

ΑΕ 25-12-2021

ΓΥ

$$x_1 = 10 \cdot \sin(10t + \frac{\pi}{3})$$

$$x_2 = 10 \cdot \sin(10t - \frac{\pi}{6})$$

$$x = x_1 + x_2$$

$$x = A \cdot \sin(\omega t + \theta) \quad (1)$$

ο θ είναι η φάση της ταλάντωσης x ως προς Α και ως προς Α₁

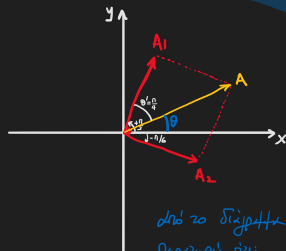
$$\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{3} - (-\frac{\pi}{6}) = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\phi} = \sqrt{10^2 + 10^2 + 2 \cdot 10 \cdot 10 \cdot \cos\frac{\pi}{2}} = \sqrt{2 \cdot 10^2} = 10\sqrt{2} \text{ m} \quad (2)$$

$$\text{Eq } \theta' = \frac{A_2 \cdot \sin\Delta\phi}{A_1 + A_2 \cos\Delta\phi} = \frac{10 \cdot \sin\frac{\pi}{2}}{10 + 10 \cdot \cos\frac{\pi}{2}} = \frac{10}{20} = \frac{1}{2} \Rightarrow \theta' = \frac{\pi}{4} \text{ rad}$$

→ ο θ' είναι η φάση της ταλάντωσης x ως προς Α και ως προς Α₁

$$(1)/(2)/(3): x = 10\sqrt{2} \cdot \sin(10t + \frac{\pi}{4})$$



από το διανύσμα των ριζών παρατηρείται ότι:

$$\theta = \frac{\pi}{2} - \theta' = \frac{\pi}{2} - \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{4} \text{ rad} \quad (3)$$

2ος τρόπος:

$$x_1 = 10 \cdot \sin(10t + \frac{\pi}{3})$$

$$x_2 = 10 \cdot \sin(10t - \frac{\pi}{6})$$

$$\sin\alpha + \sin\beta = 2 \cdot \sin\frac{\alpha+\beta}{2} \cdot \cos\frac{\alpha-\beta}{2}$$

$$x = x_1 + x_2 = 10 \cdot \sin(10t + \frac{\pi}{3}) + 10 \cdot \sin(10t - \frac{\pi}{6}) = 10 \cdot [\sin(10t + \frac{\pi}{3}) + \sin(10t - \frac{\pi}{6})] =$$

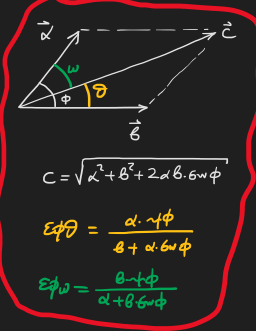
$$= 10 \cdot 2 \cdot \sin\frac{(10t + \frac{\pi}{3}) + (10t - \frac{\pi}{6})}{2} \cdot \cos\frac{(10t + \frac{\pi}{3}) - (10t - \frac{\pi}{6})}{2} =$$

$$= 20 \cdot \sin\frac{10t + \frac{\pi}{3} + 10t - \frac{\pi}{6}}{2} \cdot \cos\frac{10t + \frac{\pi}{3} - 10t - \frac{\pi}{6}}{2} =$$

$$= 20 \cdot \sin\frac{10t + \frac{\pi}{2}}{2} \cdot \cos\frac{10t + \frac{\pi}{3} - 10t - \frac{\pi}{6}}{2} =$$

$$= 20 \cdot \sin\frac{10t}{2} \cdot \cos\frac{10t + \frac{\pi}{3} - 10t - \frac{\pi}{6}}{2} = 20 \cdot \sin\frac{10t}{2} \cdot \cos\frac{10t + \frac{\pi}{3} - 10t - \frac{\pi}{6}}{2} =$$

$$\Rightarrow x = 10\sqrt{2} \cdot \sin(10t + \frac{\pi}{4})$$



$$c = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

$$\text{Eq } \theta = \frac{a \cdot \sin\phi}{b + a \cos\phi}$$

$$\text{Eq } \omega = \frac{b \cdot \sin\phi}{a + b \cos\phi}$$

$$x_1 = \sqrt{3} \cdot \sin(10t + \frac{\pi}{3})$$

$$x_2 = 1 \cdot \sin(10t - \frac{\pi}{6})$$

$$\Delta\phi = \frac{\pi}{3} - (-\frac{\pi}{6}) = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2} \text{ rad}$$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\phi} =$$

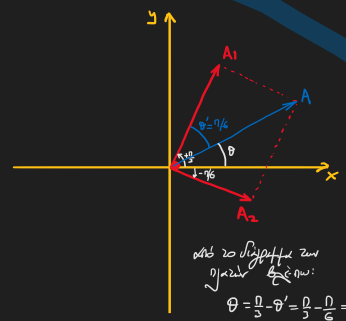
$$= \sqrt{(\sqrt{3})^2 + 1^2 + 2 \cdot \sqrt{3} \cdot 1 \cdot \cos\frac{\pi}{2}} =$$

$$= \sqrt{3+1} \Rightarrow A = 2 \text{ m}$$

$$\text{Eq } \theta' = \frac{A_2 \cdot \sin\Delta\phi}{A_1 + A_2 \cos\Delta\phi} = \frac{1 \cdot \sin\frac{\pi}{2}}{\sqrt{3} + 1 \cdot \cos\frac{\pi}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \Rightarrow \theta' = \frac{\pi}{6} \text{ rad}$$

→ φάση Α-Α₁

από τον τετράγωνο φάσης και διανύσματος ριζών δίνει έναν 2ο τρόπο λύσης



από το διανύσμα των ριζών παρατηρείται ότι:

$$\theta = \frac{\pi}{2} - \theta' = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{6} \text{ rad}$$

$$x = x_1 + x_2 = A \cdot \sin(\omega t + \theta) \Rightarrow$$

$$\Rightarrow x = 2 \cdot \sin(10t + \frac{\pi}{6})$$