta\mu x}$$

$$10. \quad \frac{\eta\mu x}{1 + \sigma\upsilon\nu x} + \frac{1 + \sigma\upsilon\nu x}{\eta\mu x} = \frac{2}{\eta\mu x}$$

$$11. \quad 3\sigma\upsilon\nu^2 x + 2\eta\mu^2 x = \frac{3 + 2\varepsilon\varphi^2 x}{1 + \varepsilon\varphi^2 x}$$

$$12. \quad \varepsilon\varphi x \cdot \varepsilon\varphi y \cdot (\sigma\varphi x + \sigma\varphi y) = \varepsilon\varphi x + \varepsilon\varphi y$$

$$13. \quad \frac{\eta\mu x}{1 + \sigma\upsilon\nu x} - \frac{1 + \sigma\upsilon\nu x}{\eta\mu x} + 2\sigma\varphi x = 0$$

$$14. \quad \frac{\sigma\upsilon\nu^3 x}{\eta\mu x} + \frac{\varepsilon\varphi x}{1 + \varepsilon\varphi^2 x} = \sigma\varphi x$$

$$15. \quad \frac{\varepsilon\varphi x + \sigma\varphi y}{\varepsilon\varphi y + \sigma\varphi x} = \frac{\varepsilon\varphi x}{\varepsilon\varphi y}$$

$$16. \quad \frac{1}{\sigma\upsilon\nu x} - \frac{\sigma\upsilon\nu x}{1 + \eta\mu x} = \varepsilon\varphi x$$

$$17. \quad \frac{\sigma\varphi x}{1 - \sigma\varphi x} - \frac{\varepsilon\varphi x}{1 - \varepsilon\varphi x} = \frac{\eta\mu x + \sigma\upsilon\nu x}{\eta\mu x - \sigma\upsilon\nu x}$$

$$18. \quad \frac{\varepsilon\varphi x + \eta\mu x}{\varepsilon\varphi x - \eta\mu x} = \left(\frac{1}{\eta\mu x} + \sigma\varphi x \right)^2$$

$$19. \quad \frac{\varepsilon\varphi x - \eta\mu x}{\eta\mu^3 x} = \frac{1}{\sigma\upsilon\nu x + \sigma\upsilon\nu^2 x}$$

$$20. \quad \varepsilon\varphi x - \sigma\varphi x = \frac{1 - 2\sigma\upsilon\nu^2 x}{\eta\mu x \cdot \sigma\upsilon\nu x}$$

$$21. \quad \frac{\sigma\varphi x}{1 + \sigma\varphi x} + \frac{\varepsilon\varphi x}{1 + \varepsilon\varphi x} = 1$$

$$22. \quad \sigma\varphi x + \frac{\eta\mu x}{1 + \sigma\upsilon\nu x} = \frac{1}{\eta\mu x}$$

$$23. \quad \eta\mu^2 x \cdot (1 + \sigma\varphi^2 x) + \sigma\upsilon\nu^2 x \cdot (1 + \varepsilon\varphi^2 x) = 2$$

$$24. \quad \sigma\upsilon\nu^4 x + \eta\mu^2 x \cdot \sigma\upsilon\nu^2 x + \eta\mu^2 x = 1$$

3. Αναγωγή στο 1^ο τεταρτημόριο

1. Συμπληρώστε τον παρακάτω πίνακα

$\eta\mu(-x) = -\eta\mu x$	$\eta\mu(\pi - x) =$	$\eta\mu\left(\frac{3\pi}{2} + x\right) =$
$\sigma\upsilon\nu(-x) =$	$\sigma\upsilon\nu(\pi - x) =$	$\sigma\upsilon\nu\left(\frac{3\pi}{2} + x\right) =$
$\varepsilon\varphi(-x) =$	$\varepsilon\varphi(\pi - x) =$	$\varepsilon\varphi\left(\frac{3\pi}{2} + x\right) =$

$\sigma\varphi(-x) =$	$\sigma\varphi(\pi - x) =$	$\sigma\varphi\left(\frac{3\pi}{2} + x\right) =$
$\eta\mu\left(\frac{\pi}{2} - x\right) =$	$\eta\mu(\pi + x) =$	$\eta\mu(2\pi - x) =$
$\sigma\upsilon\nu\left(\frac{\pi}{2} - x\right) =$	$\sigma\upsilon\nu(\pi + x) =$	$\sigma\upsilon\nu(2\pi - x) =$
$\varepsilon\varphi\left(\frac{\pi}{2} - x\right) =$	$\varepsilon\varphi(\pi + x) =$	$\varepsilon\varphi(2\pi - x) =$
$\sigma\varphi\left(\frac{\pi}{2} - x\right) =$	$\sigma\varphi(\pi + x) =$	$\sigma\varphi(2\pi - x) =$
$\eta\mu\left(\frac{\pi}{2} + x\right) =$	$\eta\mu\left(\frac{3\pi}{2} - x\right) =$	$\eta\mu(2\pi + x) =$
$\sigma\upsilon\nu\left(\frac{\pi}{2} + x\right) =$	$\sigma\upsilon\nu\left(\frac{3\pi}{2} - x\right) =$	$\sigma\upsilon\nu(2\pi + x) =$
$\varepsilon\varphi\left(\frac{\pi}{2} + x\right) =$	$\varepsilon\varphi\left(\frac{3\pi}{2} - x\right) =$	$\varepsilon\varphi(2\pi + x) =$
$\sigma\varphi\left(\frac{\pi}{2} + x\right) =$	$\sigma\varphi\left(\frac{3\pi}{2} - x\right) =$	$\sigma\varphi(2\pi + x) =$

2. Να απλοποιηθούν οι παραστάσεις:

α. $\frac{\eta\mu(3\pi + \alpha) \cdot \sigma\varphi(7\pi + \alpha) \cdot \sigma\upsilon\nu\alpha}{\sigma\upsilon\nu(3\pi + \alpha) \cdot \sigma\varphi(4\pi + \alpha) \cdot \eta\mu\alpha}$

β. $\frac{2 \cdot \eta\mu(\pi - \theta) + \sigma\upsilon\nu\left(\frac{\pi}{2} - \theta\right) - 2 \cdot \eta\mu(2\pi - \theta)}{\eta\mu\left(\frac{\pi}{2} - \theta\right) - 3 \cdot \sigma\upsilon\nu(\pi - \theta) + \sigma\upsilon\nu(2\pi - \theta)}$

3. Να εκφράσετε συναρτήσει των $\sigma\upsilon\nu x$ και $\eta\mu x$ την παράσταση:

$$A = \sigma\upsilon\nu(-x) + \eta\mu(-x) + \eta\mu(\pi + x) + \sigma\upsilon\nu(\pi - x)$$

4. Εάν A, B και Γ είναι γωνίες τριγώνου, να αποδείξετε ότι:

α. $\eta\mu(B + \Gamma) = \eta\mu A$

β. $\sigma\upsilon\nu(B + \Gamma) = -\sigma\upsilon\nu A$

γ. $\varepsilon\varphi(B + \Gamma) = -\varepsilon\varphi A$

5. Αν $0 < \alpha < \frac{\pi}{2}$, να αποδειχθεί ότι:

$$\frac{\sin\left(\frac{\pi}{2} - \alpha\right) \cdot \varepsilon\phi\left(\frac{9\pi}{2} + \alpha\right) \cdot \eta\mu\left(\frac{3\pi}{2} - \alpha\right) \cdot \sigma\upsilon\nu\alpha}{\eta\mu\left(\frac{3\pi}{2} + \alpha\right) \cdot \sigma\phi\left(\frac{6\pi}{2} - \alpha\right) \cdot \sin(\pi - \alpha) \cdot \eta\mu\alpha} = -1$$

6. Να αποδειχθεί ότι:

$$\frac{\eta\mu^2 212^\circ \cdot \sin 302^\circ - \sin^3 148^\circ}{\eta\mu 58^\circ \cdot \sin 32^\circ + \eta\mu 392^\circ \cdot \eta\mu 148^\circ - \eta\mu 58^\circ \cdot \eta\mu 148^\circ} = \sin 32^\circ + \eta\mu 32^\circ$$

7. Έστω $A = \frac{\varepsilon\phi\left(\frac{3\pi}{2} - x\right) \cdot \sin\left(\frac{3\pi}{2} + x\right)}{\sin(2\pi - x)}$

$$B = \sin\left(x - \frac{\pi}{2}\right) \cdot \eta\mu(\pi + x)$$

$$\Gamma = \sin(\pi + x) \cdot \eta\mu\left(x - \frac{\pi}{2}\right)$$

Να αποδειχθεί ότι:

α. $\Gamma = B + 1$ **β.** $B + A = \sin^2 \alpha$ **γ.** $A + B = \Gamma$

4. Βασικές τριγωνομετρικές εξισώσεις

1. Να λύσετε τις εξισώσεις:

i. $\eta\mu x = \eta\mu \frac{\pi}{7}$

ii. $\eta\mu x = \eta\mu 70^\circ$

iii. $\sin x = \sin \frac{3\pi}{15}$

iv. $\sin x = \sin 25^\circ$

v. $\varepsilon\phi x = \varepsilon\phi \frac{\pi}{10}$

vi. $\varepsilon\phi x = \varepsilon\phi 42^\circ$

vii. $\sigma\phi x = \sigma\phi \frac{2\pi}{11}$

viii. $\sigma\phi x = \sigma\phi 88^\circ$

2. Να λύσετε τις εξισώσεις:

i. $\eta\mu x = 0$

ii. $\eta\mu x = 1$

iii. $\eta\mu x = -1$

iv. $\sin x = 0$

v. $\sin x = 1$

vi. $\sin x = -1$

vii. $\varepsilon\phi x = 0$

viii. $\varepsilon\phi x = 1$

ix. $\varepsilon\phi x = -1$

x. $\sigma\phi x = 0$

xi. $\sigma\phi x = 1$

xii. $\sigma\phi x = -1$

3. Να λύσετε τις εξισώσεις:

$$\text{i.} \quad \eta_{\mu\chi} = \frac{1}{2}$$

$$\text{ii.} \quad \eta_{\mu\chi} = \frac{\sqrt{2}}{2}$$

$$\text{iii.} \quad \eta_{\mu\chi} = \frac{\sqrt{3}}{2}$$

$$\text{iv.} \quad \sigma_{\nu\chi} = \frac{1}{2}$$

$$\text{v.} \quad \sigma_{\nu\chi} = \frac{\sqrt{2}}{2}$$

$$\text{vi.} \quad \sigma_{\nu\chi} = \frac{\sqrt{3}}{2}$$

$$\text{vii.} \quad \varepsilon_{\varphi\chi} = \frac{\sqrt{3}}{3}$$

$$\text{viii.} \quad \varepsilon_{\varphi\chi} = 1$$

$$\text{ix.} \quad \varepsilon_{\varphi\chi} = \sqrt{3}$$

$$\text{x.} \quad \sigma_{\varphi\chi} = \frac{\sqrt{3}}{3}$$

$$\text{xi.} \quad \sigma_{\varphi\chi} = 1$$

$$\text{xii.} \quad \sigma_{\varphi\chi} = \sqrt{3}$$

4. Να λύσετε τις εξισώσεις:

$$\text{i.} \quad \eta_{\mu\chi} = -\frac{\sqrt{2}}{2}$$

$$\text{ii.} \quad \sigma_{\nu\chi} = -\frac{1}{2}$$

$$\text{iii.} \quad \varepsilon_{\varphi\chi} = -1$$

$$\text{iv.} \quad \sigma_{\varphi\chi} = -\sqrt{3}$$

$$\text{v.} \quad \eta_{\mu\chi} = -\frac{1}{2}$$

$$\text{vi.} \quad \sigma_{\nu\chi} = -\frac{\sqrt{3}}{2}$$

$$\text{vii.} \quad \varepsilon_{\varphi\chi} = -\frac{\sqrt{3}}{3}$$

$$\text{viii.} \quad \sigma_{\varphi\chi} = -1$$

$$\text{ix.} \quad \eta_{\mu\chi} = -\eta_{\mu} \frac{\pi}{10}$$

$$\text{x.} \quad \sigma_{\nu\chi} = -\sigma_{\nu} \frac{\pi}{5}$$

$$\text{xi.} \quad \varepsilon_{\varphi\chi} = -\varepsilon_{\varphi} 55^0$$

$$\text{xii.} \quad \sigma_{\varphi\chi} = -\sigma_{\varphi} 17^0$$

5. Να λύσετε τις εξισώσεις:

$$\text{i.} \quad \sqrt{2} \eta_{\mu\chi} - 1 = 0$$

$$\text{ii.} \quad 2\sqrt{3} \eta_{\mu\chi} + 3 = 0$$

$$\text{iii.} \quad 2\sigma_{\nu\chi} + \sqrt{3} = 0$$

$$\text{iv.} \quad 6\sigma_{\nu\chi} - 3\sqrt{2} = 0$$

$$\text{v.} \quad \sqrt{3} \varepsilon_{\varphi\chi} - 1 = 0$$

$$\text{vi.} \quad \sqrt{6} \sigma_{\varphi\chi} + \sqrt{18} = 0$$

$$\text{vii.} \quad \eta_{\mu\chi} - \eta_{\mu} \frac{2\pi}{9} = 0$$

$$\text{viii.} \quad \sigma_{\nu\chi} - \sigma_{\nu} \frac{\pi}{7} = 0$$

$$\text{ix.} \quad \varepsilon_{\varphi\chi} + \varepsilon_{\varphi} \frac{\pi}{8} = 0$$

$$\text{x.} \quad \sigma_{\varphi\chi} + \sigma_{\varphi} \frac{\pi}{9} = 0$$

6. Να λύσετε τις εξισώσεις:

$$\text{i.} \quad \eta_{\mu\chi} \cdot (\sqrt{2} \sigma_{\nu\chi} - 1) = 0$$

$$\text{ii.} \quad \sigma_{\nu\chi} \cdot (2\eta_{\mu\chi} - \sqrt{3}) = 0$$

$$\text{iii.} \quad \varepsilon_{\varphi\chi} \cdot (\sqrt{3} \sigma_{\varphi\chi} + 1) = 0$$

$$\text{iv.} \quad \sigma_{\varphi\chi} \cdot (\varepsilon_{\varphi\chi} - 1) = 0$$

$$\text{v.} \quad (1 - \eta_{\mu\chi}) \cdot (\sqrt{2} + 2\sigma_{\nu\chi}) = 0$$

$$\text{vi.} \quad (\varepsilon_{\varphi\chi} + \sqrt{3}) \cdot (\sigma_{\varphi\chi} + 1) = 0$$

7. Να λύσετε τις εξισώσεις:

$$\text{i.} \quad \eta\mu\left(x + \frac{\pi}{5}\right) = \eta\mu\left(x + \frac{\pi}{10}\right)$$

$$\text{ii.} \quad \eta\mu(\pi - x) = \eta\mu\left(\frac{\pi}{4} - 2x\right)$$

$$\text{iii.} \quad \sigma\upsilon\nu\pi x = \sigma\upsilon\nu\left(2\pi x - \frac{\pi}{3}\right)$$

$$\text{iv.} \quad \sigma\upsilon\nu\left(x + \frac{3\pi}{2}\right) = \sigma\upsilon\nu\left(x - \frac{\pi}{6}\right)$$

$$\text{iii.} \quad \varepsilon\varphi\left(x - \frac{\pi}{4}\right) + \varepsilon\varphi\left(\frac{\pi}{6} - 2x\right) = 0$$

$$\text{iv.} \quad \sigma\varphi(x + 10^0) - \sigma\varphi 2x = 0$$

8. Να λύσετε τις εξισώσεις:

$$\text{i.} \quad \eta\mu x = \sigma\upsilon\nu x$$

$$\text{ii.} \quad \varepsilon\varphi x = \sigma\varphi x$$

$$\text{iii.} \quad \eta\mu 2x = \sigma\upsilon\nu\left(x - \frac{\pi}{3}\right)$$

$$\text{iv.} \quad \sigma\upsilon\nu 2x = \eta\mu\left(x + \frac{\pi}{4}\right)$$

$$\text{v.} \quad \varepsilon\varphi(\pi + 2x) = \sigma\varphi\left(x - \frac{\pi}{4}\right)$$

$$\text{vi.} \quad \sigma\varphi\left(2x - \frac{\pi}{5}\right) - \varepsilon\varphi\left(x + \frac{\pi}{10}\right) = 0$$

$$\text{vii.} \quad \eta\mu\left(3x - \frac{\pi}{4}\right) = -\sigma\upsilon\nu\left(x - \frac{\pi}{4}\right)$$

$$\text{viii.} \quad \sigma\upsilon\nu\left(x - \frac{\pi}{3}\right) = -\eta\mu\left(\frac{\pi}{4} - x\right)$$

$$\text{ix.} \quad \varepsilon\varphi\left(2x - \frac{\pi}{3}\right) = -\sigma\varphi\left(2x - \frac{\pi}{6}\right)$$

$$\text{x.} \quad \sigma\varphi\left(x - \frac{\pi}{4}\right) + \varepsilon\varphi\left(x + \frac{\pi}{8}\right) = 0$$

9. Να λύσετε τις εξισώσεις:

$$\text{i.} \quad 2\eta\mu^2 x = \sqrt{3}\eta\mu x$$

$$\text{ii.} \quad \eta\mu^2 x - \sqrt{2}\eta\mu^3 x = 0$$

$$\text{iii.} \quad \sigma\upsilon\nu x - \sqrt{2}\sigma\upsilon\nu^2 x = 0$$

$$\text{iv.} \quad 2\sigma\upsilon\nu^2 x = -\sigma\upsilon\nu x$$

$$\text{v.} \quad \varepsilon\varphi^3 x - \varepsilon\varphi x = 0$$

$$\text{vi.} \quad \sqrt{3}\sigma\varphi x - \sigma\varphi^2 x = 0$$

10. Να λύσετε τις εξισώσεις:

$$\text{i.} \quad \eta\mu^2 x = \frac{1}{2}$$

$$\text{ii.} \quad \eta\mu^2(2x) - \frac{3}{4} = 0$$

$$\text{iii.} \quad \sigma\upsilon\nu^2 x - 1 = 0$$

$$\text{iv.} \quad 4\sigma\upsilon\nu^2\left(x - \frac{\pi}{3}\right) = 1$$

$$\text{v.} \quad 3\varepsilon\varphi^2 x - 1 = 0$$

$$\text{vi.} \quad \sigma\varphi^2\left(\frac{x}{2}\right) - 1 = 0$$

$$\text{vii.} \quad 8\eta\mu^3 x + 1 = 0$$

$$\text{viii.} \quad 9\sigma\varphi^4 x - 1 = 0$$

11. Να λύσετε τις εξισώσεις:

$$\text{i.} \quad \eta\mu x \cdot \sigma\upsilon\nu x - 1 = \sigma\upsilon\nu x - \eta\mu x$$

$$\text{ii.} \quad 2\eta\mu x + 2\sqrt{2}\eta\mu x \cdot \sigma\upsilon\nu x = 1 + 2\eta\mu x$$

$$\text{iii.} \quad \eta\mu^2 x - \sigma\upsilon\nu^2 x = 0$$

iv. $\sqrt{3} \varepsilon\varphi x - \varepsilon\varphi^2 x - \sqrt{3} \sigma\varphi x + 1 = 0$

v. $(1 - 2\eta\mu x)^2 + 2\eta\mu x - 1 = 0$

iv. $\sigma\varphi^4 x + \sigma\varphi x = 0$

12. Να λύσετε τις εξισώσεις:

i. $\eta\mu^2 x - 2\eta\mu x + 1 = 0$

ii. $2\eta\mu^2 x + \eta\mu x - 1 = 0$

iii. $4\sigma\upsilon\nu^2 x - 4\sigma\upsilon\nu x + 1 = 0$

iv. $\sigma\upsilon\nu^2 x - 2\sigma\upsilon\nu x - 3 = 0$

v. $\eta\mu^2 x - \frac{4 + \sqrt{3}}{2} \eta\mu x + \sqrt{3} = 0$

vi. $\sigma\upsilon\nu^2 x - \frac{3\sqrt{2}}{2} \sigma\upsilon\nu x + 1 = 0$

vii. $4\sigma\upsilon\nu^2 x - 2(\sqrt{3} + 1)\sigma\upsilon\nu x + \sqrt{3} = 0$

viii. $\varepsilon\varphi^2 x - (1 - \sqrt{3})\varepsilon\varphi x - \sqrt{3} = 0$

ix. $3\sigma\varphi^2 x - 4\sqrt{3} \sigma\varphi x + 3 = 0$

13. Να λύσετε τις εξισώσεις:

i. $\eta\mu x = \sqrt{3} \sigma\upsilon\nu x$

ii. $2\eta\mu x + \sqrt{12} \sigma\upsilon\nu x = 0$

iii. $\eta\mu\left(x - \frac{\pi}{4}\right) + \sigma\upsilon\nu\left(x - \frac{\pi}{4}\right) = 0$

iv. $\sqrt{3} \eta\mu 3x = \sigma\upsilon\nu 3x$

15. Να λύσετε τις εξισώσεις:

i. $\sigma\varphi x = \sigma\upsilon\nu x$

ii. $\varepsilon\varphi x = \eta\mu x$

iii. $\varepsilon\varphi x - \sigma\upsilon\nu x + \frac{1}{\sigma\upsilon\nu x} = 0$

iv. $\eta\mu x - \frac{1}{\sigma\upsilon\nu x} = 1 - \varepsilon\varphi x$

v. $2\varepsilon\varphi^2 x + \frac{\varepsilon\varphi x}{\sigma\upsilon\nu x} = 1$

vi. $\sigma\varphi^2 x \cdot \eta\mu x - \sigma\upsilon\nu x = 0$

16. Να λύσετε τις εξισώσεις:

i. $\varepsilon\varphi x = \frac{\sqrt{3}}{3}, x \in [0, 2\pi]$

ii. $3\sigma\upsilon\nu x + \sqrt{3} = 0, x \in (-\pi, \pi)$

iii. $\varepsilon\varphi\left(\frac{\pi}{3} - x\right) = 1, x \in \left(\frac{\pi}{2}, \frac{5\pi}{2}\right]$

iv. $\sqrt{2} \eta\mu(\pi x) - 1 = 0, x \in [-1, 2]$

17. Να λύσετε τις εξισώσεις:

i. $\sigma\upsilon\nu x (1 + \varepsilon\varphi^2 x) - \varepsilon\varphi x = \sigma\upsilon\nu x$

ii. $\varepsilon\varphi^2 x = 4\sigma\upsilon\nu^2 x - 1$

iii. $\frac{1}{\sigma\upsilon\nu^2 x} = 2\varepsilon\varphi x$

iv. $\frac{1}{2\sigma\upsilon\nu^2 x} = \sqrt{3} \varepsilon\varphi x - 1$

18. Να βρείτε για ποιες τιμές του x , καθεμιά από τις επόμενες συναρτήσεις έχει τη μέγιστη και για ποιες την ελάχιστη τιμή της.



i. $f(x) = 2 - \eta\mu\left(2x - \frac{\pi}{6}\right), \quad 0 \leq x \leq 2\pi$

ii. $f(x) = 4\sigma\upsilon\nu\left(\frac{x}{2} + \frac{\pi}{4}\right), \quad 0 \leq x \leq 2\pi$

iii. $f(x) = 3 + 3\sigma\upsilon\nu(\pi x - \pi), \quad -1 \leq x \leq 1$