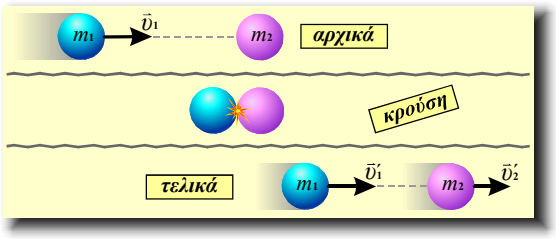
ΚΕΝΤΡΙΚΗ ΕΛΑΣΤΙΚΗ ΚΡΟΥΣΗ ΔΥΟ ΣΦΑΙΡΩΝ **ΜΕ ΤΗΝ ΜΙΑ ΑΚΙΝΗΤΗ**

➢ ΔΕΔΟΜΕΝΑ: $\boxed{m_1}\ \boxed{\upsilon_1}$, $\boxed{m_2}\ \boxed{\upsilon_2=0}$

➢ ΖΗΤΟΥΜΕΝΑ: $\boxed{\upsilon_1'\ \&\ \upsilon_2'}$

$$\boxed{\upsilon_1' = \frac{m_1 - m_2}{m_1 + m_2}\upsilon_1}$$

$$\boxed{\upsilon_2' = \frac{2m_1}{m_1 + m_2}\upsilon_1}$$



$\hookrightarrow$ ΑΔΟ: $\vec{p}_{\text{ΟΛΙΚΟ}} = \vec{p}'_{\text{ΟΛΙΚΟ}} \Rightarrow \vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2' \ \overset{(\upsilon_2=0)}{\Rightarrow}\ p_1 = p_1' + p_2' \Rightarrow m_1\upsilon_1 = m_1\upsilon_1' + m_2\upsilon_2' \Rightarrow$

$\Rightarrow m_1\upsilon_1 - m_1\upsilon_1' = m_2\upsilon_2' \Rightarrow \boxed{m_1(\upsilon_1 - \upsilon_1') = m_2\upsilon_2'}$ (1)

$\hookrightarrow$ ΑΔΚΕ: $K_{\text{ΟΛΙΚΟ}} = K'_{\text{ΟΛΙΚΟ}} \Rightarrow K_1 + K_2 = K_1' + K_2' \ \overset{(\upsilon_2=0)}{\Rightarrow}\ \frac{1}{2}m_1\upsilon_1^2 = \frac{1}{2}m_1\upsilon_1'^2 + \frac{1}{2}m_2\upsilon_2'^2 \Rightarrow$

$\Rightarrow m_1\upsilon_1^2 = m_1\upsilon_1'^2 + m_2\upsilon_2'^2 \Rightarrow m_1\upsilon_1^2 - m_1\upsilon_1'^2 = m_2\upsilon_2'^2 \Rightarrow m_1(\upsilon_1^2 - \upsilon_1'^2) = m_2\upsilon_2'^2 \Rightarrow$

$\Rightarrow \underbrace{m_1(\upsilon_1 - \upsilon_1')(\upsilon_1 + \upsilon_1')}_{\lambda\acute{o}\gamma\omega\ \tau\eta\varsigma\ (1)} = m_2\upsilon_2'\upsilon_2' \Rightarrow \boxed{\upsilon_1 + \upsilon_1' = \upsilon_2'}$ (2)

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(1)/(2): $m_1(\upsilon_1 - \upsilon_1') = m_2(\upsilon_1 + \upsilon_1') \Rightarrow m_1\upsilon_1 - m_1\upsilon_1' = m_2\upsilon_1 + m_2\upsilon_1' \Rightarrow m_1\upsilon_1 - m_2\upsilon_1 = m_2\upsilon_1' + m_1\upsilon_1' \Rightarrow$

$\Rightarrow (m_1 - m_2)\upsilon_1 = (m_2 + m_1)\upsilon_1' \Rightarrow \boxed{\upsilon_1' = \frac{m_1 - m_2}{m_2 + m_1}\upsilon_1}$ (A)

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(1)/(A): $\upsilon_2' = \upsilon_1 + \frac{m_1 - m_2}{m_2 + m_1}\upsilon_1 = \left(1 + \frac{m_1 - m_2}{m_2 + m_1}\right)\upsilon_1 = \frac{m_2 + m_1 + m_1 - m_2}{m_2 + m_1}\upsilon_1 \Rightarrow \boxed{\upsilon_2' = \frac{2m_1}{m_2 + m_1}\upsilon_1}$
