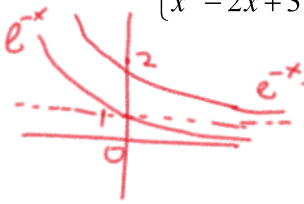
ΘΕΜΑ 5ο Να παραστήσετε γραφικά τις συναρτήσεις

$$f(x)=\begin{cases} e^{x}-1\,,\,x<0 \\ \sqrt{x+1}\,,\,x\geq 0 \end{cases} \qquad g(x)=\begin{cases} e^{-x}+1\,,\,x<0 \\ x^{2}-2x+3\,,\,x\geq 0 \end{cases} \qquad h(x)=\begin{cases} \dfrac{1}{x-1}+2\,,\,x<2\,,\,x\neq 1 \\ 3\,,\,x\geq 2 \end{cases}$$

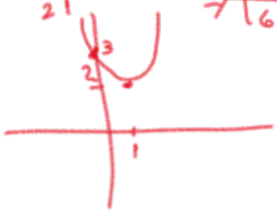


$e^{-x}+1 \rightarrow$ έχει ασύμπτωτη τη $y=1$

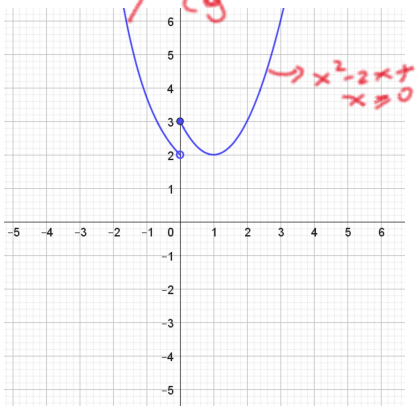
Η $y=x^2-2x+3$ είναι παραβολή με κορυφή $K\left(-\frac{\beta}{2\alpha},-\frac{\Delta}{4\alpha}\right)$

$-\frac{\beta}{2\alpha}=-\frac{-2}{2\cdot 1}=1$

x	-1	0	1	2	3
y	6	3	2	3	6

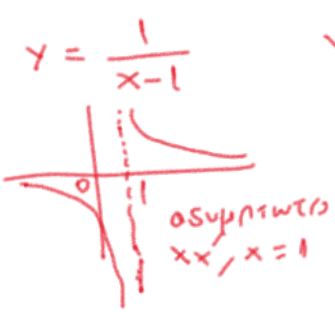


$e^{-x}+1, x<0$ (g

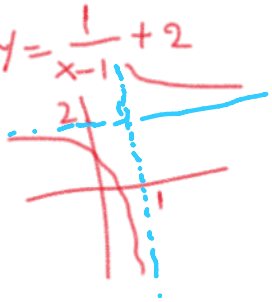


$\rightarrow x^2-2x+3$, $x\geq 0$

ασύμπτωτες xx', yy'

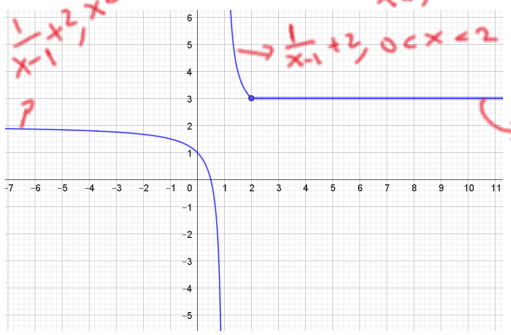


ασύμπτωτες xx', $x=1$



ασύμπτωτες $y=2$, $x=1$

$\frac{1}{x-1}, x<0$



$\rightarrow \frac{1}{x-1}+2$, $0<x<2$

$\rightarrow y=3$, $x\geq 2$

ΘΕΜΑ 6ο Να παραστήσετε γραφικά τις συναρτήσεις

$$f(x)=\sqrt{4-x^2}\ ,\ g(x)=-\sqrt{4-x^2}\ ,\ k(x)=\sqrt{3-2x-x^2}\ ,\ q(x)=-\sqrt{3-2x-x^2}\ ,$$

$$h(x)=\sqrt{9-9x^2}\ ,\ \varphi(x)=-\sqrt{9-9x^2}\ ,\ s(x)=\sqrt{4x^2-4}\ ,\ t(x)=-\sqrt{4x^2-4}\ ,$$

Θέτω $f(x)=y$

$y=\sqrt{4-x^2}$

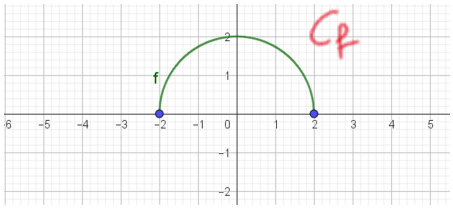
Θέτω $g(x)=y$

Έστω $f(x)=y$

$y=\sqrt{4-x^2}$, $y\geq 0$

$y^2=4-x^2$ ημικύκλιο με

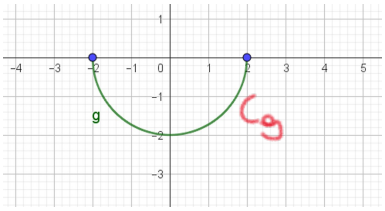
$x^2+y^2=4$ κέντρο $(0,0)$, $\rho=2$



Έστω $g(x)=y$

$y=-\sqrt{9-x^2}$, $y\leq 0$

$x^2+y^2=4$ ημικύκλιο



Έστω $k(x)=y$

$y=\sqrt{3-2x-x^2}$, $y\geq 0$

$y^2=3-2x-x^2 \Rightarrow$

$x^2+y^2+2x-3=0$

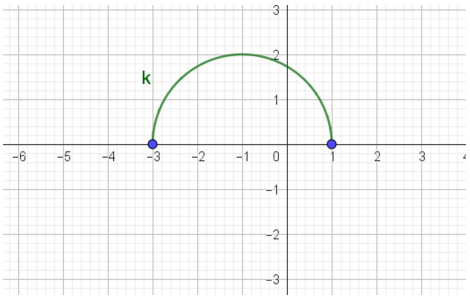
$\rightarrow A=2,\ B=0,\ \Gamma=-3$

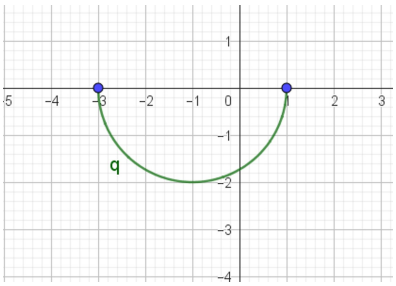
$A^2+B^2-4\Gamma=4+12=16>0$ ημικύκλιο

$K\left(-\frac{A}{2},-\frac{B}{2}\right)$

$k(-1,0)$

$\rho=\frac{\sqrt{A^2+B^2-4\Gamma}}{2}=\frac{\sqrt{16}}{2}=2$





Έστω $h(x)=y$

$y=\sqrt{9-9x^2}$, $y\geq 0$

$y^2=9-9x^2$

$9x^2+y^2=9$

$x^2+\frac{y^2}{9}=1$

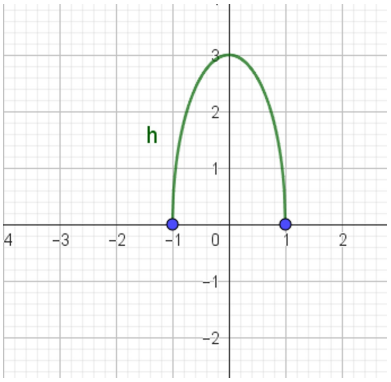
$a=3,\ \beta=1,\ \gamma=\sqrt{a^2-\beta^2}=\sqrt{8}$

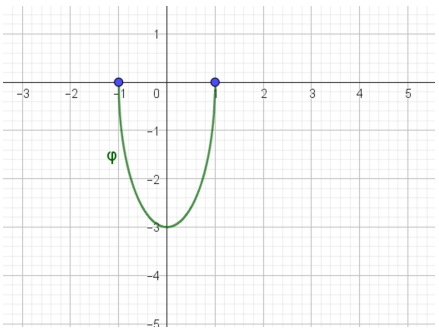
ημικύκλιο με εστίες $E(0,\sqrt{8})$, $E'(0,-\sqrt{8})$

$y=\varphi(x)$

$y=-\sqrt{9-9x^2}$, $y\leq 0$

$x^2+\frac{y^2}{9}=1$





Έστω $y = s(x)$

$y = \sqrt{4x^2-4}$, $y \geq 0$

$y^2 = 4x^2 - 4$

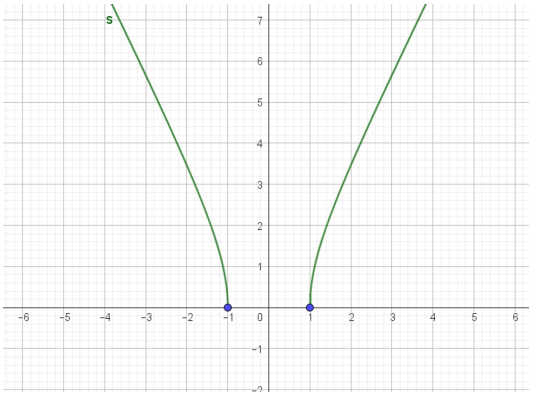
$4x^2 - y^2 = 4$ υπερβολή με εστίες

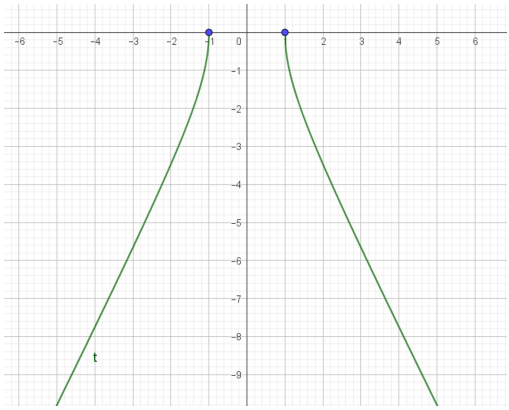
$x^2 - \frac{y^2}{4} = 1$ $E(\sqrt{5},0)$, $E'(-\sqrt{5},0)$

$\alpha = 1$, $\beta = 2$, $\gamma = \sqrt{\alpha^2+\beta^2} = \sqrt{5}$

$y = t(x)$

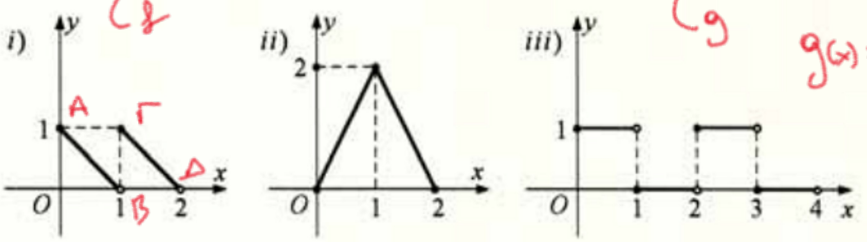
$y = \sqrt{4x^2-4}$, $y \leq 0$





Β΄ ΟΜΑΔΑΣ

1. Να προσδιορίσετε τη συνάρτηση f της οποίας η γραφική παράσταση είναι:



Cf

Cg

$g(x) = \begin{cases} 1, & x \in [0,1) \cup [3,3] \\ 0, & x \in [1,2) \cup (3,4] \end{cases}$

i) $f(x) = \begin{cases} -x+1, & 0 \leq x < 1 \\ -x+2, & 1 \leq x \leq 2 \end{cases}$

Εξίσωση της ΑΒ

$\lambda_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{0-1}{1-0} = -1$

ΑΒ: $y - y_A = \lambda_{AB}(x - x_A)$

$y - 1 = -1(x-0)$

$y = -x + 1$

Εξίσωση της ΓΔ

$\lambda_{\Gamma\Delta} = \frac{y_\Delta - y_\Gamma}{} = \frac{0-1}{} = -1$

$$\lambda_{\Gamma\Delta} = \frac{y_\Delta - y_\Gamma}{x_\Delta - x_\Gamma} = \frac{0-1}{2-1} = -1$$

$$\Gamma\Delta: y - y_\Delta = \lambda_{\Gamma\Delta}(x - x_\Delta) \Rightarrow y - 0 = -1(x-2) \Rightarrow y = -x+2$$