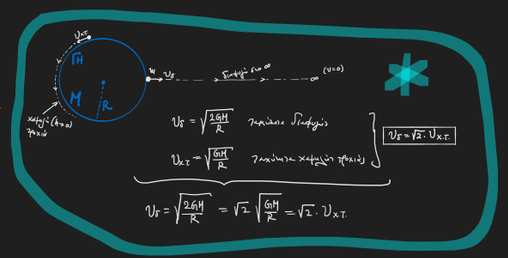
ΤΡ2-3-2021

Β+2



$v_{\delta} = \sqrt{\frac{2GM}{R}}$ ταχύτητα διαφυγής

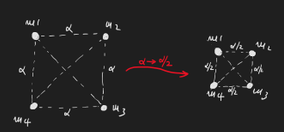
$v_{\kappa\tau} = \sqrt{\frac{GM}{R}}$ ταχύτητα κυκλοφορίας τροχιάς

$\left.\begin{array}{l} v_{\delta} = \sqrt{\frac{2GM}{R}} \\ v_{\kappa\tau} = \sqrt{\frac{GM}{R}} \end{array}\right\}$ $\boxed{v_{\delta} = \sqrt{2}\cdot v_{\kappa\tau}}$

$v_{\delta} = \sqrt{\frac{2GM}{R}} = \sqrt{2}\sqrt{\frac{GM}{R}} = \sqrt{2}\cdot v_{\kappa\tau}$

Μεταβολή έναν Δυν. Βαρύτητας Συστήματος

$m_1 = m_2 = m_3 = m_4 = m$



$\alpha \cdot \sqrt{2}$ (από Π.Θ.)

$U_{\alpha\rho\chi}^{\tau\epsilon\tau} = -G\frac{m_1m_2}{\alpha} - G\frac{m_1m_3}{\alpha} - G\frac{m_2m_4}{\alpha} - G\frac{m_3m_4}{\alpha} - G\frac{m_1m_3}{\alpha\sqrt{2}} - G\frac{m_2m_4}{\alpha\sqrt{2}} =$

$= -G\frac{m^2}{\alpha} - G\frac{m^2}{\alpha} - G\frac{m^2}{\alpha} - G\frac{m^2}{\alpha} - G\frac{m^2}{\alpha\sqrt{2}} - G\frac{m^2}{\alpha\sqrt{2}} =$

$= -\frac{Gm^2}{\alpha}\left(1+1+1+1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right) = -\frac{Gm^2}{\alpha}\left(4+\frac{2}{\sqrt{2}}\right) = -G\frac{m^2}{\alpha}(4+\sqrt{2})$

αφού:

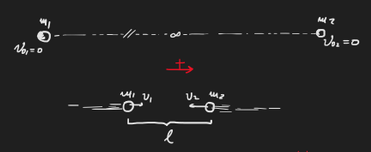
$U_{\alpha\rho\chi}^{\tau\epsilon\lambda} = -G\frac{m^2}{\alpha/2}(4+\sqrt{2}) = -2G\frac{m^2}{\alpha}(4+\sqrt{2})$

$\Delta U = U_{\alpha\rho\chi}^{\tau\epsilon\lambda} - U_{\alpha\rho\chi}^{\tau\epsilon\tau} = -2G\frac{m^2}{\alpha}(4+\sqrt{2}) + G\frac{m^2}{\alpha}(4+\sqrt{2}) =$

$= -G\frac{m^2}{\alpha}(4+\sqrt{2})(2-1) = -G\frac{m^2}{\alpha}(4+\sqrt{2})$

Παράδειγμα S.13/σελ.178/Εκ.Βιβλίο

m_1
m_2
$v_{01} = v_{02} = 0$
∞
$v_1 = ?$
$v_2 = ?$ (ε)
G



Το σύστημα είναι απομονωμένο άρα ισχύει η ΑΔΟ:

$\vec{P}_{\Sigma\upsilon\sigma\tau}^{(\epsilon)} = \vec{P}_{\Sigma\upsilon\sigma\tau}^{(\alpha)} \Rightarrow \vec{P}_1^{(\epsilon)} + \vec{P}_2^{(\epsilon)} = \vec{P}_1^{(\alpha)} + \vec{P}_2^{(\alpha)} \Rightarrow$ (αρχική ακινησία)

$0 = P_1^{(\epsilon)} - P_2^{(\epsilon)} \Rightarrow P_1^{(\epsilon)} = P_2^{(\epsilon)} \Rightarrow$

$\Rightarrow m_1v_1 = m_2v_2 \Rightarrow \boxed{v_2 = \frac{m_1v_1}{m_2}}$ (1)

ΑΔΜΕ: $E_{(\alpha)} = E_{(\epsilon)} \Rightarrow K_{1(\alpha)} + K_{2(\alpha)} + U_{(\alpha)} = K_{1(\epsilon)} + K_{2(\epsilon)} + U_{(\epsilon)} \Rightarrow$ ($K_{(\alpha)} = 0$ αρχική ακινησία)

$\Rightarrow 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - G\frac{m_1m_2}{\ell} \Rightarrow$

(1) $\Rightarrow 0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2\frac{m_1^2v_1^2}{m_2^2} - G\frac{m_1m_2}{\ell} \Rightarrow$

$\Rightarrow 0 = \frac{1}{2}v_1^2 + \frac{1}{2}\frac{m_1}{m_2}v_1^2 - G\frac{m_2}{\ell} \Rightarrow G\frac{m_2}{\ell} = \frac{1}{2}v_1^2\left(1+\frac{m_1}{m_2}\right) \Rightarrow$

$\Rightarrow G\frac{m_2}{\ell} = \frac{1}{2}v_1^2\frac{m_2+m_1}{m_2} \Rightarrow v_1^2 = \frac{2Gm_2^2}{\ell(m_1+m_2)} \Rightarrow \boxed{v_1 = m_2\sqrt{\frac{2G}{\ell(m_1+m_2)}}}$

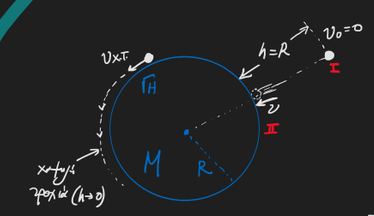
(1): $v_2 = \frac{m_1}{m_2}m_2\sqrt{\frac{2G}{\ell(m_1+m_2)}} \Rightarrow \boxed{v_2 = m_1\sqrt{\frac{2G}{\ell(m_1+m_2)}}}$

όρισμα: $Z = \sqrt{\frac{2G}{\ell(m_1+m_2)}}$

$v_1 = m_2 Z$

$v_2 = m_1 Z$

ΕΛΛΕΙΨΗ για ύψος $h = R$



ΑΔΜΕ (I → II):

$E_I = E_{II} \Rightarrow K_I + U_I = K_{II} + U_{II} \Rightarrow$ ($K_I = 0$)

$\Rightarrow -G\frac{Mm}{R+h} = \frac{1}{2}mv^2 - G\frac{Mm}{R} \Rightarrow$

$\Rightarrow -G\frac{Mm}{R+R} = \frac{1}{2}mv^2 - G\frac{Mm}{R} \Rightarrow$

$\Rightarrow -G\frac{Mm}{2R} + G\frac{Mm}{R} = \frac{1}{2}mv^2 \Rightarrow$

$\Rightarrow G\frac{Mm}{2R} = \frac{1}{2}mv^2 \Rightarrow \boxed{v = \sqrt{\frac{GM}{R}}}$ $(= v_{x.\tau.})$

$\hookrightarrow 8000\ m/s$