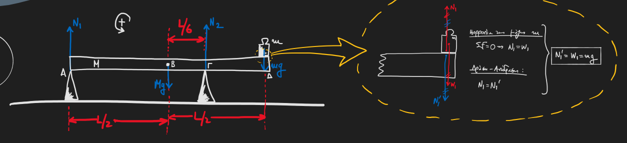
Τρ 30-3-2021

ΓΥ

$N_1 = ?$
$N_2 = ?$



Ισορροπία του βάρους m: $\Sigma F = 0 \rightarrow N_1 = w_1$

Δράση - Αντίδραση: $N_1 = N_1'$

$\boxed{N_1' = W_1 = mg}$

- $\Sigma F_y = 0 \Rightarrow N_1 + N_2 = Mg + \underbrace{mg}_{N_1'} \Rightarrow N_1 + N_2 = (M+m)g \rightarrow \underline{\overline{N_1 = (M+m)g - N_2}}$ (1)
- $\underset{\tau_{N_2(\Gamma)}=0}{\Sigma \tau_{(\Gamma)}} = 0 \Rightarrow -N_1 \cdot \underbrace{\left(\frac{L}{2}+\frac{L}{6}\right)}_{(A\Gamma)} + Mg\underset{(B\Gamma)}{\frac{L}{6}} - \underbrace{mg}_{N'}\underbrace{\left(\frac{L}{2}-\frac{L}{6}\right)}_{(\Gamma\Delta)} = 0 \Rightarrow -N_1\frac{2\cancel{L}}{3} + Mg\frac{\cancel{L}}{6} - mg\frac{\cancel{L}}{3} = 0 \Rightarrow$

$$\Rightarrow \frac{Mg}{6} - \frac{mg}{3} = N_1\frac{2}{3} \rightarrow$$

$$\xrightarrow{\cdot 6} Mg - 2mg = 4N_1 \rightarrow \boxed{N_1 = \frac{Mg}{4} - \frac{mg}{2}} \;(2)$$

⟹ μπορεί και να μηδενιστεί ⟹ $N_1 = 0 \Rightarrow \frac{M\cancel{g}}{4_2} = \frac{m\cancel{g}}{\cancel{2}} \Rightarrow \boxed{m = \frac{M}{2}}$

↑ αν ισχύει αυτό τότε έχω οριακή απώλεια επαφής στο A

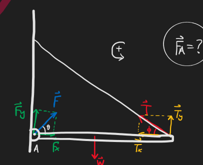
$$(1)/(2):\; \frac{Mg}{4} - \frac{mg}{2} = Mg + mg - N_2 \rightarrow \boxed{N_2 = \frac{3Mg}{4} + \frac{3mg}{2}}$$

⟹ πάντα θετική

$N_2 = \frac{3Mg}{4} + \frac{3\frac{M}{2}g}{2} \Rightarrow N_2 = \frac{3Mg}{2} \neq 0$

$\left.\begin{matrix} N_1 = 0 \\ N_2 \neq 0 \end{matrix}\right\} \Rightarrow$ οριακή περιστροφή ως προς το Γ με φορά ↻

ΑΝΑΤΡΟΠΗ



$\vec{F_A} = ?$

$$\Sigma\vec{\tau}_A = 0 \Rightarrow \cancelto{0}{\vec{\tau}_{F_A}} + \vec{\tau}_w + \vec{\tau}_T = 0 \Rightarrow$$

$$\Rightarrow -\tau_w + \tau_T = 0 \Rightarrow$$

$$\Rightarrow \tau_w = \tau_T \Rightarrow$$

$$\rightarrow w\frac{\cancel{L}}{2} = T\cdot\cancel{L}\cdot\eta\mu\phi \Rightarrow$$

$$\rightarrow \boxed{T = \frac{W}{2\cdot\eta\mu\phi}} \;(1)$$

$$T_x = T\cdot\sigma\upsilon\nu\phi = \frac{W}{2\cdot\eta\mu\phi}\cdot\sigma\upsilon\nu\phi = \frac{W}{2\cdot\varepsilon\varphi\phi}$$

$$T_y = T\cdot\eta\mu\phi = \frac{W}{2\cdot\cancel{\eta\mu\phi}}\cdot\cancel{\eta\mu\phi} = \frac{W}{2}$$

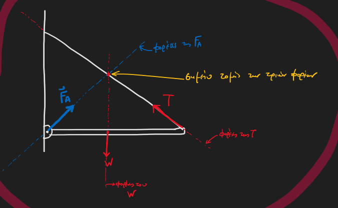
$$\Sigma F_x = 0 \Rightarrow F_x = T_x \Rightarrow \boxed{F_x = \frac{W}{2\cdot\varepsilon\varphi\phi}}$$

$$\Sigma F_y = 0 \Rightarrow F_y + T_y = W \rightarrow F_y = W - T_y \Rightarrow$$

$$\Rightarrow F_y = W - \frac{W}{2} \rightarrow \boxed{F_y = \frac{W}{2}}$$

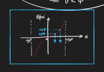
$$F_A = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(\frac{W}{2\varepsilon\varphi\phi}\right)^2 + \left(\frac{W}{2}\right)^2} \Rightarrow \boxed{F_A = \frac{W}{2}\sqrt{\frac{1}{\varepsilon\varphi^2\phi} + 1}}$$

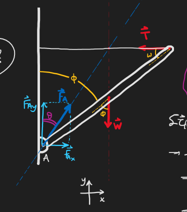
$$\varepsilon\varphi\theta = \frac{F_y}{F_x} = \frac{\frac{W}{2}}{\frac{W}{2\varepsilon\varphi\phi}} \Rightarrow \varepsilon\varphi\theta = \varepsilon\varphi\phi \Rightarrow \boxed{\theta = \phi}$$



$\vec{F_A} = ?$

$\varepsilon\varphi\theta = \frac{1}{2}\varepsilon\varphi\phi \Rightarrow \varepsilon\varphi\theta < \varepsilon\varphi\phi \Rightarrow$
$\Rightarrow \theta < \phi$





$\omega = 90^\circ - \phi \rightsquigarrow \eta\mu\omega = \eta\mu(90^\circ - \phi) = \sigma\upsilon\nu\phi$ (*)

$$\Sigma\vec{\tau}_{(A)} = 0 \Rightarrow \cancelto{0}{\vec{\tau}_{F_A}} + \vec{\tau}_w + \vec{\tau}_T = 0 \Rightarrow$$

$$\rightarrow -\tau_w + \tau_T = 0 \rightarrow \tau_w = \tau_T \Rightarrow$$

$$\Rightarrow W\cdot\frac{L}{2}\cdot\eta\mu\phi = T\cdot L\cdot\eta\mu\omega \Rightarrow$$

(*)

$$\Rightarrow W\frac{\cancel{L}}{2}\cdot\eta\mu\phi = T\cdot\cancel{L}\cdot\sigma\upsilon\nu\phi \Rightarrow$$

$$\Rightarrow \frac{W}{2}\cdot\eta\mu\phi = T\cdot\sigma\upsilon\nu\phi \Rightarrow \boxed{T = \frac{W}{2}\cdot\varepsilon\varphi\phi}$$

$$\Sigma F_x = 0 \Rightarrow F_{Ax} = T \rightarrow \boxed{F_{Ax} = \frac{W}{2}\cdot\varepsilon\varphi\phi}$$

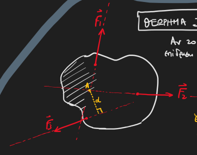
$$\Sigma F_y = 0 \Rightarrow \boxed{F_{Ay} = W}$$

$$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = \sqrt{\left(\frac{W}{2}\cdot\varepsilon\varphi\phi\right)^2 + W^2} =$$

$$= \sqrt{\frac{W^2}{4}\varepsilon\varphi^2\phi + W^2} \Rightarrow$$

$$\Rightarrow \boxed{F_A = W\sqrt{\frac{1}{4}\cdot\varepsilon\varphi^2\phi + 1}}$$

$$\varepsilon\varphi\theta = \frac{F_{Ax}}{F_{Ay}} = \frac{\frac{W}{2}\cdot\varepsilon\varphi\phi}{W} \Rightarrow \boxed{\varepsilon\varphi\theta = \frac{1}{2}\varepsilon\varphi\phi} \rightsquigarrow \boxed{\theta \neq \phi}$$



ΘΕΩΡΗΜΑ 3 συντρεχόντων δυνάμεων

Αν το στερεό ισορροπεί υπό την επίδραση ΜΟΝΟ των 3 συντρεχόντων δυνάμεων τότε αυτές έχουν φορείς που διέρχονται από το ίδιο σημείο

Έστω ότι η $\vec{F_3}$ έχει φορέα που δεν διέρχεται από το A (σημείο τομής των φορέων των F_1 κ' F_2).

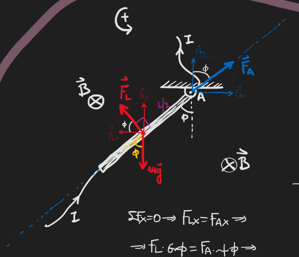
Τότε: $\Sigma\vec{\tau}_{(A)} = \underset{0}{\cancel{\vec{\tau}_{F_1(A)}}} + \underset{0}{\cancel{\vec{\tau}_{F_2(A)}}} + \vec{\tau}_{F_3(A)}$

όμως έχουμε ισορροπία: $\Sigma\vec{\tau}_{(A)} = 0$

$\rightarrow \vec{\tau}_{F_3(A)} = 0 \Rightarrow F_3\cdot d = 0 \xrightarrow{F_3 \neq 0} \boxed{d = 0}$

άρα ο φορέας της F_3 δεν απέχει από το A, είναι πάνω στο A.

Θ.Α.



$$\Sigma\vec{\tau}_A = 0 \Rightarrow \vec{\tau}_{F_L} + \vec{\tau}_{mg} + \cancelto{0}{\vec{\tau}_{F_A}} = 0$$

$$\Rightarrow -\tau_{F_L} + \tau_{mg} = 0 \Rightarrow$$

$$\rightarrow \tau_{mg} = \tau_{F_L} \Rightarrow$$

$$\Rightarrow mg\frac{\cancel{L}}{\cancel{2}}\eta\mu\phi = F_L\cdot\frac{\cancel{L}}{\cancel{2}} \Rightarrow$$

$$\Rightarrow \underline{\overline{mg\cdot\eta\mu\phi = BIL}} \;(1)$$

$$\boxed{I = \frac{mg\,\eta\mu\phi}{B\cdot L}}$$

$$\Sigma F_x = 0 \Rightarrow F_{Lx} = F_{Ax} \Rightarrow$$

$$\Rightarrow F_L\cdot\sigma\upsilon\nu\phi = F_A\cdot\eta\mu\phi \Rightarrow$$

$$\Rightarrow F_A = \frac{F_L\cdot\sigma\upsilon\nu\phi}{\eta\mu\phi} \overset{(1)}{=} \frac{mg\cdot\cancel{\eta\mu\phi}\cdot\sigma\upsilon\nu\phi}{\cancel{\eta\mu\phi}} \rightarrow$$

$$\rightarrow \boxed{F_A = mg\cdot\sigma\upsilon\nu\phi}$$

αν δεν είχαμε κάνει την υπόθεση των 3 συντρεχόντων δυνάμεων, δηλ. αν είχαμε βάλει την $\vec{F_A}$ σε τυχαία γωνία θ, τότε με $\Sigma\tau_A = 0$, $\Sigma F_x = 0$ κ' $\Sigma F_y = 0$ θα αποδεικνύαμε ότι $\theta = \phi$