Σύνθεση Ταλαντώσεων 1ου Είδους

Έστω σώμα (Σ) που εκτελεί ταυτόχρονα 2 ταλαντώσεις:

$x_1 = A_1 \cdot \eta\mu\omega t$

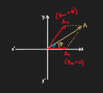
$x_2 = A_2 \cdot \eta\mu(\omega t + \theta)$

Αρχή της Επαλληλίας:

$x = x_1 + x_2 \Rightarrow x = A_1 \cdot \eta\mu\omega t + A_2 \cdot \eta\mu(\omega t + \theta) \Rightarrow$ ΑΑΤ $\Rightarrow x = A \cdot \eta\mu(\omega t + \theta)$

$\varepsilon\varphi\theta = \frac{A_2 \cdot \eta\mu\theta}{A_1 + A_2 \cdot \sigma\upsilon\nu\theta}$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\sigma\upsilon\nu\theta}$



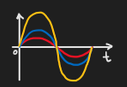
Ειδικές Περιπτώσεις Σύνθεσης ΑΑΤ 1ου Είδους

$\theta = 0$

$x_1 = A_1 \cdot \eta\mu\omega t$

$x_2 = A_2 \cdot \eta\mu(\omega t + 0) = A_2 \cdot \eta\mu\omega t$

$x = x_1 + x_2 = A_1 \cdot \eta\mu\omega t + A_2 \cdot \eta\mu\omega t = (A_1 + A_2) \cdot \eta\mu\omega t$



$\theta = \pi$

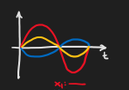
$x_1 = A_1 \cdot \eta\mu\omega t$

$x_2 = A_2 \cdot \eta\mu(\omega t + \pi) = -A_2 \cdot \eta\mu\omega t$

$x = x_1 + x_2 = A_1 \cdot \eta\mu\omega t + (-A_2 \cdot \eta\mu\omega t) = (A_1 - A_2) \cdot \eta\mu\omega t$

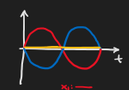
$A_1 > A_2$

$x = (A_1 - A_2) \cdot \eta\mu\omega t$



$A_1 = A_2$

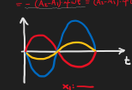
$x = 0$



$A_1 < A_2$

$x = (A_1 - A_2) \cdot \eta\mu\omega t = -|A_1 - A_2| \cdot \eta\mu\omega t =$

$= -(A_2 - A_1) \cdot \eta\mu\omega t = (A_2 - A_1) \cdot \eta\mu(\omega t + \pi)$



Ενέργεια στην Σύνθεση Ταλαντώσεων 1ου Είδους

$x_1 = A_1 \cdot \eta\mu\omega t$

$x_2 = A_2 \cdot \eta\mu\omega t$

$\Big\} \; x = x_1 + x_2 = A \cdot \eta\mu(\omega t + \theta)$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\sigma\upsilon\nu\varphi}$

$\varepsilon\varphi\theta = \frac{A_2 \cdot \eta\mu\varphi}{A_1 + A_2 \cdot \sigma\upsilon\nu\varphi}$

$D_1 = D_2 = D = m\omega^2$

με ω = ίδια

$\omega_1 = \omega_2 = \omega$ κοινή

- $E_1 = \frac{1}{2}DA_1^2 \Rightarrow 2E_1 = D \cdot A_1^2 \Rightarrow A_1^2 = \frac{2E_1}{D} \Rightarrow A_1 = \sqrt{\frac{2E_1}{D}}$
- $E_2 = \frac{1}{2}DA_2^2 \Rightarrow 2E_2 = D \cdot A_2^2 \Rightarrow A_2^2 = \frac{2E_2}{D} \Rightarrow A_2 = \sqrt{\frac{2E_2}{D}}$
- $E = \frac{1}{2}DA^2 = \frac{1}{2}D(A_1^2 + A_2^2 + 2A_1A_2\sigma\upsilon\nu\varphi) =$

$= \frac{1}{2}DA_1^2 + \frac{1}{2}DA_2^2 + \frac{1}{2}D\,2A_1A_2\sigma\upsilon\nu\varphi =$

$= E_1 + E_2 + D \cdot \sqrt{\frac{2E_1}{D}} \cdot \sqrt{\frac{2E_2}{D}} \cdot \sigma\upsilon\nu\varphi =$

$= E_1 + E_2 + D\sqrt{\frac{4E_1E_2}{D^2}} \cdot \sigma\upsilon\nu\varphi =$

$= E_1 + E_2 + D\,\frac{2\sqrt{E_1E_2}}{D} \cdot \sigma\upsilon\nu\varphi \Rightarrow$

$\Rightarrow \boxed{E = E_1 + E_2 + 2\sqrt{E_1E_2} \cdot \sigma\upsilon\nu\varphi}$

Άσκηση:

$x = 10\sqrt{3} \cdot \eta\mu(2\pi t + \frac{\pi}{2})$ → $A = 10\sqrt{3}$ m, $\omega = 2\pi$ rad/s, $\varphi = \pi/2$ rad

$x_1 = 10 \cdot \eta\mu 2\pi t$ → $A_1 = 10$ m

$x_2 = A_2 \cdot \eta\mu(2\pi t + \phi)$

$A_2 = ?$

$\phi = ?$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\sigma\upsilon\nu\phi} \Rightarrow$

$10\sqrt{3} = \sqrt{10^2 + A_2^2 + 2 \cdot 10A_2\sigma\upsilon\nu\phi} \Rightarrow$

$\overset{\uparrow^2}{\Rightarrow} 300 = 100 + A_2^2 + 20A_2\sigma\upsilon\nu\phi \Rightarrow$

$\Rightarrow \boxed{200 = A_2^2 + 20 \cdot A_2 \cdot \sigma\upsilon\nu\phi}$ (1)

$\varepsilon\varphi\theta = \frac{A_2 \cdot \eta\mu\phi}{A_1 + A_2\sigma\upsilon\nu\phi} \Rightarrow \varepsilon\varphi\frac{\pi}{2} = \frac{A_2 \cdot \eta\mu\phi}{10 + A_2\sigma\upsilon\nu\phi} \Rightarrow \infty = \frac{A_2 \cdot \eta\mu\phi}{10 + A_2\sigma\upsilon\nu\phi}$

$\varepsilon\varphi\frac{\pi}{2} = \frac{\eta\mu\pi/2}{\sigma\upsilon\nu\pi/2} = \frac{1}{0}$

$10 + A_2\sigma\upsilon\nu\phi = 0 \Rightarrow$

$\Rightarrow 10 = -A_2\sigma\upsilon\nu\phi \Rightarrow$

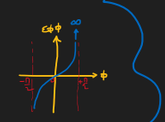
$\Rightarrow \boxed{A_2\sigma\upsilon\nu\phi = -10}$ (2)

2 αγκάθια...

(1)/(2): $200 = A_2^2 + 20(-10) \Rightarrow 200 = A_2^2 - 200 \Rightarrow A_2^2 = 400 \Rightarrow \boxed{A_2 = 20\ m}$

(2): $20\sigma\upsilon\nu\phi = -10 \Rightarrow \sigma\upsilon\nu\phi = -\frac{1}{2} \overset{(*)}{\Rightarrow} \boxed{\phi = \frac{2\pi}{3}\ rad}$

$\varepsilon\varphi\frac{\pi}{2} \to \infty$



$\varepsilon\varphi\frac{\pi}{2} = \frac{\eta\mu\pi/2}{\sigma\upsilon\nu\pi/2} = \frac{1}{0} = \infty$

(*) Γενικά $\sigma\upsilon\nu\varphi = -\frac{1}{2} \Rightarrow \varphi = \frac{2\pi}{3}$ ή $\varphi = -\frac{2\pi}{3}$ ή $\frac{4\pi}{3}$

Προσοχή στις διευθύνσεις !!!

$\sigma\upsilon\nu\varphi = -\frac{1}{2} = -\sigma\upsilon\nu\frac{\pi}{3} = \sigma\upsilon\nu(\pi - \frac{\pi}{3}) = \sigma\upsilon\nu\frac{2\pi}{3} \Rightarrow$

$\varphi = 2k\pi + \frac{2\pi}{3} \xrightarrow{k=0} \varphi = \frac{2\pi}{3}$ rad

$\varphi = 2k\pi - \frac{2\pi}{3} \xrightarrow{k=1} \varphi = -\frac{2\pi}{3}$ rad ή $\varphi = \frac{4\pi}{3}$ rad