Τρ 16-3-2021

B+2

(112) σελ. 205

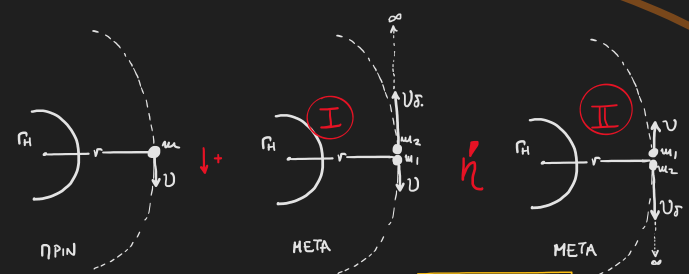
m, υ (δορ. τρ.)

m_1 (δορ. τρ.)

m_2 (διαφυγή)

$m_1 = ;$

$m_2 = ;$



ΘΕΛΟΥΝ ΑΠΟΔΕΙΞΗ

- Δορυφορική Ταχύτητα: $\upsilon = \sqrt{\frac{GM}{r}}$
- Ταχύτητα Διαφυγής: $\upsilon_\delta = \sqrt{\frac{2GM}{r}} = \sqrt{2}\cdot\upsilon$

αφού το m_1 συνεχίζει στην ίδια δορυφορική τροχιά (ακτίνας r) θα συνεχίσει και με την ίδια ταχύτητα $\upsilon = \sqrt{GM/r}$

(I) ΑΔΟ: $m\upsilon = m_1\cdot\upsilon - m_2\upsilon_\delta \Rightarrow$

$m\upsilon = m_1\upsilon - m_2\cdot\upsilon\cdot\sqrt{2} \Rightarrow$

$\boxed{m = m_1 - m_2\sqrt{2}}$

αλλά: $m = m_1 + m_2 \Rightarrow m_1 = m - m_2$

$m = m - m_2 - m_2\sqrt{2} \Rightarrow$

$\Rightarrow m_2 = -m_2\sqrt{2} \Rightarrow$

$\Rightarrow \boxed{1 = -\sqrt{2}}$ ΑΤΟΠΟ!

(II) ΑΔΟ: $m\upsilon = m_2\upsilon_\delta - m_1\cdot\upsilon \Rightarrow$

$m\upsilon = m_2\cdot\upsilon\sqrt{2} - m_1\cdot\upsilon \Rightarrow$

$\boxed{m = m_2\sqrt{2} - m_1}$

αλλά: $m = m_1 + m_2 \Rightarrow m_1 = m - m_2$

$m = m_2\sqrt{2} - m + m_2 \Rightarrow$

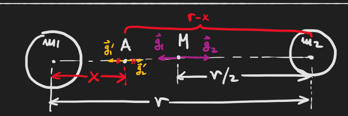
$2m = m_2(\sqrt{2}+1) \Rightarrow$

$\Rightarrow \boxed{m_2 = \frac{2m}{\sqrt{2}+1}}$

και: $m_1 = m - m_2 = m - \frac{2m}{\sqrt{2}+1} = m\left(1 - \frac{2}{\sqrt{2}+1}\right) =$

$= m\frac{\sqrt{2}+1-2}{\sqrt{2}+1} \Rightarrow \boxed{m_1 = m\frac{\sqrt{2}-1}{\sqrt{2}+1}}$

Άλλη Εφαρμογή (SOS για διαγώνισμα)



$m_1 = m$

$m_2 = 4m$

$m_3 = 2m$

α) $F_{12} = ;$

β) $V_M = ;$

γ) $g_M = ;$

δ) $x = ;\ (g_A = 0)$

ε) $W^{m_3}_{M\to A} = ;$

α) $F_{12} = G\frac{m_1 m_2}{r^2} = G\frac{m\cdot 4m}{r^2} = \frac{4Gm^2}{r^2}$ ∴

β) $V_M = V_M^{(m_1)} + V_M^{(m_2)} = -G\frac{m_1}{r/2} - G\frac{m_2}{r/2} =$

$= -\frac{G2m_1}{r} - G\frac{2m_2}{r} = -\frac{G2m}{r} - \frac{G2\cdot 4m}{r} =$

$= -\frac{2Gm}{r} - \frac{8Gm}{r} \Rightarrow V_M = -\frac{10Gm}{r}$ ∴

γ) $g_M = g_2 - g_1 = G\frac{m_2}{(r/2)^2} - G\frac{m_1}{(r/2)^2} = G\frac{4m}{\frac{r^2}{4}} - G\frac{m}{\frac{r^2}{4}} = \frac{16Gm}{r^2} - \frac{4Gm}{r^2} = \frac{12Gm}{r^2}$ ∴

με φορά προς τα δεξιά

δ) $g_A = 0 \Rightarrow g_1' = g_2' \Rightarrow G\frac{m_1}{x^2} = G\frac{m_2}{(r-x)^2} \Rightarrow \frac{m}{x^2} = \frac{4m}{(r-x)^2} \Rightarrow$

$\Rightarrow (r-x)^2 = 4x^2 \xrightarrow{\sqrt{\dots}} r - x = \pm 2x$

$r - x = 2x$

$r = 3x$

$\boxed{x = r/3}$ ∴

$r - x = -2x$

$r = -x$

$x = -r$ απορρίπτεται (βρίσκεται πριν το m_1 όπου $\vec{g}_1' \uparrow\uparrow \vec{g}_2'$)

ε) $V_A = V_A^{(m_1)} + V_A^{(m_2)} = -G\frac{m_1}{x} - G\frac{m_2}{r-x} = -G\frac{m}{r/3} - G\frac{4m}{r - r/3} = -G\frac{3m}{r} - G\frac{4m}{2r/3} =$

$= -G\frac{3m}{r} - G\frac{6m}{r} = -\frac{9Gm}{r}$

$W^{m_3}_{M\to A} = m_3\cdot V_{MA} = m_3(V_M - V_A) = 2m\left(-\frac{10Gm}{r} - \left(-\frac{9Gm}{r}\right)\right) = 2m\left(-\frac{Gm}{r}\right) = -\frac{2Gm^2}{r}$ ∴