Πε 27.1.2022

1ο ΓΕΛ ΣΑΛΑΜΙΝΑΣ

Συνέχεια από την Δευτέρα 24.1.22

$\boxed{x_1 = 10\,\eta\mu 2\pi t}$, $\boxed{x_2 = A_2\,\eta\mu(2\pi t + \phi)}$ $\Big\}$ $+$ $x = x_1 + x_2$ $\Big\}$ $\boxed{x = 10\sqrt{3}\,\eta\mu\left(2\pi t + \frac{3\pi}{2}\right)}$

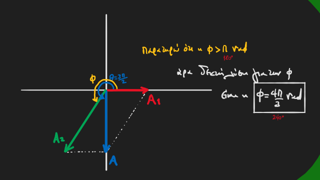
$A_1 = 10\,cm$, $\omega = 2\pi\ rad/s$, $A_2 = ?$, $\phi = ?$, $A = 10\sqrt{3}\ cm$, $\theta = \frac{3\pi}{2}\ rad$

$A_2 = ?$ $\quad \phi = ?$

$\boxed{A_2 = 20\,cm}$

(2): $20 \cdot \sigma\upsilon\nu\phi = -10 \Rightarrow \sigma\upsilon\nu\phi = -\frac{10}{20} \to \boxed{\sigma\upsilon\nu\phi = -\frac{1}{2}}$ $\Big\}$ $\phi = ?$

Δίνεται ότι: $\sigma\upsilon\nu\frac{2\pi}{3} = \sigma\upsilon\nu\frac{4\pi}{3} = -\frac{1}{2}$ (120°, 240°)



Παρατηρώ ότι η $\phi > \pi$ rad (180°) άρα δεκτή τιμή για την ϕ είναι η $\boxed{\phi = \frac{4\pi}{3}\ rad}$ (240°)

'Ασκηση: $\boxed{x_1 = 2\,\eta\mu 5t}$, $\boxed{x_2 = 2\sqrt{3}\,\eta\mu\left(5t + \frac{5\pi}{6}\right)}$ $\Big\}$ $x = x_1 + x_2 = ?$

$A_1 = 2\,m$, $A_2 = 2\sqrt{3}\,m$, $\omega = 5\ rad/s$, $\phi = \frac{5\pi}{6}\ rad$

$\sigma\upsilon\nu\frac{5\pi}{6} = \sigma\upsilon\nu\left(\pi - \frac{\pi}{6}\right) = -\sigma\upsilon\nu\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

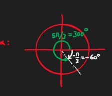
$\eta\mu\frac{5\pi}{6} = \eta\mu\left(\pi - \frac{\pi}{6}\right) = \eta\mu\frac{\pi}{6} = \frac{1}{2}$ Ooo.

γνωρίζουμε ότι: $\boxed{x = A\,\eta\mu(\omega t + \theta)}$, $\boxed{A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\sigma\upsilon\nu\phi}} = \sqrt{2^2 + (2\sqrt{3})^2 + 2\cdot 2\cdot 2\sqrt{3}\cdot\sigma\upsilon\nu\frac{5\pi}{6}} = \sqrt{4 + 12 + 8\sqrt{3}\cdot\left(-\frac{\sqrt{3}}{2}\right)} = \sqrt{16 - 12} = \sqrt{4} \Rightarrow \boxed{A = 2\,m}$

$\boxed{\varepsilon\varphi\theta = \frac{A_2\,\eta\mu\phi}{A_1 + A_2\,\sigma\upsilon\nu\phi}} = \frac{2\sqrt{3}\cdot\frac{1}{2}}{2 + 2\sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)} = \frac{\sqrt{3}}{2 - 3} \Rightarrow \boxed{\varepsilon\varphi\theta = -\sqrt{3}} \Rightarrow \varepsilon\varphi\theta = -\varepsilon\varphi\frac{\pi}{3} \Rightarrow \varepsilon\varphi\theta = \varepsilon\varphi\left(-\frac{\pi}{3}\right) \longrightarrow \theta = k\pi + \left(-\frac{\pi}{3}\right)$

- $k=0$: $\theta = -\frac{\pi}{3}$ (*)
- $k=1$: $\theta = \frac{2\pi}{3}$
- $k=2$: $\theta = 2\pi - \frac{\pi}{3} = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$ (*)

ίδια γωνία:



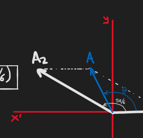
$\varepsilon\varphi\theta = \varepsilon\varphi\alpha \Rightarrow \theta = k\pi + \alpha$, $k = 0, 1, \ldots$

Επιλέγω: $\boxed{\theta = \frac{2\pi}{3}}$ (120°) ή $\boxed{\theta = \frac{5\pi}{3}}$ (300°) ?

$\boxed{\varepsilon\varphi\theta = -\sqrt{3}}$

$\boxed{x_1 = 2\,\eta\mu 5t}$ (2)

$\boxed{x_2 = 2\sqrt{3}\,\eta\mu(5t + 5\pi/6)}$ ($\approx 3{,}47$, 150°)



Παρατηρώ ότι $\theta < \pi$ rad άρα δεκτή τιμή για την θ είναι η $\boxed{\theta = \frac{2\pi}{3}\ rad}$

$$\boxed{x = 2\,\eta\mu\left(5t + \frac{2\pi}{3}\right)}$$

Ενέργεια Σύνθετης Ταλάντωσης 1ου είδους

$x_1 = A_1\,\eta\mu\omega t$

$x_2 = A_2\,\eta\mu(\omega t + \phi)$ $\Big\}$ $x = x_1 + x_2 = A\,\eta\mu(\omega t + \theta)$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\sigma\upsilon\nu\phi}$

$\varepsilon\varphi\theta = \frac{A_2\,\eta\mu\phi}{A_1 + A_2\,\sigma\upsilon\nu\phi}$

$D = m\omega^2$, ω = κοινό για x_1, x_2, x άρα D θα είναι κοινό για x_1, x_2, x

$E_1 = \frac{1}{2}DA_1^2 \Rightarrow A_1^2 = \frac{2E_1}{D} \Rightarrow A_1 = \sqrt{\frac{2E_1}{D}}$

$E_2 = \frac{1}{2}DA_2^2 \Rightarrow A_2^2 = \frac{2E_2}{D} \Rightarrow A_2 = \sqrt{\frac{2E_2}{D}}$

$E = \frac{1}{2}DA^2 = \frac{1}{2}D(A_1^2 + A_2^2 + 2A_1A_2\sigma\upsilon\nu\phi) =$

$= \frac{1}{2}DA_1^2 + \frac{1}{2}DA_2^2 + DA_1A_2\sigma\upsilon\nu\phi =$

$= E_1 + E_2 + D\sqrt{\frac{2E_1}{D}}\cdot\sqrt{\frac{2E_2}{D}}\,\sigma\upsilon\nu\phi =$

$= E_1 + E_2 + D\sqrt{\frac{2E_1\,2E_2}{D\cdot D}}\,\sigma\upsilon\nu\phi =$

$= E_1 + E_2 + D\frac{2}{D}\sqrt{E_1\cdot E_2}\cdot\sigma\upsilon\nu\phi \Rightarrow$

$$\Rightarrow \boxed{E = E_1 + E_2 + 2\sqrt{E_1\cdot E_2}\cdot\sigma\upsilon\nu\phi}$$

αν $\phi = \frac{\pi}{2}$ $\rightsquigarrow$ $A = \sqrt{A_1^2 + A_2^2 + \cancel{2A_1A_2\sigma\upsilon\nu\frac{\pi}{2}}} \Rightarrow A = \sqrt{A_1^2 + A_2^2}$

$\rightsquigarrow$ $E = E_1 + E_2 + \cancel{2\sqrt{E_1\cdot E_2}\cdot\sigma\upsilon\nu\frac{\pi}{2}} \Rightarrow E = E_1 + E_2$

'Ασκηση με Δύο Αρχικές Φάσεις

- $x_1 = 2\,\eta\mu\left(5t + \frac{\pi}{6}\right)$
- $x_2 = 2\sqrt{3}\,\eta\mu\left(5t + \frac{2\pi}{3}\right)$

$\Big\}$ $x = x_1 + x_2 = ?$

$A_1 = 2\,m$, $A_2 = 2\sqrt{3}\,m$, $\omega = 5\ rad/s$, $\phi_1 = \pi/6\ rad$, $\phi_2 = 2\pi/3\ rad$

$\Delta\phi = \phi_2 - \phi_1 = \frac{2\pi}{3} - \frac{\pi}{6} = \frac{4\pi}{6} - \frac{\pi}{6} \Rightarrow \Delta\phi = \frac{3\pi}{6} = \frac{\pi}{2}\ rad$

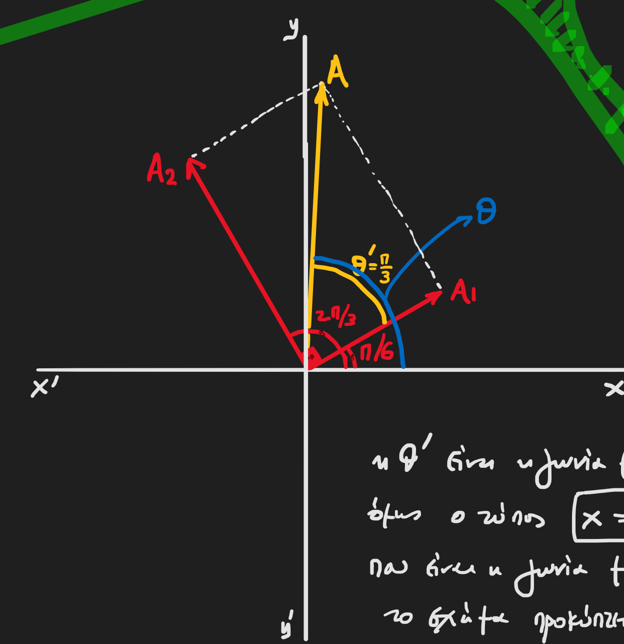
$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\sigma\upsilon\nu\Delta\phi} = \sqrt{2^2 + (2\sqrt{3})^2 + \cancel{2\cdot 2\cdot 2\sqrt{3}\cdot\sigma\upsilon\nu\frac{\pi}{2}}} =$

$= \sqrt{4 + 12} = \sqrt{16} \Rightarrow \boxed{A = 4\,m}$

$\varepsilon\varphi\theta' = \frac{A_2\cdot\eta\mu\Delta\phi}{A_1 + A_2\,\sigma\upsilon\nu\Delta\phi} = \frac{2\sqrt{3}\cdot\eta\mu\,\pi/2}{2 + \cancel{2\sqrt{3}\,\sigma\upsilon\nu\,\pi/2}} = \frac{2\sqrt{3}\cdot 1}{2} = \sqrt{3} \Rightarrow \boxed{\theta' = \frac{\pi}{3}\ rad}$

ΠΡΟΣΟΧΗ

ΔΕΝ ΕΙΝΑΙ: $x = 4\,\eta\mu\left(5t + \frac{\pi}{3}\right)$



η θ' είναι η γωνία μεταξύ του A και του A₁.

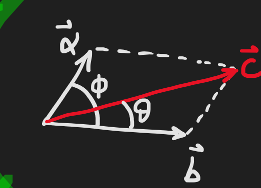
όμως ο τύπος $\boxed{x = A\,\eta\mu(\omega t + \theta)}$ έχει την θ

που είναι η γωνία μεταξύ του $x'x$ – A. Άρα από

το σχήμα προκύπτει ότι: $\theta = \theta' + \frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}\ rad$

Άρα: $\boxed{x = 4\,\eta\mu\left(5t + \frac{\pi}{2}\right)}$

ΜΑΘΗΜΑΤΙΚΑ



$c = \sqrt{\alpha^2 + b^2 + 2\alpha b\,\sigma\upsilon\nu\phi}$

$\varepsilon\varphi\theta = \frac{\alpha\cdot\eta\mu\phi}{b + \alpha\cdot\sigma\upsilon\nu\phi}$

η θ είναι η γωνία μεταξύ του c και του διανύσματος που είναι κάτω δεξιότερα στον τύπο της εφαπτομένης